Stability Analysis of Dual Motor Ball and Beam System Using Fuzzy Logic Controller

Dr. Lakshmanan M[1], Dr. Kamatchi Kannan V[2]
[1]*Associate Professor, Department of EEE, GSSS Institute of Engineering and Technology for Women, Metagalli, Mysuru -570 016, Karnataka, India*
[2]*Associate Professor, Department of EEE, Bannari Amman Institute of Technology, Sathyamangalam-638 402, Tamil Nadu, India*

Abstract: **The control of Dual Motor Ball and Beam (DMBB) system is proposed which is a best example for a non-linear system. In the DMBB system, the output (the ball position) increases without limit for a fixed input (beam angle). The DC servo motor connected to the each end of the beam through a gear mechanism is used to control the position of the ball on the beam. The open loop response of ball and beam system is unstable. The controller's job is to regulate the position of the ball on the beam by changing the angle of the beam. A controller is designed for this system so that the ball's position can be manipulated. In this work, we have adopted a linear model and each motor's angular position is controlled by PD controller which is designed using root locus technique, while the ball position is controlled by an outer loop which is designed using Fuzzy logic controller. The control goal is to bring the ball to desired position on the beam and keep it there. In general, a cooperative control system comprises multiple dynamic entities that share information or tasks to accomplish a common objective.**

Key words: **Ball and Beam System, PID Controller, Fuzzy Logic Controller, Root Locus Technique, Cooperative Logic.**

I. INTRODUCTION

The ball and beam system is one of the most enduringly popular and important model for understanding control systems engineering. The most important parts of a ball and beam control system are a small ball, a beam, a motor, lever arm and a controller. The single motor ball and beam (SMBB) system is simply called "BALL AND BEAM SYSTEM" ([1], [5]-[7],[10]). There are two variants of the single motor ball and beam system, one end of the beam is hinged to a fixed base, while the other end is driven by a motor and the beam pivots on the motor shaft that is located at the beam's centre of gravity. In contrast to the single motor ball and beam (SMBB) system, in dual motor ball and beam (DMBB) system, each end of the beam is driven by a motor. The control job is to automatically regulate the position of the ball by suitably adjusting the slope of the beam using the motors. The DMBB and SMBB systems are unstable in open loop because the ball does not stay in any desired position on the beam, when the beam is tilted and no control effort is being applied. The DMBB system resembles two SMBB systems working together such that the net slope of the beam is the sum of the angles contributed by the individual motors. This proposed work aims to prevent the problem of unstable response due to the nature of double integrator presents in the DMBB system itself. The controller's job is to regulate the position of the ball on the beam by changing the angle of the beam. Controllers are designed for this DMBB system so that the ball's position can be manipulated. To maintain the ball position in the beam, each motor's angular position is controlled by a PD controller ([1],[3],[5],[8]) which is designed using root locus technique, while the ball position is controlled by an outer loop which is designed using Fuzzy logic controller[4]. The cooperative control in the case of our DMBB system is accomplished through predefined workload sharing.

II. DUAL MOTOR BALL AND BEAM SYSTEM

From figure 1, the DMBB system comprises a steel ball rolling on top of a rail. The rail is made up of two parallel bars one of negligibly low resistance and the other of a much larger resistance. The nicrome wire runs along the entire length of a plexi glass rod. A voltage is applied to the ends of the steel rod. As the ball rolls on the rail, it contacts the steel rod and the nichrome wire, forming a voltage divider. This voltage divider is used to sense the location of the ball on the beam. Each end of the beam hangs from an arm. A rack and pinion gear arrangement on both ends of the beam is used to control the vertical movement of the beam. The arm is connected to the rack and the pinion gear is mounted on the shaft of each of the two brushed dc motors ([1]-[3],[5]).

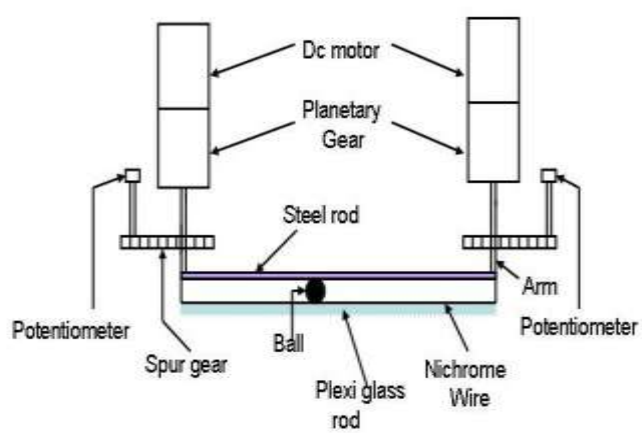

Fig. 1. Top view of the DMBB system

2.1 Modeling of Dual Motor Ball and Beam System

Throughout this paper, all physical quantities are in SI units. The kinematics diagram of the DMBB system as shown in the figure 2. The basic mathematical description of the DMBB system consists of the DC servo motor dynamics and the ball on the beam model. For each motor unit, the transfer function from the armature voltage E_i to the angular displacement θ_i of the shaft is, given by the equation, ([1],[2],[5])

