Determination of Mean Depth of Flow Using Finite Difference Method for the Equation Obtained with Respect to Maximum Value of Manning's Roughness Coefficient

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Abstract: **The mean depth of flow obtained from fourth order differential equation for the relationship of mean depth of flow and mean velocity of flow with respect to Manning's roughness coefficient is found. The mean depth of flow obtained using finite difference method is nearly equal to the mean depth of flow given in 0.75-inch roughness bed data. Hence, finite difference method is appropriate.**

Keywords: **Mean velocity of flow, Mean depth of flow, Manning's roughness coefficient, Differential equation, Finite difference method.**

I. INTRODUCTION

The relationship for mean depth of flow and mean velocity of flow obtained with respect to maximum value of Manning's roughness coefficient & 4th order differential equation is obtained and analysis is made using Finite difference method. The mean depth of flow obtained using finite difference method is nearly equal to the mean depth of flow given in 0.75 inch roughness bed data hence finite difference method is appropriate.

II. EXPERIMENTAL SETUP & PROCEDURES

FlumeData were obtained for 0.75-inch roughness bed.

Flume: -The flume is open and it is 1.168m wide and 9.54m long. Roughness bed was obtained by smearing masonite boards with fiberglass resin. The boards were screwed to the bed of the flume.

Experimental Procedure: -Five to seven flows were taken for three different slopes (2, 5 and 8%). At each flow, depth was measured at a single cross section, so that channel properties could be calculated.

Data Availability: - Please refer the observation table section for the data collected

III. OBSERVATIONS

The relationship for mean depth of flow and mean velocity of flow with respect to maximum value of Manning's roughness coefficient is given by: -

y= 0.0161 (x)$^{0.3153}$ where y corresponds to mean depth of flow and V corresponds to x.

0.75 inch roughness bed data

x

V ↗ 0.02 m/sec 0.5 m/sec 0.08 m/sec 1.1 m/sec ≈1.064 m/sec

d

y ↙ 0.0223 meter 0.0439 meter 0.0365 meter 0.0392 meter

$$\frac{d^4 y}{dx^4} = -0.0158(x)^{-3.6847}$$

Replacing y^{iv} by its central difference approximation, the differential equation becomes:-

$$\frac{1}{h^4}(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}) = -0.0158(x)^{-3.6847}$$

h = 0.3

Hence

$$y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}) = -0.0002(x)^{-3.6847}$$

i = 1,2

i = 1 we have

$$y_3 - 4y_2 + 6y_1 - 4y_0 + y_{-1} = -0.0002(x_1)^{-3.6847}$$

At i = 2 we have

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = -0.0002(x_2)^{-3.6847}$$

Using y_0 = 0.0223 meter and y_3 = 0.0392 meter we have

For i =1

$6y_1 - 4y_2 + y_{-1} = 0.0474$ - (i)

At i =2

$y_4 + 6y_2 - 4y_i = 0.1340$ - (ii)

Regarding the conditions y" at $x_0 = -0.0527$

$$y_i'' = \frac{1}{h^2}(y_{i+1} - 2y_i + y_{i-1})$$

At i = 0,

$$y_0'' = \frac{1}{0.09}(y_{i+1} - 2y_i + y_{i-1}) = -0.0527$$

$$or \quad y_{i+1} - 2y_i + y_{i-1} = -0.0047$$

or $y_{-1} = 0.0339 - y_i$ - (iii)

y_3" at $x_3 = 1.1$ m/sec

$$= \frac{1}{0.09}(y_{i+1} - 2y_i + y_{i-1}) = 0.0035[1.1]^{-1.6847}$$

At i = 3

$y_4 + y_2 = 0.0781$ - (iv)

Using (iv) the equation (ii) reduced to

$5y_2 - 4y_1 = 0.0559$ - (v)

Using (iii) the equation (i) becomes

$5y_1 - 4y_2 = 0.0075$ - (vi)

Solving v & vi equation

y_2 = 0.0344 meter

and y_1 = 0.0290 meter

From the 0.75 inch roughness bed data:-

y_1= 0.0439 meter and y_2 = 0.0365 meter

And from the computation using Finite difference method

We have $y_1 = 0.0290$ meter $\approx$ 0.0439 meter

And $y_2 = 0.0344$ meter $\approx$ 0.0365 meter

Hence the value of mean depth of flow obtained from Finite difference method provides nearly equal value of depth of flow given in 0.75 inch roughness bed flume data and the equation between mean depth of flow and mean velocity of flow is obtained with respect to maximum value of Manning's roughness coefficient.

