# Reconstructing a Nonminimum Phase Response From Amplitude-Only Data of an Electromagnetic System

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***Abstract*—A general method is presented for reconstructing nonminimum phase from amplitude only data. The nonminimum phase is generated utilizing the nonparametric method.The advantage of this method is that no priori information is needed and no such choice of basis function is required as the solution procedure develops the nature of the solution. This is accomplished by the Hilbert Transform which is a very fundamental property of nature that the real and imaginary part of the nonminimum phase transfer function can satisfy the relationship. The application of this method has been applied to the some antenna radiation pattern and scattering parameters of microwave filters.**

***Keywords*—Nonminimum Phase, Amplitude-only, Hilbert Transform.**

## I. Introduction

IN an electromagnetic system, the magnitude response can easily be measured but the phase response is hard to obtain. Thereforeitis importanttoreconstructthe phaseresponsefrom amplitude-onlydata.For minimum phase systems, the reconstruction of phase from amplitude-only data is relatively straightforward as the phase response is given by the Hilbert transform of the log of the magnitude of the amplitude data . In other words, the minimum phase as a function of frequency is given by and is expressed as It can be shown that the minimum phase and the log magnitude of the amplitude have a Hilbert transform relationship. In other words, the minimum phase as a function of frequency *w* can be expressed as

$$\arg\,[X(w)] = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\ln |X(\lambda)|}{\lambda - w} d\lambda \quad (1)$$

where P denotes a principal-value integral, as the integrand has a singularity and is not integrable. The integral in (1) only exists in a principal-value sense. However, this property given by (1) of a linear-time invariant system does not hold if the system is not minimum phase. The minimum-phase property of a transfer function refers only to all the zeros of $X(s)$ . The zeros must lie in the left half ($s = \sigma + jw$, $\sigma < 0$)of the -plane. If the system is not minimum phase (i.e., when some of the zeros of the transfer function may be on the right half-plane), then (1) does not hold. From a practical stand point a minimum phase representation between a single input and single output implies that there is a single path of propagation between the input and the output and that the input arrives at the output instantaneously without a delay. For a real system, which may be distributed implies there are many paths between the input and the output and that the energy arrives at the output after some delay at which it has been applied to the input. Since most electromagnetic systems are distributed and have a nonminimum phase response, the Cepstrum method given by (1) is not much useful for practical nonminimum phase reconstruction. Hence, (1) has very little use for the practical problems. However, there is a more general result of the Hilbert transform, which is based on causality. This result is valid for a nonminimum phase system. We utilize the principle of causality to do nonminimum phase realizations. The principle of causality implies that the function for and is nonzero otherwise. It is important at the onset to point out that the phase realization (be it minimum or nonminimum phase) is not a unique problem. A linear-phase term may be added to any phase function without altering its amplitude spectrum. This is because the addition of a linearphase to the phase of the transfer function with a uniform amplitude is equivalent to a pure delay in the time domain. Since we are dealing with linear-shift invariant systems (as the response of the system is the same independent of the time origin), changing the impulse response of the system by a time shift does not alter the transfer function of the original system, except that the phase spectrum is modified by a linear-phase function. The slope of this linear-phase function is equivalent to the time delay. For an antenna problem this is equivalent to a shift in the spatial coordinates

## II. Causality and Properties of the Transfer Function

A function $x(t)$ is said to be “causal” if

$x(t) = 0$ whenever $t < 0$. (2)

These types of functions arise in the study of causal systems and are of obvious importance in describing phenomena that have well-defined starting points.Let $x(t)$be a real causal function with Fourier transform $X(\omega)$, and let $R(\omega)$ and $I(\omega)$ be the real and the imaginary parts of $X(\omega)$.Then $X(\omega)=R(\omega)+jI(\omega)=|X(\omega)|e^{j\phi(w)}$. Since $x(t)$ is real, $R(\omega)$ is even and $I(\omega)$ is an odd function of $\omega$. A general question of whether a specified amplitude characteristic can be realized as a causal system response is answered by the Paley–Wiener criterion.Let Consider a specific magnitude $|X(\omega)|$of a transfer function $X(\omega)$.It can be realized by means of a causal system if and only if the integral

