Heat Generation and Dufour Influences on MHD Convective Flow through an Inclined Channel

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Abstract: The theoretical assessment of the impacts of velocity, temperature, concentration varieties and magnetic fields on convective intermittent stream was studied on an electrically directing, viscous and incompressible fluid through a permeable medium in a slanted plane. A bunch of coupled partial differential equations emerging from the issue were converted to space dependent ordinary differential equations with a single term perturbation techniques and solved systematically by the technique of undetermined coefficients. The answers for the temperature, concentration and velocity were shown in plots. From the plots, the accompanying outcomes have been drawn; it is seen that expansion in the Prandtl number declines the temperature, expansion in the Reynolds number reduces the temperature and concentration of the fluid, expansion in the Schmidt number abates the concentration making it more critical at the centre of the flow region, expansion in penetrability prompts expansion in the speed and expansion in the magnetic field prompts decline in the speed, a decrease in the temperature profile is noted owing to the increase in the heat generation, expansion in Dufour number increases both the temperature and speed profiles.

Keywords: Dufour, Free convection, magnetohydrodynamics, porous medium, inclined channel, thermal radiation

I. INTRODUCTION

Permeable medium is a medium that has interconnected pores where liquids can move through. It is valuable as it is very well utilized in the powerful insurance of some underlying parts of turbojet and rocket motors, for example, ignition chamber dividers, fumes spouts or gas turbine sharp edges from hot gases. Eckert and Drake(1958) and Jain and Bansal(1973) portrayed warmth move decrease of counte stream of incompressible liquid infused into the stream field from a plate that is fixed opposite the expulsion of warmth from a plate that is moving. It has a two dimensional issue in capsulated by uniform infusion and suction applied at the permeable plate. Gersten and Gross(1974) checked warmth move along a plane wall with occasional suction velocity.

MHD meaning magnetohydrodynamic fluid is a fluid that conducts electricity in electric and magnetic fields. It incorporates fluid dynamics and electromagnetic assertions to describe concurrent effects of magnetic field on the flow and vice versa. Its concern is on gases that are ionized and liquids that are electrically conducting. Varieties of papers have evolved over the years on this concept. Take for instance; Singh and Mathew(2008) studied the effects that injection/suction has on oscillating hydrodynamic magnetic flow in a horizontal channel that is rotating. Attia and Kotb(1996) examined magneto hydrodynamic flow between parallel plates having heat transfer. Swapna et al.(2017) studied mass transfer on mixed convective periodic flow through porous medium in an inclined channel. Achogo et al.(2020) examined magnetohydrodynamic convective periodic flow through a porous medium in an inclined channel with thermal radiation and chemical reaction.

The concept of natural convective heat transfer occurs owing to difference in temperature in an enclosure or near a heated or cooled flat plate. Much attention has been given to natural convection on horizontal and vertical channel but a few attention has been given to inclined plates despite the frequent occurrence of this geometric configuration in engineering and natural environment. Amongst the few researchers that made research on inclined surface are Ganesan and Palani(2003) and Sparrow and Husar(1969) who studied natural convection on inclined plate. Said et al.(2005) investigated turbulent natural convection between inclined isothermal plates. Chen(2004) studied natural convection flow over an inclined surface that is permeable having variable wall temperature and concentration. Hossain et al.(1996) examined the free convection from evolving from inclined at small angle to the plate that is isothermal. The numerical solution of free convection flow past an inclined surface was studied by Anghel et al.(2001). Exact solution analysis of radiative convective flow of heat and mass transfer over inclined plate in a porous medium was examined Bhuvaneswari et al.(2010) deduced MHD flow, heat and mass transfer on an inclined stretching sheet having thermal radiation and hall effect that is permeable. Achogo et al.(2020) studied mutual influences of heat and mass transfer on mhd flow through a channel with periodic wall concentration and temperature.

The study of thermal radiation in channels of different geometries has received attention from researchers owing to its significance in free convection which is useful in the heating of rooms and buildings by the use of radiators. Ahmed and Sarmah(2009) studied thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed vertical plate. Alabraba et al.(1992) examined free convection interaction with thermal radiation in a hydrodynamic boundary layer taking into account the binary chemical reaction and the less attended Soret and Dufour effects. Alagoa et al.(1998) looked into the radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time-dependent suction.

