Sensitivity Analysis of Overtime using Capacity Planning Levers

Ashish Pandey

***Abstract*---Demand, Throughput rate, Utilization and Overtime are pillars of Capacity planning. Overtime is a function of Demand, Throughput rate and Utilization. A firm's objective is to determine the distribution of the Overtime risk estimate using the independent marginal distributions and dependency structure between the capacity levers. This additional operating cost can be managed using capacity levers and cross training efforts in order to help the firm manage labor expenses to meet seasonal demand patterns. If not managed efficiently it can lead to higher than expected costs.Unlike a manufacturing firm the capacity planning for a services firm would vary significantly. Throughput rate for a services firm would vary in parallel with the seasonal demand because of the high dependency on human effort, in high volume period associates complete more transactions in the given time while in a lean period the processing time for a transaction would increase leading to decline in throughput rate. Given this dependency among capacity levers the paper describes the application of t-copula as a joint density estimation technique to model the correlation among Demand, Throughput rate and Utilization. Data is hypothecated for illustration purpose and reflects the business case to model the dependency among capacity levers.**

***Keywords:* Capacity planning, Copula, Overtime, Throughput rate, Utilization**

I. INTRODUCTION

Sensitivity analysis is the study of uncertainty in the dependent variable apportioned to uncertainty in the independent variables. Understanding the variability and predictability of Overtime hours becomes important in the Capacity planning domain. Demand, Throughput & Utilization are key pillars of Capacity planning that are used as levers to meet the incoming demand and thus to effectively manage the overtime cost.

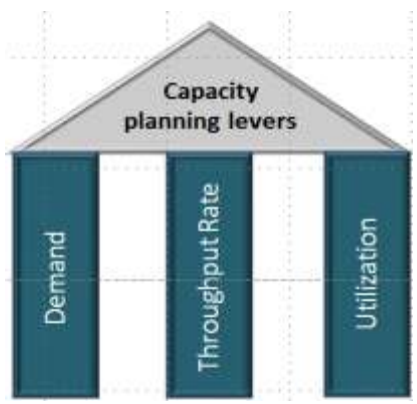

Figure 1: Capacity planning levers

The following example is the case of a back office operations within a financial services firm where Volume is the demand in terms of transactions or requests to be processed. In this case, Volumes are driven primarily through market conditions and a firm's investment decisions, which might be difficult to predict and could be random in nature. Throughput is the number of transactions completed in a given unit of time, Utilization relates to the proportion of time spent on core work, and Overtime is the excess of man-hours spent over and above the available hours in a given timeframe. Core work is the time spent on processing the transactions that are billable to the client.

Understanding the distribution of overtime is crucial to manage the staffing strategy. It is important to study how capacity levers vary under different scenarios and how dependency among them impacts the Overtime distribution. If they are independent then each lever can be studied independently and can be modeled to study the distribution of Overtime, while this will not be the case in a scenario where there is some dependency and it becomes important to study the joint density of the Capacity planning levers.

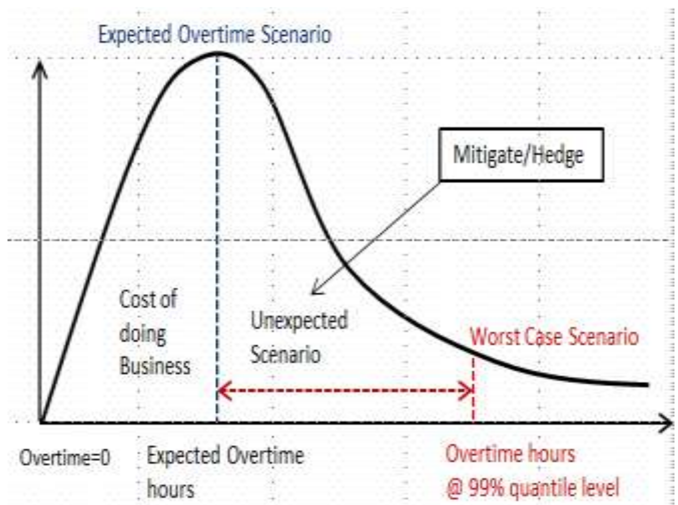

Figure 2: Overtime distribution, Expected, Unexpected and Worst-case scenario

Overtime distribution derived through sensitivity analysis then enables to quantify Expected and Unexpected Overtime costs, and model the Worst-case.