$$\frac{\theta_i(s)}{E_i(s)} = \frac{K_m}{s(\tau_m s+1)}, i = 1,2, \qquad (1)$$

Where K_m and τ_m are the gain constant and time constant of the motor unit.

The model of a SMBB system is given by the equation,

$$\ddot{r} = -\frac{5}{7} g \sin\alpha \qquad (2)$$

Since α remains small, $\sin\alpha \approx \alpha$ .The approximate linear model for the ball and beam system becomes,

$$\frac{R(s)}{\alpha(s)} = -\frac{5g}{7s^2} \qquad (3)$$

Where R(s) = L{r(t)},the Laplace transform of r(t).Let us define angle θ as the angle through which the shaft of the motor unit in an equivalent SMBB setup would turn in order to provide an angular displacement of α to its beam. This SMBB setup is equivalent to DMBB setup in the sense that the former is obtained from the latter by fixing the shaft one of the two motors. It can be shown that $\theta = \theta_1 + \theta_2$, where the angles are considered positive in the counter clock wise direction.

The relation between α and θ is given by the equation,

$$\alpha = l\theta / L \qquad (4)$$

Based on equations (3) and (4) the transfer function of the Dual motor ball and beam system is given by the equation, (e.g. [1], [5])

$$G(s) = \frac{R(s)}{\theta(s)} = -\frac{5lg}{7L}\frac{1}{s^2} \qquad (5)$$

Table 1. Dual motor ball and beam system specification

Notations	Parameters	Values
α	Beam angle in radians	-----
r	Ball position in meter	-----
g	Gravitational acceleration	9.8 m/s^2
L	Length of the beam	0.5 m
l	Length of the arm	0.025m
θ	Servo gear angle	-----
K_m	Motor Gain constant	0.6318
T_m	Motor Time constant	0.0158 s

2.2 Cooperative Logic

In general, a cooperative control system comprises multiple dynamic entities that share information or tasks to accomplish a common objective. The cooperative control in the case of DMBB system is accomplished through predefined workload sharing. (e.g. [1], [5])

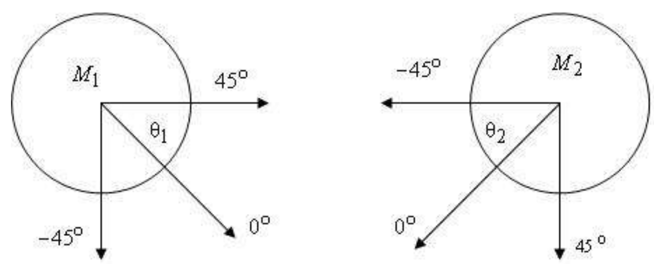

Fig. 3. Cooperative logic

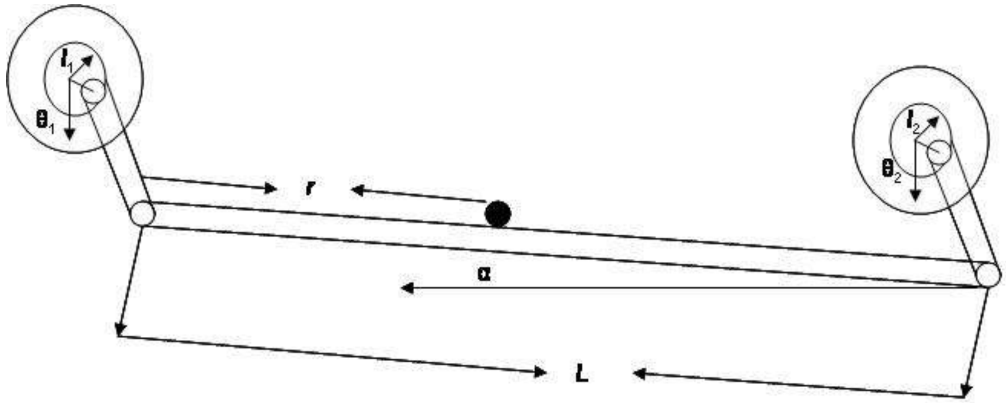

Fig. 4. Kinematics diagram of DMBB System

In this method, the arm is considered as initialized when it is at 45^0 to the normal to the ground as shown in figure 3. The two motors will act simultaneously in the same direction such that the net angle contribution to the beam is equal to the desired angle θ.