$\int_{-\infty}^{\infty} \frac{\ln|X(w)|}{1+w^2} dw < \infty$ (3)

is bounded. So the phase function associated with $|X(w)|$ exists such that x(t) is causal. If $X(w)$ has a causal representation it can be stated that

$$x(t) = \frac{2}{\pi}\int_0^{\infty} R(w)\cos(wt)\, dw \qquad 0<t \qquad (4)$$

$$x(t) = -\frac{2}{\pi}\int_0^{\infty} I(w)\sin(wt)\, dw \qquad 0<t \qquad (5)$$

and

$$\int_0^{\infty} |x(t)|^2 dt = \frac{1}{\pi}\int_{-\infty}^{\infty} |R(w)|^2 dw = \frac{1}{\pi}\int_{-\infty}^{\infty} |I(w)|^2 dw \quad (6)$$

If $x(t)$ is bounded in the origin then we have

$$R(w) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{I(s)}{w-s}\, ds = -H[I(w)] \quad (7)$$

$$I(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{R(s)}{w-s}\, ds = -H[R(w)] \qquad (8)$$

Where $H$[.]defines the Hilbert transform which constitutes a convolution operation with function $\frac{1}{\pi w}$, which is not defined at w = 0. The last two integral (7) and (8) are defined in terms of Hilbert transform and Cauchy Principle value sense.

## III. APPLICATION OF THE PROPERTY OF THE HILBERT TRANSFORM TO RECONSTRUCT THE PHASE

In the previous sections the analysis of the transfer function $R(w) + jI(W)$ defined over the $-\infty < w < \infty$. But in the practical application $R(w)$ and $I(w)$ are generally define over a finite segment. Now if we modified the previous assumption by assuming $R(w)$ and $I(w)$ are periodic functions with a period of $2\pi$ denoted by $R_p(w)$ and $I_p(w)$, one can write

$$R_p(w) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nw + \phi_n) \qquad for \quad 0<w<\pi \qquad (9)$$

Where $a_i$ for $i = 0,1\ldots\ldots\ldots\infty$ are the discreate Fourier cosine transform of $R(w)$ and $\phi_n$ is the certain phase associated with it.

Now if we take the Hilbert transform of $R_p(w)$ , the one can obtain from (7)

$$I_p(w) = -\sum_{n=1}^{\infty} a_n \sin(nw + \phi_n) \qquad for \quad 0<w<\pi \qquad (10)$$

It is important to note that the same Fourier coefficient are used in (9) are also used in (10). So (9) and (10) are related through the coefficients $a_i$ and $\phi_i$. The equations (9) and (10) are very important in the sense of modulation and demodulation scheme implemented on all the hardware instrument.When a low frequency signal $f_m$ modulated with carrier frequency $f_0$ two side band is generated, one located at $f_m$- $f_0$and other located at $f_m$+ $f_0$. The modulated signal translated up in the frequency and effective bandwidth of the total waveform with respect to the carrier frequency decreases. So when the signal travel to the desired destination one have to demodulate the signal and eliminate the carrier frequency to recover the baseband signal. But it is not very easy to demodulate the original signal at high frequency which cannot be beat with a local oscillator, as the two side band simultaneously create interference. So, in the first step of the demodulation process one generate an analytic signal through I (in-phase) and the Q(quadraturephase) components of the modulated signal so that the signal is defined only for positive frequencies and the negative frequencies will be eliminated through. Now, the analytic signal beats with the local oscillator generating a single sideband generating a translated version of the baseband signal of interest. It is very important to note that the economic cost goes almost double for using two different channel. A more effective way is to generate a ***I*** channel only, then digitized it, apply the Hilbert transform and create a ***Q*** channel. The next step can be done by the software . This way the cost can be reduce by half. Some ground probing radar has used this innovative technique.