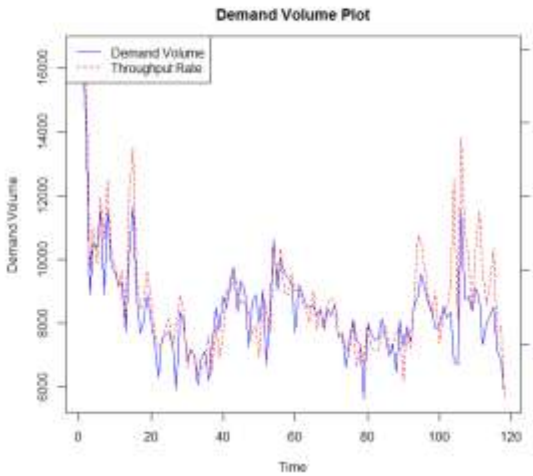

Figure 3: Demand Vs Throughput Rate

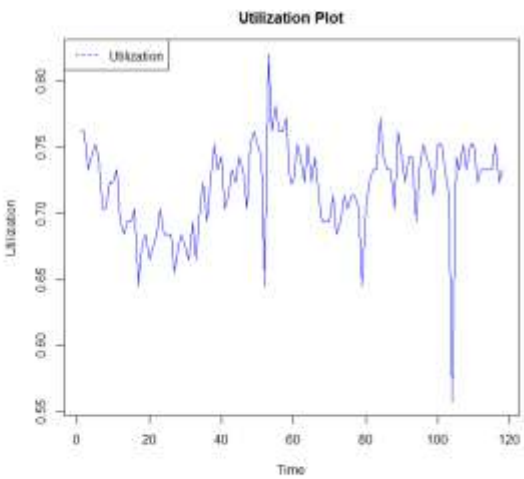

Figure 4: Utilization plot: There is some relation of Utilization with the Demand trend.

Figure 3& 4 describes a classic case of Parkinson's Law that states “work expands so as to fill the time available for its completion”. Demand and the Throughput rate vary in parallel to each other. High peak seasons are managed by an increase in Throughput rate and low Volume period brings the Throughput rate down. Utilization varies in relation to the demand but the relation is not highly correlated.

These key levers will not be independent of each other and there will always be some level of correlation between them, thus modeling the joint density estimation becomes crucial.

The sensitivity analysis method described in this paper thus uses a copula based methodology to model the joint density estimation of these levers and then uses simulation to derive the overtime distribution. Copula is a multivariate probability distribution used to model the dependence between random variables using their marginal distributions

Capacity planning as a domain involves a firm’s strategic goals towards staffing at a tactical level, with a short-term horizon and long-term horizon. At a tactical level (i.e. one day or a week) these metrics would generally vary in a random manner while on a medium to long-term there may be some level of stability and possible trends in the data.

This paper also describes the application of Brownian motion and Monte Carlo simulation to derive staffing strategies in the short-term level (i.e. intraday/daily/weekly) where demand may follow a random pattern.

II. STATISTICAL THEORY

If $(X_1, X_2, X_3 \ldots . X_n)$is a random vector with continuous marginal distributions given$\mathrm{F(X)} = \mathrm{P(X \leq x)}$then the random vector

$$(U_1, U_2, U_3 \ldots . U_n) = (F_1(X_1), F_2(X_2), F_3(X_3), \ldots . F_n(X_n))$$

exhibits a uniform marginal distribution. A copula of$(X_1, X_2, X_3 \ldots . X_n)$is defined below

$$\mathrm{C}(u_1, u_2, u_3 . u_n) = \mathrm{P}[(U_1 \leq u_1), (U_2 \leq u_2), .. (U_n \leq u_n)]$$

Copulas are useful in studying the joint outcomes of random variables.