For the most part, it was realized that heat and mass transitions were made from temperature and concentration gradient individually. Nonetheless, heat motion is made because of concentration gradient; this known as Soret impact. Same goes to the mass transition where the motion happened by the temperature gradient and is called Dufour impact. Design including the heat and mass exchange is a significant subject because of a wide scope of utilization, for example, the hardening of parallel co-coating, substance reactors, geosciences multi-part softens, oil supplies, isotope division and in blend between gases. Due to these significance; many researchers have done research in this light. Nor(2017) studied soret and dufour effects on boundary layer flows. Rama(2019) investigated the dual effects of dufour and radiation on mhd convection of cassan fluid.

The aim of this paper is to examine heat generation and dufour effects on mhd free convection through a porous inclined channel.

II. FORMULATION OF THE PROBLEM.

We consider the periodic flow of an electrically conducting, viscous and incompressible fluid through a inclined medium. The two plates are at a distance d apart. The coordinate system is chosen such that x – axis lies along the centerline and the y – axis along the magnetic field. The fluid is injected through the lower stationary porous plate and sucked through the upper porous plate in oscillatory motion in its own plane. The injection and suction velocity is $V'$. The magnetic field is applied perpendicular to the parallel plates. The temperature difference of the plates is assumed high enough to induce radiation. All the physical parameters are independent of x for this problem of fully developed flows that is laminar. The flow is governed by the following equations:

$$\frac{\partial T'}{\partial t} + v \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'}{\partial y'} - \frac{Q_0 T'}{T_m} \frac{\partial^2 C'}{\partial y'^2}$$  \hspace{1cm} (3)

$$\frac{\partial C'}{\partial t} + v \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K'C'$$  \hspace{1cm} (4)

The boundary conditions expedient to this problem are

$$u' = 0, \quad v' = V, \quad T' = 0, \quad C' = 0 \text{ at } y = -\frac{d}{2} \quad (5)$$

$$u' = U \cos \omega t', \quad v' = V, \quad T' = T_0 \cos \omega t', \quad C' = C_0 \cos \omega t' \text{ at } y = \frac{d}{2} \quad (6)$$

where $u(y', t')$ axial velocity, $t'$ is the time, $v'$ is the kinematic viscosity, $\sigma$ is electrical conductivity, $k$ is the thermal conductivity, $C_p$ is the specific heat at constant pressure, $\rho$ is the fluid density, $\omega$ is the frequency of oscillation, $T'$ is the temperature of the fluid, $C'$ is the concentration of the fluid, $B_0$ is the magnetic field, $T_0$ and $C_0$ are reference temperature and concentration respectively, $D$ is mass diffusivity, $P'$ is the pressure, $V$ is the oscillating velocity, $g$ is the acceleration due to gravity, $K'$ is the chemical reaction term, $q'$ is the radiation flux, $K'$ is the permeability of the porous medium, $B_0$ and $B_0$ are thermal and concentration volume expansion coefficients.

We assumed that the fluid is optically thin having a relatively low density. Hence the heat flux according to Cogley et al. (1968) is expressed as;

$$\frac{\partial q'}{\partial y'} = 4\alpha^2 T'$$  \hspace{1cm} (7)

where $\alpha$ is the mean absorption coefficient.

Going by the internal flow of the oscillation in the channel; the pressure gradient variations is assumed as

$$-\frac{1}{\rho} \frac{\partial P'}{\partial x'} = P \cos \omega t'$$  \hspace{1cm} (8)

Substituting equation (7) into equation (3); we get

$$\frac{\partial T'}{\partial t} + v \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{4\alpha^2}{\rho c_p} T' - Q_0 T' \frac{\partial^2 C'}{T_m} \frac{\partial y'^2}{\partial y'^2}$$

Equation (1) integrates to $v' = V$ on the assumption that there is constant injection and suction velocity $V$ at the upper and lower plates.