2.3 Closed Loop Control of DMBB System

The model of dual motor ball and beam system with controllers is shown in figure 4. A PD controller (K_p+ K_ds) is used for each motor since PD controller will reduce the overshoot. The Fuzzy logic controller is used to produce desired angle of the beam and the two motors work cooperatively to achieve this angle.

III. PD CONTROLLER DESIGN FOR MOTOR

PD Controller design for motor which is designed using root locus technique. In this technique, a controller can be introduced in cascade with open loop system so that, it can have a pair of dominant closed loop poles, to satisfy a specified time domain specifications damping ratio and Natural frequency of oscillation. ([1],[9],[11],[12])

The dominant pole,

$$S_d = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \qquad (6)$$

The transfer function of the PID controller is given by,

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \qquad (7)$$

For design of a PD controller put K_i=0,

$$K_p = \frac{\sin\varphi_d}{DA_d \sin\beta} \qquad (8)$$

$$K_d = \frac{-\sin(\beta+\theta_d)}{A_d \sin\beta} \qquad (9)$$

IV. RESULTS AND DISCUSSION

From the figure 5, we observe that the open loop step response of DMBB system is unstable. The ball didn't reach the stable state i.e. it does not settle at the set point.

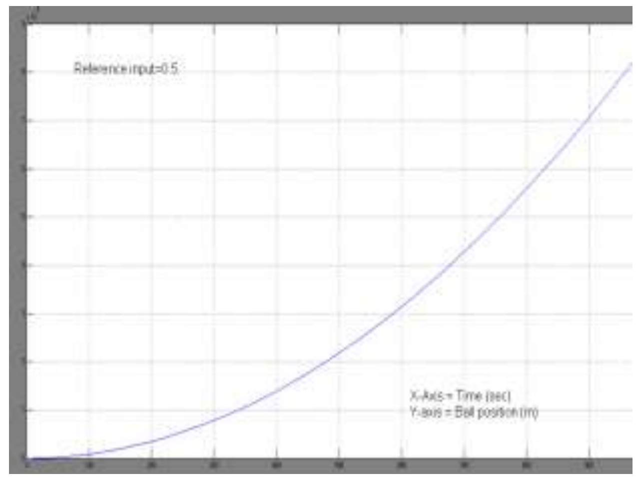

Fig. 5. Open loop Step Response of DMBB System

4.1 Design of PD Controller for Motor

The model of dual motor ball and beam system with controllers is shown in figure 6. A PD

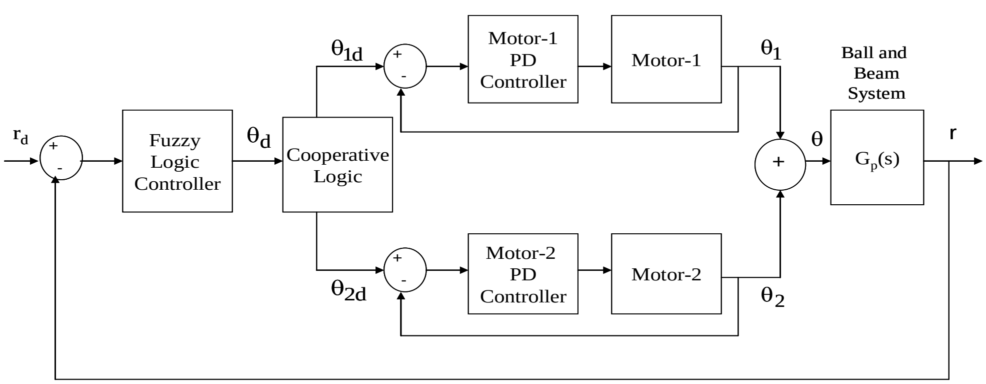

Fig. 6. Block diagram of closed loop control of DMBB system

controller of the form (K_p+ K_ds) is used for each motor since PD controller will reduce the overshoots. The K_p and K_d values of the PD controller obtained for different peak over shoot (%) and Settling time (T_s) are given in table -2.