Geometric Brownian motion follows the stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t d\mathrm{W_t}$$

$\mathrm{W_t}$is a Wiener process, μ is the percentage drift andσ is the percentage volatility.The above stochastic differential equation using Itô calculus can be written as

$$S_t = S_o \exp[\left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t]$$

III. SENSITIVITY ANALYSIS OF OVERTIME USING CAPACITY PLANNING LEVERS

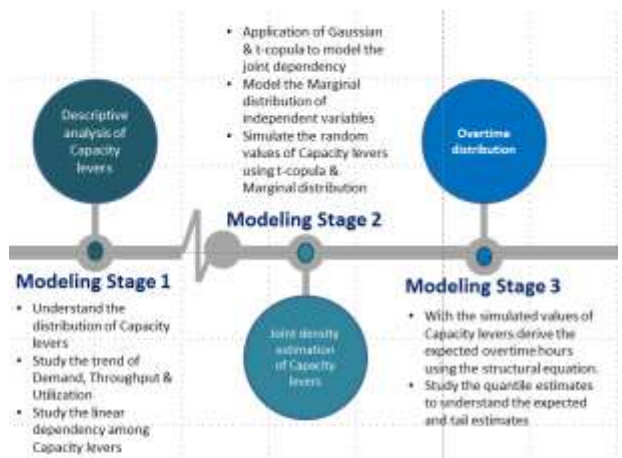

Figure 5: Stages of sensitivity analysis.

3.1 Descriptive analysis of Capacity levers

$$Utilization = ((Time\ spent\ on\ core\ work)) / ((Total\ hours))$$

A higher Utilization means that the time spent on core work (excluding internal deliverables or meetings that are not billable to the client) contributes to a larger portion of the total billable hours.

$$Throughput\ Rate = ((Volume\ of\ transactions\ processed)) / ((Time\ spent\ on\ core\ work)\ )$$

Throughput rate in isolation as a point estimate will not help in deriving any conclusion. It needs to be compared on relative terms on a time scale or along the service lines with similar type of work.

$$Overtime\ hours = [((Demand\ Volume)) / ((Target\ Rate)\ ) - (Staffing\ supply * Available\ hours * Utilization * (Throughput\ rate)/(Target\ rate)) * staffing\ adjustment\ ]$$

If the total available hours in a week stands at 40 (8 working hours/day), then any additional time spent over and above 40 is categorized as overtime (additional cost to the firm). Data for this analysis uses a weekly interval scale to model the Capacity levers.