Introducing the following dimensionless variables:

$$x = \frac{x}{d}, \quad y = \frac{y}{d}, \quad u = \frac{u}{V}, \quad T = \frac{T}{T_0}, \quad v' = V, \quad C = \frac{C}{C_0}, \quad Sc = \frac{\sigma}{\rho}, \quad P = \frac{P'}{\rho V}, \quad \omega = \frac{\omega d^2}{g}, \quad Re = \frac{V d}{g},$$

$$\frac{\partial T'}{\partial t} + v \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'}{\partial y'} - \frac{Q_0 T'}{T_m} \frac{\partial^2 C'}{\partial y'^2}$$  \hspace{1cm} (3)

$$\frac{\partial C'}{\partial t} + v \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K'C'$$  \hspace{1cm} (4)
\[ P_r = \frac{\mu_C}{\kappa_0}, \quad K = \frac{\kappa}{d^2}, \quad Gr = \frac{\rho g B Y d^2 T_0}{\nu}, \quad M = \frac{B_0 d}{\sqrt{\nu}}, \quad Gm = \frac{g B^2 d^2 C_0}{\nu}, \quad \rho = \frac{\mu}{\nu}, \quad Du = \frac{B_n K e C_0}{\nu \tau_{mT_0}}, \quad Sc = \frac{\mu}{\nu} \]

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} + \frac{\partial}{\partial x} \left[ \frac{1}{2} \frac{\partial u}{\partial y} \right] - \frac{1}{k Re} u + \frac{Gr}{Re} \sin \omega T y \\
+ \frac{Gm}{Re} \sin \alpha C y
\end{align*}
\]

(11)

\[
\begin{align*}
\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} + \frac{1}{Re Pr} \frac{\partial^2 T}{\partial y^2} - \frac{N^2}{Re Pr} T - ST - \frac{Du}{Re} T
\end{align*}
\]

(12)

\[
\begin{align*}
\frac{\partial C}{\partial t} + \frac{\partial C}{\partial y} = \frac{1}{Sc Re} \frac{\partial^2 C}{\partial y^2} - \frac{K_e}{Sc} C
\end{align*}
\]

(13)

where \( u \) is the dimensionless velocity, \( y \) is the dimensionless co-ordinate axis normal to the plates, \( t \) is the dimensionless time, \( T \) is the dimensionless temperature, \( C \) is the dimensionless concentration, \( Gr \) is the thermal Grashof number, \( Gm \) is the concentration Grashof number, \( Pr \) is the Prandt number, \( M \) is the magnetic parameter, \( Sc \) is the Schmidt number, \( K \) porosity, \( Du \) is the Dufour number and \( Kr \) chemical reaction.

The corresponding boundary conditions are non-dimensionalized to:

\[
\begin{align*}
u &= 0, \quad T = 0, \quad C = 0 \quad \text{at} \quad y = - \frac{1}{2} \\
u &= 1, \quad T = 1, \quad C = 1 \quad \text{at} \quad y = \frac{1}{2}
\end{align*}
\]

(14a)

(14b)

### III. METHOD OF SOLUTION

Equations (10) – (13) are second order coupled partial differential equations, we therefore assumed the solution of the form:

\[
\begin{align*}
u(y) &= u_0(y) e^{it} \\
T(y,t) &= \theta_0(y) e^{it} \\
C(y,t) &= \varphi_0(y) e^{it}
\end{align*}
\]

(15)

(16)

(17)

Applying (14 – 18) into the relevant equations in (10 - 14), we obtain

\[
\begin{align*}
\frac{d^2 u_0}{dy^2} - Re Pr \frac{du_0}{dy} - (M^2 + \frac{1}{K} + i\omega) u_0 = -Re P \sin \alpha C y \theta_0 - Gm \sin \alpha C \varphi_0
\end{align*}
\]

(19)
\[
\tau = \left( \frac{\partial u_0}{\partial y} \right)_{y=0} = \alpha_5 D_7 + \alpha_6 D_8 + \alpha_3 D_{10} + \alpha_4 D_{11} + \alpha_1 D_{12} + \alpha_2 D_{13}
\]  
(29)

\[
Nu = -\left( \frac{\partial \theta_0}{\partial y} \right)_{y=0} = -(\alpha_3 D_3 + \alpha_4 D_4 + \alpha_1 D_5 + \alpha_2 D_6)
\]  
(30)

\[
Sh = -\left( \frac{\partial \psi_0}{\partial y} \right)_{y=0} = -A(\alpha_2 + \alpha_1 e^{-y^2})
\]  
(31)

IV. RESULTS AND DISCUSSION

Figures 1, 2, 3, 4 and 5 depict the impacts of Reynolds number, Schmidt number, chemical reaction, frequency of oscillation and time increment on the concentration of the fluid. In figures 1, 2 and 3; it is shown that increasing the Reynolds number, Schmidt number and chemical reaction parameter decrease the fluid concentration. Meanwhile variation in figures 4 and 5 produce no change in the fluid concentration. Swapna et al (2017) obtained the same results.