Now the magnitude response of the system is given by

$$|X(w)|^2 = |R(w)|^2 + |I(w)|^2$$

$$\cong |R_p(w)|^2 + |I_p(w)|^2$$

$$= |a_0 + \sum_{n=1}^{\infty} a_n \cos(nw + \phi_n)|^2$$

$$+ |\sum_{n=1}^{\infty} a_n \sin(nw + \phi_n|^2 \quad (11)$$

So from the given power spectrum of the system the coefficients can be calculated. Once the coefficients from the (11) is known one can obtain the nonminimum phase function from

$$\Phi(w) = tan^{-1}[\frac{I(w)}{R(w)}]$$

$$= tan^{-1}[\frac{-\sum_{n=1}^{\infty} a_n \sin(nw + \phi_n)}{a_0 + \sum_{n=1}^{\infty} a_n \cos(nw + \phi_n)}] \quad (12)$$

By utilizing the minimum phase function the real part of the transfer function can be derived from

$$R_{min}(w) = |H(w)|\cos[\phi_{min}(w)] \quad \text{for } 0<w<\pi \qquad (13)$$

Then we obtain the discreate cosine transform to calculate the coefficients

## IV. PHASE SYNTHESIS

Any element of current or charge located in a medium produce magnetic and electric fields. Now the amount of finite energy transmitted to infinity can be term as radiation of the small current element. So the far fields is related to the radiation that fields at infinity. It is important to note that a static charge may generate near fields but the field at infinity due to this charge is zero. Far field of an electromagnetic system is proportional to the current distribution .As all the antennas are of finite spatial size one can say that the spatial current distribution is causal. Because of the causal current distribution the real and imaginary part of the magnetic vector potential is related by the Hilbert transform.So one can obtain the far-field phase pattern from the magnitude of the magnetic vector potentials. This phase is nonminimum phase.

Now any z-directed current distribution can be expressed as

$$J_x(x,y,z)\begin{Bmatrix} x \in [-a;a] \\ z \in [-c;c] \\ y \in [-b;b] \end{Bmatrix} \qquad (14)$$

And the magnetic vector potential is given by

$$A_z = \frac{e^{-jkr}}{4\pi r}\int_{-a}^{a} dx \int_{-b}^{b} dy \int_{-c}^{c} dz\, J_x(x,y,z)\,\hat{z}\,.exp[jkx\ \sin\theta\ \sin\emptyset + jky \sin\theta\ \cos\emptyset + jkz\ \cos\theta] \qquad (15)$$

Where $k = \frac{2\pi}{\lambda}$ and $r$ the spatial far field variable.The Fourier transform of $J_x(x,y,z)$ can be written as$A_z[k\ \sin\theta\ \sin\emptyset;\ k\ \sin\theta\ \cos\emptyset;\ k\ \cos\theta]$. If we restrict the current distribution to the y = 0 plane and also set $\emptyset = 0$, then we have

$$A_z = \frac{e^{-jkr}}{4\pi r}\int_{-a}^{a} dx \int_{-c}^{c} dz J_x(x,y).exp\,[jkz\ \cos\theta] \qquad (16)$$

As the magnetic vector potential is only reponsible for the far field and there is no contribution of the sclar field potential , one can write

$$\mathbf{E}_{\mathbf{far}}(x,y,z) = -jw\mathbf{A} \quad (17)$$

Now we can transform (17) into $E_\theta(\theta)$ and hence we have

$$E_\theta(\theta) = jw\ sin(\theta)\ f_{un}(\theta) \quad (18)$$

Therefore, the real and the imaginary parts of$A_z$are related through the Hilbert transform. In general, we are given the power field patterns and, hence, we transform the field patterns to by utilizing the following transformation.

$$E_\theta(\Omega) = j\sqrt{1-\Omega^2}^{2}\left(\sum_n a_n e^{-j(n\Omega+\phi_n)}\right)w \ (19)$$

Where $\Omega = \cos(\theta)$. The inportance of (19) is that the real an imaginart part of the magnetic vector potential are related by the Hilbert transform but it does not apply to the fileds.

One inportant thing is to note that thefrequency variable $\Omega$ is related with $\cos(\theta)$. So the $E$- field data we get need to interpolate the in $\cos(\theta)$ space i.e $\cos(\theta)$ = -1,……0,*1/N, 2/N, 3/N,…..*1 before one can do the optimization.