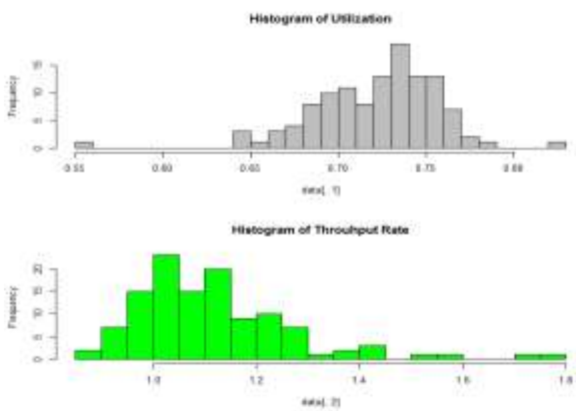

Figure 6: Histogram of Utilization & Throughput Rate

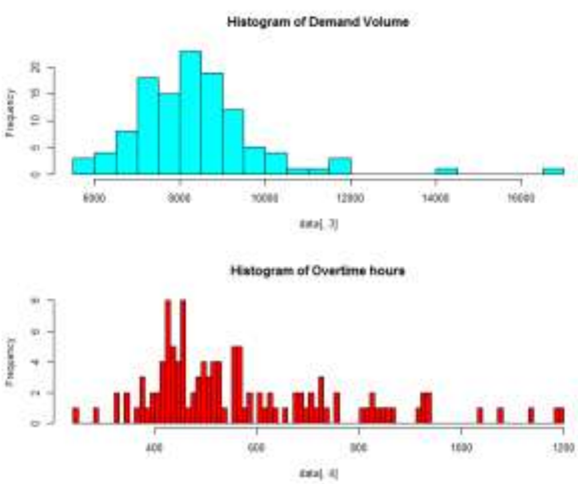

Figure 7: Histogram of Demand Volume and Overtime hours

Utilization	Throuhput.Rate	Demand.Volume	Overtime.hours
Min. :0.5572	Min. :0.8506	Min. : 5630	Min. : 240.2
1st Qu.:0.6965	1st Qu.:1.0302	1st Qu.: 7465	1st Qu.: 435.9
Median :0.7234	Median :1.0963	Median : 8244	Median : 513.0
Mean :0.7200	Mean :1.1203	Mean : 8381	Mean : 573.5
3rd Qu.:0.7429	3rd Qu.:1.1861	3rd Qu.: 8923	3rd Qu.: 685.9
Max. :0.8211	Max. :1.7957	Max. :16541	Max. :1194.4

Figure 8: Summary of Capacity planning levers

Histogram of Demand, Overtime & Throughput rate suggests that the data is skewed to the right indicating a less frequent but higher demand accompanied by increase in Throughput and Overtime hours.

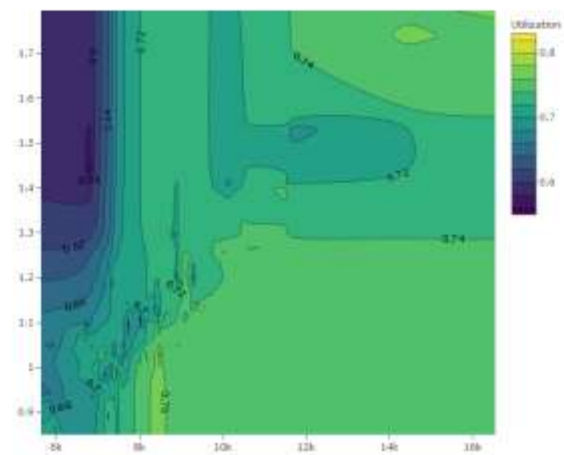

Figure 9: Contour plot of Demand, Throughput Rate & Utilization

The above contour plot gives a graphical view of the data in three dimensions.Axis X, Y & Z respectively describe Demand Volume, Throughput Rate and Utilization.

3.2 Fitting copula distribution to the Independent variables

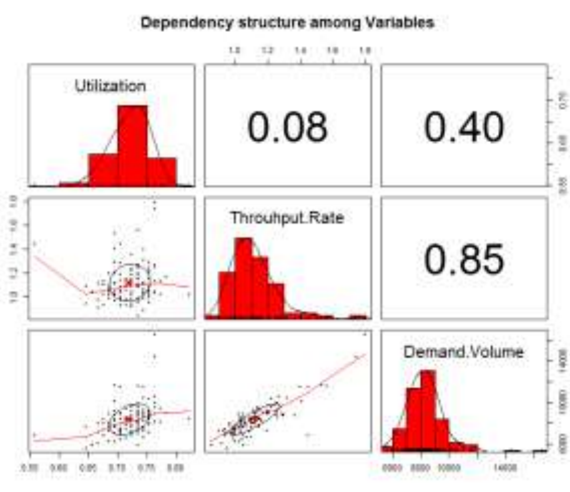

Figure 10: Dependency structure: Correlation & Histogram

The above dependency analysis shows a strong positive relation between Demand vs. Throughput Rate and Demand vs. Utilization. The model uses t-Copula to derive joint density among variables.The lower AIC & BIC values of the fitted t-Copula function suggest that it is a good fit in comparison to a Gaussian copula.