Since the mean depth of flow obtained for 4th order differential equation derived for the relationship of mean depth of flow and mean velocity of flow with respect to maximum Manning's roughness coefficient for y_2 obtained by using Finite difference method provides nearly equal value as compared to data given in 0.75 inch roughness bed hence this value of y_2 will be used to provide thickness of side pitching along bank of river and also for thickness of apron.

Corresponding to mean depth of flow y_2= 0.0344 meter which is nearly equal to 0.0365 meter from 0.75 inch roughness bed flume data & corresponding to mean depth of flow = 0.0365 meter the value of discharge of flow Q = 0.0333 m^3/sec

Now using

$$\frac{Q_m}{Q_p} = \frac{1}{n^{2.5}}$$

where n = scale factor = 130

where Q_m= Discharge of flow for model or laboratory channel and Q_p= Discharge of flow for prototype or taken for river hydraulics.

$$Now \quad \frac{0.0333}{Q_p} = \frac{1}{(130)^{2.5}} = \frac{1}{192690}$$

$\therefore Q_p$= 6417 m^3/sec

Hence thickness of stone pitching provided along bank of river

$$t = 0.06(Q_p)^{1/3} = 0.06(6417)^{1/3} = 1.112\, meter$$

and thickness of apron = 1.9 t = 2.112 meter.

IV. CONCLUSION

The relationship for mean depth of flow and mean velocity of flow obtained with respect to maximum value of Manning's roughness coefficient & 4th order differential equation is obtained and analysis is made using Finite difference method. And it is found value of y_2 obtained i.e. mean depth of flow y_2 is nearly equal to the value of y_2 given in 0.75 inch roughness bed data and corresponding to this mean depth of flow thickness of stone side pitching along bank of river is found so that erosion cannot take place

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Notations

The following symbols are used in this paper:-

A = Flow cross sectional area = Wd.

d = Mean depth of flow in meters.

Q = Discharge of flow in cubic meters per second.

S = Channel slope.

V = Mean velocity of flow.

W = Width of the channel = 1.168m

V. OBSERVATION TABLES

Table 1: 0.75-inch roughness bed flume data.

Sl. No. (1)	Channel Slope (2)	Discharge of flow in cubic meters per second (3)	Mean depth of flow d in meters (4)	Mean velocity of flow in meters per second (5)
1.	0.02	0.00580	0.0223	0.222
2.	0.02	0.01181	0.0290	0.348
3.	0.02	0.02482	0.0439	0.484
4.	0.02	0.04047	0.0591	0.586
5.	0.02	0.05348	0.0698	0.656
6.	0.05	0.00381	0.0141	0.230
7.	0.05	0.00843	0.0199	0.363
8.	0.05	0.02037	0.0299	0.583
9.	0.05	0.03333	0.0365	0.782
10.	0.05	0.04586	0.0434	0.904
11.	0.05	0.05460	0.0477	0.979
12.	0.08	0.00207	0.0095	0.186
13.	0.08	0.00631	0.0142	0.380
14.	0.08	0.01007	0.0200	0.430
15.	0.08	0.02825	0.0299	0.807
16.	0.08	0.04518	0.0375	1.032
17.	0.08	0.04879	0.0392	1.064

Table 2: Flume data for 0.75-inch roughness bed.

Sl. No. (1)	Manning's roughness coefficient n (2)	Chezy's resistance factor C (3)	Hydraulic radius R in meter (4)
1.	0.071	10.832	0.021
2.	0.055	14.706	0.028
3.	0.050	17.112	0.040
4.	0.051	17.758	0.054
5.	0.050	18.481	0.063
6.	0.078	9.021	0.013
7.	0.065	11.777	0.019
8.	0.053	15.310	0.029
9.	0.045	18.693	0.035
10.	0.043	19.966	0.041
11.	0.042	20.872	0.044
12.	0.096	6.932	0.009
13.	0.063	11.355	0.014
14.	0.069	11.029	0.019
15.	0.049	16.754	0.029
16.	0.043	19.503	0.035
17.	0.043	19.557	0.037

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