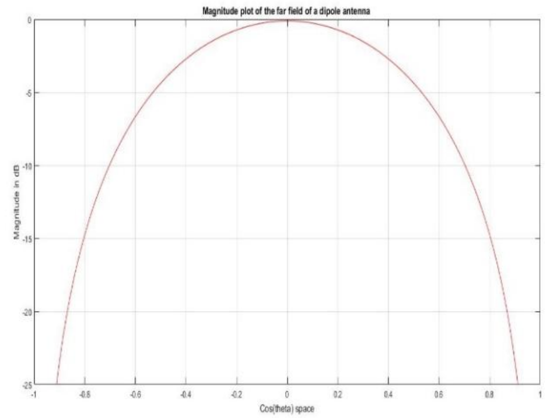


Fig. 1. Magnitude response of the far field of a dipole antenna in $\cos\theta$ space

## V. EXAMPLE

In this first example we have considered a half-wavelength long z-directed transmitting dipole of radius $0.001\lambda$. The magnitude response of the far field is given in Fig. 1.The reconstructed phase obtain from (12) shown in Fig. 2. Both the magnitude plot and reconstructed phase are equispaced in $\cos(\theta)$ space. Thishas been calculated by the MATLAB code with the help of wire antennas and scatterers(AWAS) [3]. As the dipole is centered at the origin, it is not strictly a causal function. So the delay in the time domain accounts for this spatial displacement of the origin.

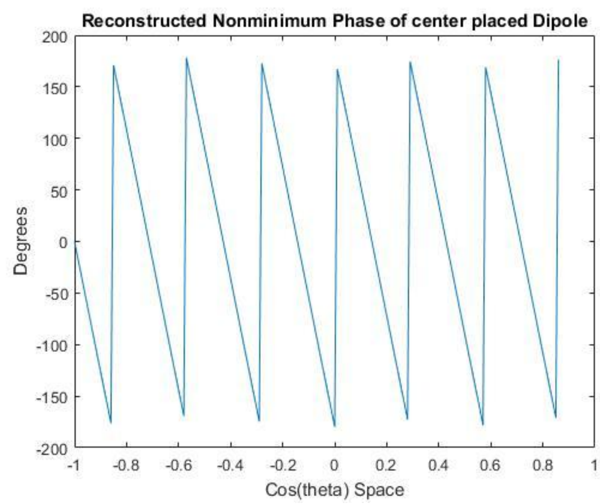


Fig.2. Reconstructed Nonminimum Phase of a center placed dipole

For the second example we have considered two z-directrd dipole with opposite phase. The length of the each dipole is $0.83\lambda$ and radius is $0.001\lambda$. The separation between two dipole is $1.25\lambda$. The magnitude plot is given in Fig. 3 which has been calculated using MATLAB code with the help of AWAS code [3]. The reconstructed phase shown in Fig. 4.

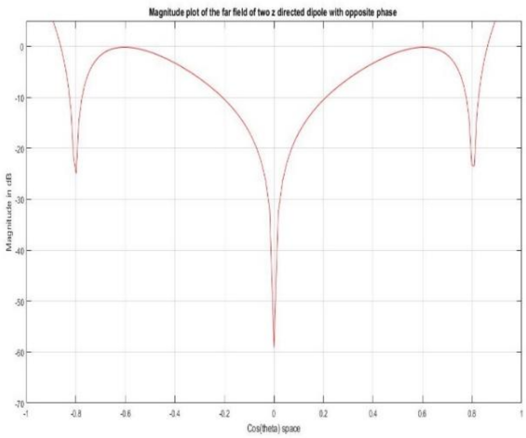


Fig. 3. Magnitude response of the far field of two z-directed dipole antenna with opposite phase in $\cos\theta$ space