Below,'t-Copula' also describes the evidence of tail dependency among variables.

```
> Unifdata=pobs(data)

> ro=cor(data)

> pairs.panels(data,hist.col="red",main="Dependency structure among Variables")

> tCop=tCopula(c(ro[2,1],ro[3,1],ro[3,2]), dim = 3, dispstr="un", df.fixed=FALSE)
```

```
t-copula, dim. d = 3
Dimension:  3
Parameters:
  rho.1   = 0.08168031
  rho.2   = 0.40234888
  rho.3   = 0.84759487
  df      = 4.00000000
dispstr:  un
```

Figure 11: t-copula output 1

```
> TCopEst=fitCopula(tCop, Unifdata, method="mpl", estimate.variance=TRUE)
```

The above function gives the estimated values of 'rho.1', 'rho.2', 'rho.3' and 'df' using maximum likelihood estimation

```
Call: fitCopula(copula, data = data, method = "mpl", estimate.variance = TRUE)
Fit based on "maximum pseudo-likelihood" and 118 3-dimensional observations.
Copula: tCopula
 rho.1  rho.2  rho.3     df
0.1928 0.4755 0.8083 9.2579
The maximized loglikelihood is 82.67
Optimization converged
```

Figure 12: t-copula output 2; rho values indicate the joint dependency structure

```
> tCop=tCopula(c(0.1928,0.4755,0.8083), dim = 3, dispstr="un", df=9.2579)

> TailDep=lambda(tCop)
```

```
    lower1     lower2     lower3     upper1     upper2     upper3
0.02447132 0.08453040 0.32098956 0.02447132 0.08453040 0.32098956
```

Figure 13: t-copula output 3, tail dependency

3.3 Marginal distributions of Independent variables and simulation

A marginal distribution is the probability distribution of individual variables contained in the subset of variables.For Utilization, Throughput Rate and Demand the log normal distribution provides the closest fit to the data.The data beingskewed to the right with a lower bound of '0' makes it an ideal candidate for log normal fit. If there is an interest in modeling extreme tail scenarios for independent variables, Extreme value theory can be incorporated along with copulas to model it. A GEV (Generalized extreme value distribution) or POT ('peak over threshold' approach) can be used to model the tails of the distributions. Diagnostic plots of the fitted log normal distribution on 'Demand Volume' are given below

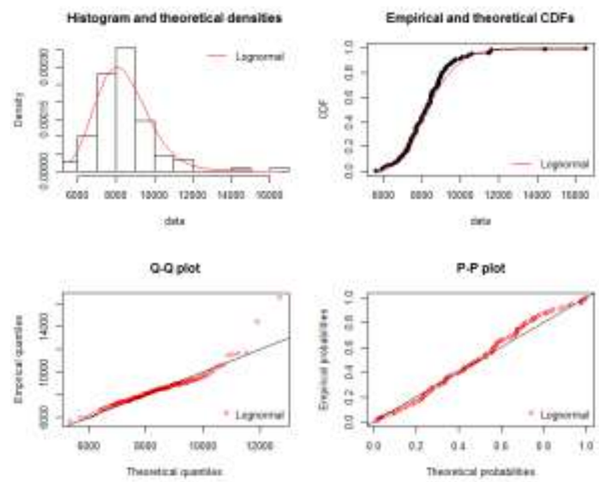

Figure 14: Diagnostic plot, Log-normal marginal distribution fit

After deriving the required t-copula estimates and marginal distributions of the independent variables the next step is to use simulation to extract the multivariate joint random values for independent variables.

```
> set.seed(123)
> r=1000 ## Total number of valuesto simulation
> t.cop_est=tCopula(c(0.1928,0.4755,0.8083), dim = 3, dispstr="un", df=9.2579)
> Sim_data_tC=rCopula(r,t.cop_est) # ## Simulated values
> sim.Util=qlnorm(Sim_data_tC[,1], fit_ln_Util$estimate[[1]], fit_ln_Util$estimate[[2]])
> sim.Rate=qlnorm(Sim_data_tC[,2], fit_ln_Rate$estimate[[1]], fit_ln_Rate$estimate[[2]])
> sim.Vol=qlnorm(Sim_data_tC[,3], fit_ln_Vol$estimate[[1]], fit_ln_Vol$estimate[[2]])
> simulated_measures=cbind(sim.Util,sim.Rate,sim.Vol)
> simulated_measures=as.matrix(simulated_measures)
> {pairs.panels(simulated_measures,hist.col="red",main="Joint density estimation among Variables",
+           bg=c("red","yellow","blue")[simulated_measures])}

> summary(simulated_measures)
```