Table-2 K_p and K_d values of the PD controller

Peak over shoot (%)	**Settling Time (T_s)**	**Proportional controller gain (K_p)**	**Derivative controller gain (K_d)**
50%	0.5s	34.4786	-1.1827
20%	0.5s	7.6988	-1.1827

10%	0.5s	4.5799	-1.1827
50%	1s	8.6196	-1.3827
20%	1s	1.9247	-1.3827
10%	1s	1.1450	-1.3827
50%	5s	0.3448	-1.5428
20%	5s	0.0770	-1.5428
10%	5s	0.0458	-1.5428

Among them, Proportional controller gain K_p = 34.4786 and derivative controller gain K_d = -1.1827 is selected for the PD controller for each motor as it gives lowest value of settling time and peak overshoot, while considering the overall system response. So, the transfer function of PD controller is given by,

$$G_c(s) = 34.4786 - 1.1827s \qquad (10)$$

4.2. PD controller designed for 50% peak over shoot and 0.5s settling time

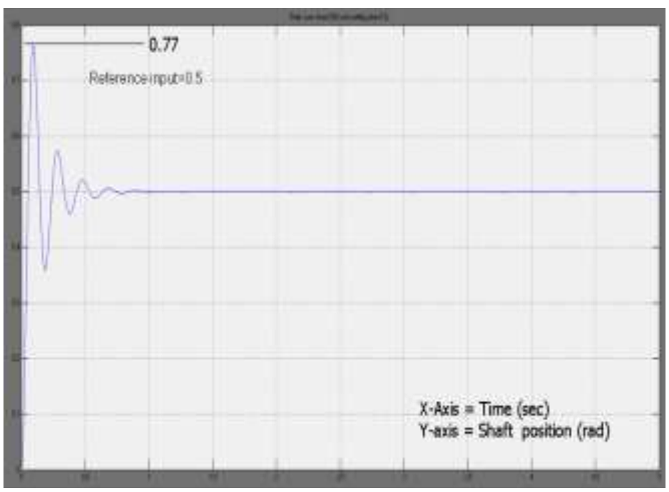

Fig. 7. Response with 50% Peak over shoot and 0.5s settling time

4.3 PD controller designed for 20% peak over shoot and 1.0s settling time

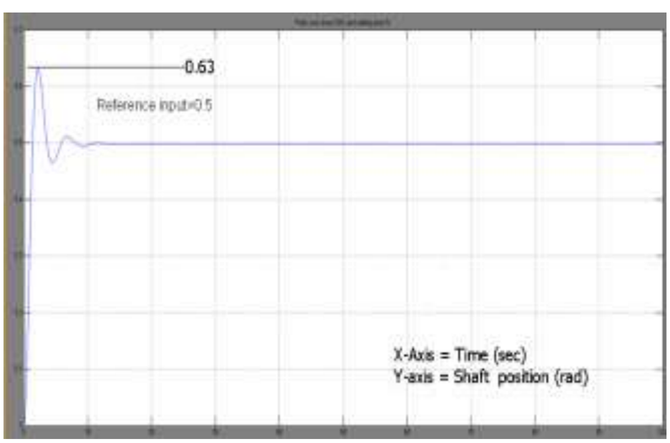

Fig.8. Response of 20% Peak over shoot and 1.0s settling time

4.5 Output Response-1 of the DMBB System

The output response of the DMBB system using various controllers for ball position control, one designed using fuzzy logic controller and another designed using lead compensator is shown in figure 9 for the desired ball position of 0.5m.

4.6 Output Response-2 of the DMBB System

The output response of the DMBB system using various controllers for ball position control, one designed using fuzzy logic controller and another designed using lead compensator is shown in figure 10 for the desired ball position of 0.3m.

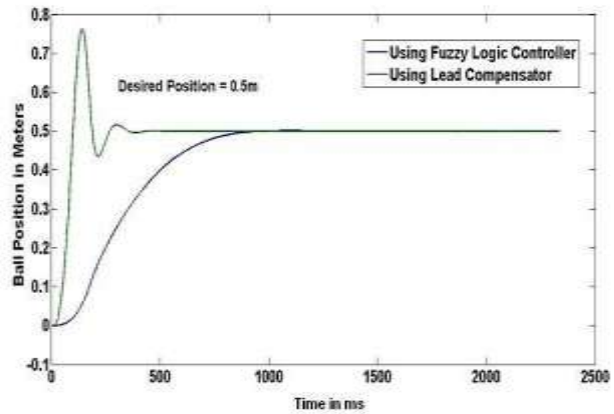

Fig..9 Output response-1 of the DMBB system using various Controllers

Performance analysis of DMBB System using various controllers for the desired ball position of .0.5m is given in Table - 2.