Figures 6, 7, 8, 9, 10, 11, 12, 13 and 14 depict the impact of rendered on the temperature reliance on the coordinate with Reynolds number, Prandtl number, thermal radiation, heat generation, frequency of oscillation, time, Schmidt number, chemical reaction and Dufour number varying. From figures 6 to 10; it is depicted that varying Reynolds number, Prandtl number, thermal radiation, heat generation and frequency of oscillation decrease fluid temperature. The decrement noted in figure 7 is due to the decrease in thermal conductivity of the fluid. In figure 8; a decrease is encountered. It is true since radiating heat at a higher value consequently cools fluid temperature. Varying the Schmidt number and Dufour number increase the fluid temperature as shown in figures 12 and 14. No change is encountered in the fluid temperature as in figures 11 and 13 in the variation of the time and chemical reaction. Figures 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26 and 27 show the reliance on the fluid velocity on the coordinate. It is seen in figures 25 and 26; the effect of Grashof number on the velocity. Increasing the Grashof number increases the velocity of the fluid. It is so because thermal buoyancy influence on the boundary layer incrementally which consequently leads to increment in the fluid velocity. Figure 24 show the effect of magnetic field parameter on the fluid velocity. It is clear that a reduction is noted as the magnetic field parameter is increased. The reason is the presence of Lorentz force in the field which retards the fluid motion. The Dufour number and porosity are seen to increase the fluid velocity on increase as shown in figure 23 and 27. The increase noted in the fluid velocity as the Dufour number is increase is due to the high concentration diffusion. The frequency of oscillation, Schmidt number decrease the fluid velocity as described by figures 19 and 21. No consequential change is noted by increasing the time, chemical reaction term, Prandtl number, thermal radiation and heat generation on the fluid velocity as noticed in figures 16, 17, 18 and 20. An increase is noted close to the lower plate and the reverse seen close to the upper plate of the fluid velocity. It is seen that at the centre of the channel the velocity tends to be constant.

Figure 1: Reliance of concentration on the coordinate with Re variation

Figure 2: Reliance of concentration on the coordinate with Sc variation

Figure 3: Reliance of concentration on the coordinate with Kr variation

Figure 4: Reliance of concentration on the coordinate with \( \omega \) variation
Figure 5: Reliance of concentration on the coordinate with t variation

Figure 6: Reliance of temperature on the coordinate with Re variation

Figure 7: Reliance of temperature on the coordinate with Pr variation

Figure 8: Reliance of temperature on the coordinate with N variation

Figure 9: Reliance of temperature on the coordinate with S variation

Figure 10: Reliance of temperature on the coordinate with \( \omega \) variation

Figure 11: Reliance of temperature on the coordinate with t variation

Figure 12: Reliance of temperature on the coordinate with Sc variation
Figure 13: Reliance of temperature on the coordinate with Kr variation

Figure 14: Reliance of temperature on the coordinate with Du variation

Figure 15: Reliance of velocity on the coordinate with Re variation

Figure 16: Reliance of velocity on the coordinate with Pr variation

Figure 17: Reliance of velocity on the coordinate with N variation

Figure 18: Reliance of velocity on the coordinate with S variation

Figure 19: Reliance of velocity on the coordinate with \( \omega \) variation

Figure 20: Reliance of velocity on the coordinate with t variation
IV. CONCLUSIONS

In this paper, we considered heat generation and dufouir influences on mhd convective flow through an inclined channel. The governing equations were solved analytically. The solutions for velocity, temperature and concentration fields were obtained in terms of exponential and complimentary functions. From the investigation, the following observations have been drawn.

1. It is observed that increase in the Prandtl number decreases the temperature.
2. It can be seen clearly that the Reynolds number decreases the concentration of the fluid.
3. It is observed that increase in permeability leads to increase in the velocity.
4. It is noted that increase in the magnetic field leads to decrease in the velocity.
5. It is observed that increase in the Grashoff number increases the velocity of the fluid.

REFERENCES