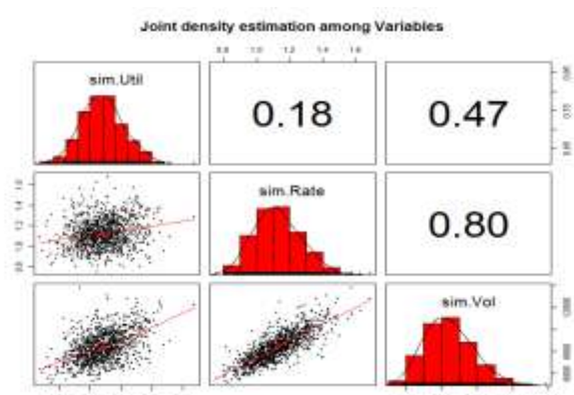

Figure 15: Joint density estimation

Below is the three-dimensional view of the correlated random variables generated after modeling the t-copula and log normal marginal distributions

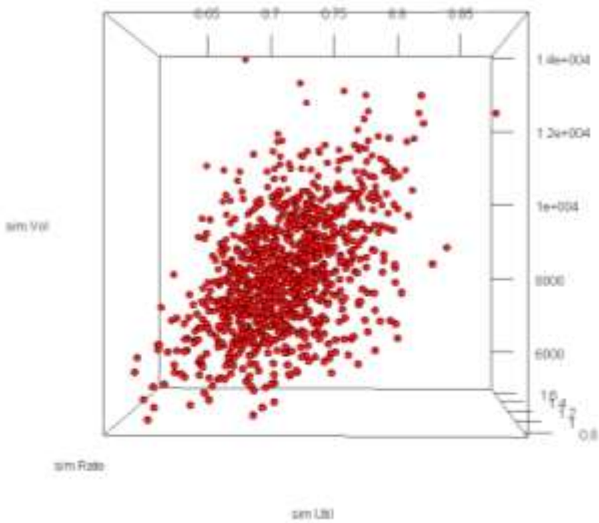

Figure 16: Three dimensional view of the dependency structure between variables

3.4 Modeling the sensitivity of Overtime given Independent variables and constants

After modeling the joint cumulative density estimation with t-copula, the next step is to derive the overtime distribution using certain assumptions. In this case,

Target Throughput Rate=1.3, Staffing supply (headcount) =250, Available hours per week=40, Supply adjusted for Vacation/absenteeism=0.87multiplier

```
> simulated_measures$Overtime_Hours=(simulated_measures$sim.Vol/TR)-
+ (HC*AH*simulated_measures$sim.Util*simulated_measures$sim.Rate/TR)*0.87
```

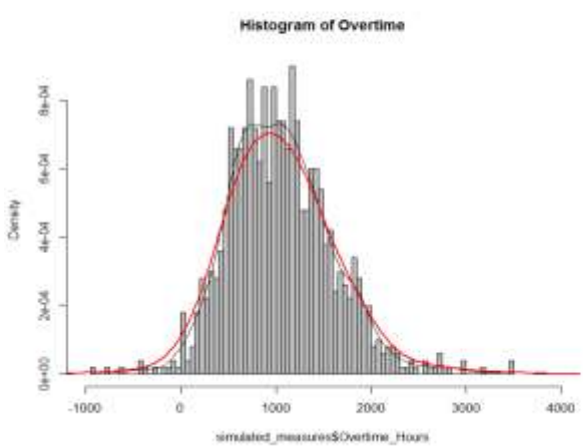

Figure 17: Histogram of simulated Overtime values

```
  Min.  1st Qu.  Median    Mean  3rd Qu.    Max.
-944.7   677.4   1014.7  1049.9  1375.5  3484.9
```

Worst-case Overtime scenario (99% quantile estimate) = 2658.4 hrs.