Table 2. Performance analysis of DMBB System using various controllers for the desired position of 0.5m

Parameters	Fuzzy logic controller	Lead compensator
Desired ball position (m)	0.5	0.5
Percent overshoot	0.3018	52.9344
Steady state error (m)	0.4997	0.4996
Settling time (ms)	30.1604	10.3424
Rise time (ms)	17.6718	0.6716

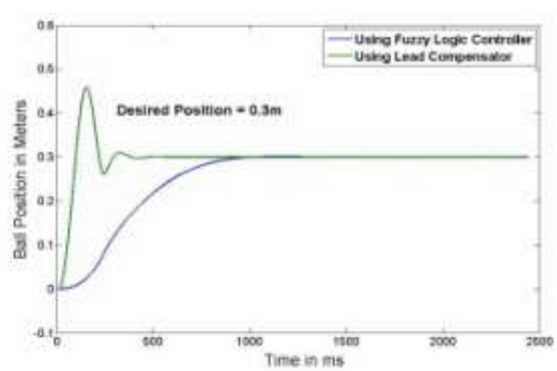

Fig..10. Output response-2 of the DMBB system using various Controllers

Performance analysis of DMBB System using various controllers for the desired ball position of 0.3m is given in Table – 3.

Table 3. Performance analysis of DMBB System using various controllers for the desired position of 0.3m

Parameters	Fuzzy logic controller	Lead compensator
Desired ball position (m)	0.3	0.3
Percent overshoot	0.5699	52.9449
Steady state error (m)	0.2994	0.2998
Settling time (ms)	28.8135	9.2859
Rise time (ms)	17.2797	0.6789

V. CONCLUSION

Dual motor Ball and Beam (DMBB) system is a test system which can be used to study the performance of cooperative control task. The DMBB system has unstable open loop step response due to the presence of two integrators in the transfer function of the system. Design of proper controllers is necessary for the stable operation of the DMBB system. A PD controller is designed for each motor control in the DMBB system, considering different time domain specifications and a best PD controller is chosen. A Fuzzy logic controller is designed for the DMBB system to control the ball position, using MATLAB SIMULINK and fuzzy logic tool box. Using fuzzy logic controller, the required ball position is obtained with less overshoot when compared to the case of using lead compensator. The steady state value of the output obtained while using both the controllers is nearly same.

REFERENCES

[1]. M.Lakshmanan, Dr.R.Ganesan, “Design of controllers for Dual Motor Ball and Beam System”, International Review of Mechanical Engineering (IREME), Vol 8, No 4 (2014).

[2]. H. E. A. Ibrahim, Ahmed A. Hakim Mahmoud,” DC Motor Control Using PID Controller Based on Improved Ant Colony Algorithm,” International Review of Automatic Control (IREACO), Vol 7, No.1 (2014).

[3]. Dharmana Simhachalam, Chanchal Dey, Member, IEEE, and Rajani K. Mudi- "An Auto-Tuning PD Controller for DC Servo Position Control System",2nd International Conference on Power, Control and Embedded Systems,2012.

[4]. Wenzhuo Chen, Xiaomei Sui, Yonghui Xing, Modeling and Modulation of Nonlinear Ball-beam System Controller Based on Matlab, 2012 9th International Conference on Fuzzy Systems and Knowledge Discovery (FSKD 2012).

[5]. Gunasekaran M, Potluri R, Cooperative control of a dual-motor ball and beam system, INDICON 2008. Annual IEEE- India Conference, 2008.

[6]. Jimenez S and Wen Yu, Stable synchronization control for two ball and beam systems, International Conference on Electrical and Electronics Engineering, pages 290-293, 2007.

[7]. Sanda Dale, Toma-Leonida Dragomir - "Interpolative-type control solutions for a “ball and beam” system",control engineering and applied informatics,Vol. 8, No. 3, pp. 3-14, 2006.

[8]. Wen Yu and Floriberto Ortiz, Stability analysis of pd regulation for ball and beam systems, Proceedings of the IEEE Conference on Control Applications, pages 517-522, 2005.

[9]. Ho, W.K., Hang, C.C. and Cao,L.S. – “Tuning of PID controllers based on gain and phase margin specifications”, Automatica, Vol.31, No.3, pp. 497, 1995.

[10]. Hauser, J., Sastry, S. and Kokotovic, P., “Nonlinear control via approximate input-output linearization: The Ball and Beam example,” IEEE Transactions on Automatic control, pp. 392-398, March 1992.

[11]. Richard C.Dorf, Robert H.Bishop, Modern Control Systems, Eighth edition, Pearson education, 2004.

[12]. Ogata.K. – “Modern Control Engineering”, Prentice-Hall, 1990.