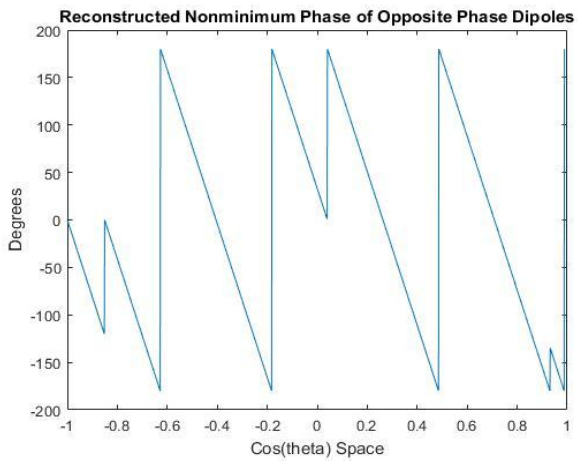


Fig.4. Reconstructed Phase of two z directed dipole with opposite phase

In the third example we have considered three z-directed planner dipole of radius $0.001\lambda$ and length of $0.5\lambda$ each. The dipole are seperated with $0.5\lambda$ distance. The magnitude plot is shown in Fig. 5. The reconstructed phase respone with $\emptyset = 90°$ is given in Fig. 6. This has been calculated using MATLAB code with the help of AWAS code [3]. Both the magnitude plot and phase response are interpolated in $\cos\theta$ space.

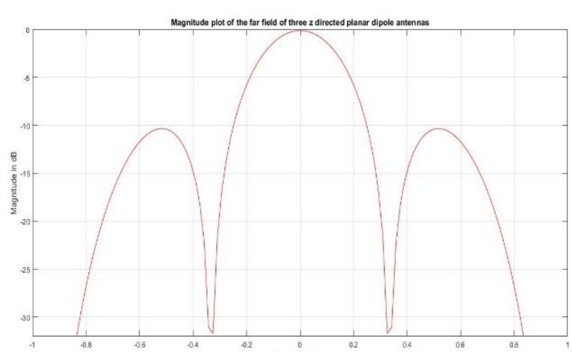


Fig. 5. Magnitude plot of the far field of three z-directed planner dipole

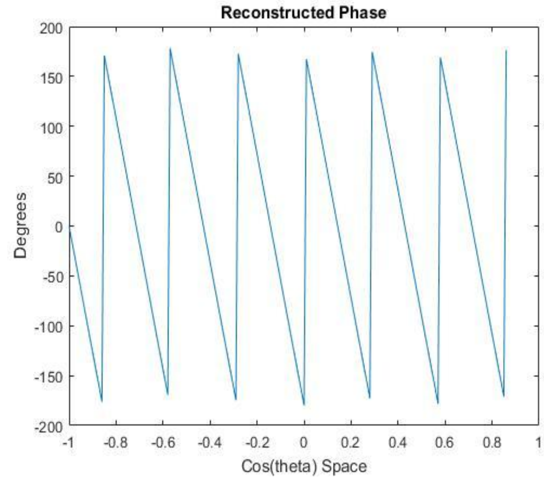


Fig.6. Reconstructed Phase of three z-directed planner dipole antenna

Atlast we test the postulate presented earlier on our measured data. Here the experimental data consist of the $S_{21}$ parameter of a coupled line microwave bandpass filter in the frequency band of 1.85 GHz to 2.15 GHz with a center frequency of 2 GHz. The magnitude response of the $S_{21}$ and $S_{11}$ parameter of the coupled line filer are shown in Fig.7 and Fig. 8 respectively.