Expected Overtime scenario (50% quantile estimate) =1014.7 hrs.

Unexpected Overtime scenario=2658.4 hrs-1014.7 hrs=1643.7

The business strategy should be to mitigate the impact of the unexpected scenario.

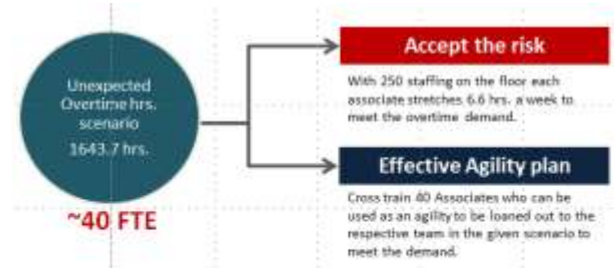

Figure 18: Line managers have two options to manage the unexpected scenario. Accept the risk of overtime hours or have an effective agility plan in place to hedge/mitigate the impact of the risk.

Utilization is a lever that can be used to manage the seasonal patterns in the demand, thus it would also be interesting to see the Overtime distribution by controlling the Utilization levels.

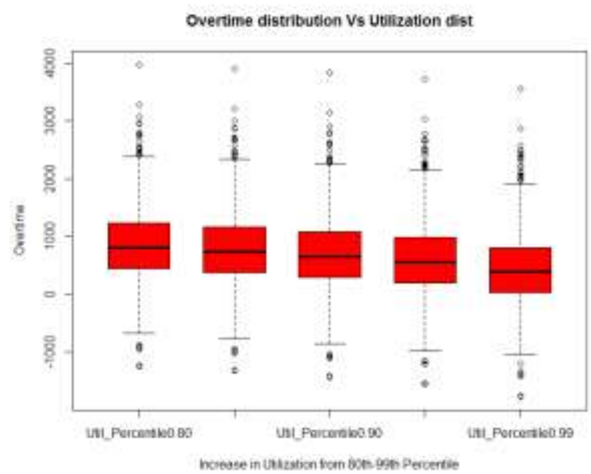

Figure 19: Box plot, Overtime distribution Vs Utilization

From the above box plot we can derive that if the firm increases Utilization from the 80th percentile level to the 99th percentile, it can reduce the mean overtime from 845.6 hours to 426.8 hours (50% decline).

IV. TACTICAL PLANNING USING GEOMETRIC BROWNIAN MOTION TO FORECAST DEMAND VOLUMES

Staffing strategy across service lines depends upon the expected demand for future time periods. It becomes difficult to accurately plan when the volumes follow a random pattern.

Geometric Brownian motion along with the use of the Monte Carlo simulation technique can help to model the random behavior of transaction volumes at a tactical level.

```
> Simultaed_GBM=function(S0,mean,volatility,T,Forecast_lead,Iterations){
+  dt=T/Forecast_lead
+  meanT=(mean-volatility^2/2)*dt
+  volatilityT=sqrt(dt)*volatility
+  pathmatrix=matrix(nrow=Iterations,ncol=Forecast_lead)
+  pathmatrix[,1]=S0
+  for (i in 1:Iterations){
+    for (j in 2:Forecast_lead){
+      pathmatrix[i,j]=
+        pathmatrix[i,j-1]*exp(rnorm(1,meanT,volatilityT))}}
+  return(pathmatrix)
+ }
```

The above algorithm fits the GBM function. For illustration, the input variables used to predict the tactical forecasts for the next 30 days are described below.

```
> S0=7870
> mean=-0.0154
> volatility=0.15
> T=1
> Forecast_lead=30
> Iterations=100
```

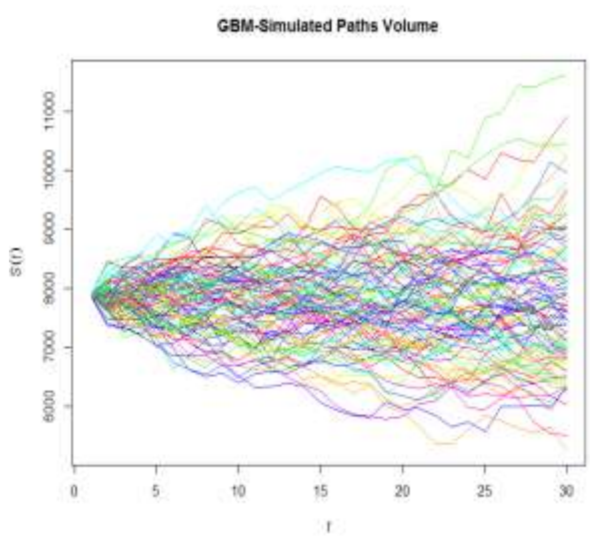

Figure 20: GBM simulated paths of Demand volume

Mean values of the above iterated forecasts at each individual value of time frame “t” can be calculated to arrive at the tactical level forecasts for the next 30 days.

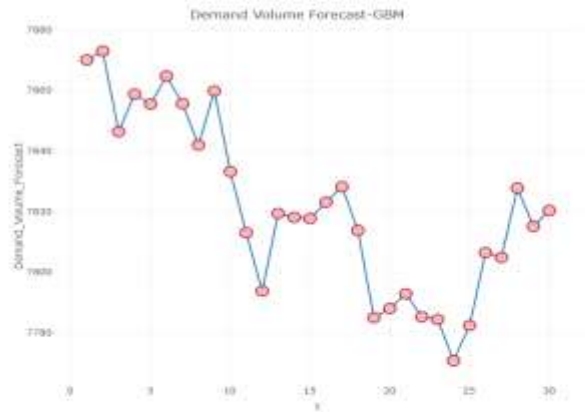

Figure 21: Demand forecast-Mean values of simulated paths

The demand forecast derived above can then be converted into supply in terms of headcount to manage that demand.

$$Required\ supply = [[((Demand\ Volume)/(Target\ rate))/(Available\ hours)]/((Utilization)\ )]$$

V. CONCLUSION

The distribution of Capacity planning levers may not be independent of each other in a services firm, in such a case the sensitivity of Overtime distribution as a function of Demand, Throughput rate & Utilization should be modeled using joint density estimation techniques. This would help the business to effectively plan for the capacity during seasonal and non-seasonal periods. In our analysis we saw that there is strong positive correlation between Demand and Throughput and low dependency between Demand and Utilization. In such a scenario from Figure 16 we saw that by increasing the Utilization level from 80th percentile to 99th percentile the firm would be able to reduce the overtime hours by approximately 50%. Thus by using the lever of Utilization and let Throughput rate follow the parallel pattern with the Demand the firm can get a realistic estimate of expected Overtime.

REFERENCES

[1]. McNeil A. J., Frey R. and Embrechts P (2006). Quantitative Risk Management: Concepts, Techniques, and Tools. Princeton University Press.

[2]. Daróczi G., Dancs V. A., Havran D., Berlinger E., Michaletzky M., Puhle M., Csóka P., Váradi K., Tulassay Z. (2013). Introduction to R for Quantitative Finance. Packt Publishing Ltd.

[3]. Bolancé, C., Guillén, M. and Padilla, A. (2015). Estimación del riesgo mediante el ajuste de cópulas. UB Riskcenter Working Papers Series 2015-01.

[4]. Bedford T., Cooke R.(2001), Probabilistic Risk Analysis: Foundations and Methods. Cambridge University Press, 2001.

[5]. Hofert M., Kojadinovic I., Maechler M., Yan J. (2017). Copula. R package

[6]. Brandimarte P. (2014). Handbook in Monte Carlo Simulation: Applications in Financial Engineering, Risk Management, and Economics. John Wiley & Sons.