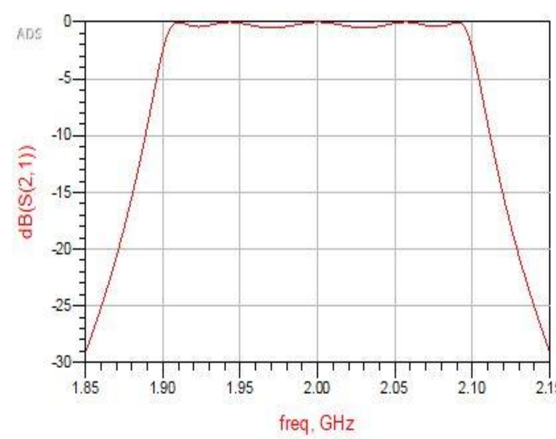


Fig. 7. Magnitude response of $s_{21}$ of a Microwave Filter

The actual phase of the filter is given in Fig. 9. This has been calculated using Advanced Design System(ADS) software. It is very important to note that $S_{21}$ is nonminimum phase function where as $S_{11}$ is minimmum phase function.The measured $S_{21}$ response below the -15 dB is discraded along with the phase. We do not consider $S_{11}$ response for phase reconstruction as it is minimum phase function. The reconstructed phase of the $S_{21}$of the coupled line band pass filter is shown in Fig.9 which has been calculated using MATLAB code with the help of AWAS code [3]. Here one can notice the phase chnges with 360° i.e -180° to +180°.

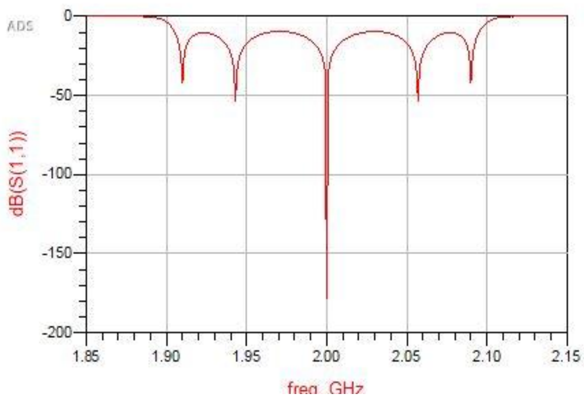


Fig. 8 Magnitude response of $s_{11}$ of a Microwave Filter

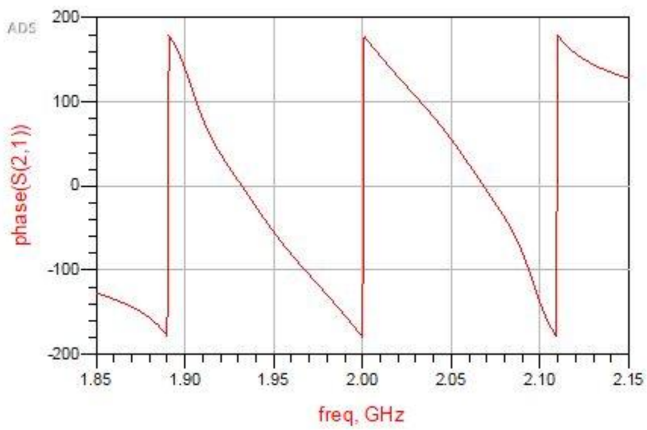


Fig. 9. Actual phase response of a Microwave Filter

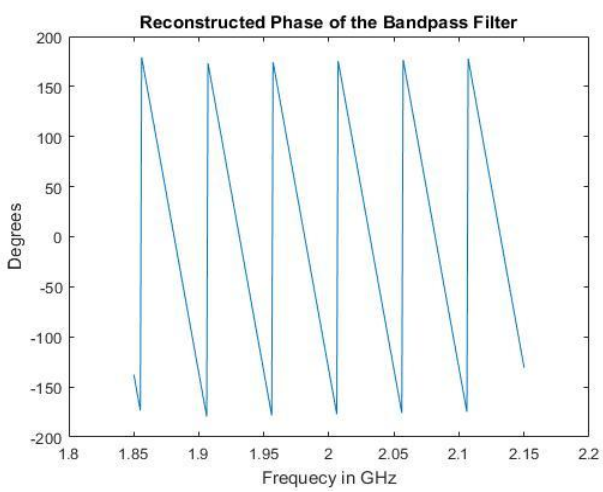


Fig. 10. Reconstucted phase of a Microwave Filter

## VI. CONCLUSION

This paper present a nonparametric method to reconstruct the nonminimum phase of an electromagnetic system. The advantage of this method is that no such choices of the basis functions need to be made, asthe solution procedure itself develops the nature of the solution and no a prioriinformation is necessary. This is done by Hilbert transform which signifies one of the fundamental property of the nature *i.e*casualty. It states that the real and imaginary part of the of any nonminimum-phase transfer function from a causal system satisfy this relationship.

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