Modelling the Impacts of Bed-Nets and Awareness Campaign on Malaria Transmission Dynamics in Hong Local Government Area, Adamawa State Nigeria

[1]Vincent Christopher., [2]Hussaini Buba Gayus., [3]Saminu Iliyasu Bala*

[1]Department of General studies, Adamawa State College of Education Hong, Nigeria,

[2]Department of Adult and none formal Education, Adamawa State College of Education, Hong, Nigeria

[3]Department of Mathematical Sciences, Bayero University Kano

[3]Department of Mathematics, Bauchi State University Gadau

DOI: https://doi.org/10.51244/IJRSI.2025.12030052

Received: 21 February 2025; Review 05 March 2025; Accepted: 07 March 2025; Published: 09 April 2025

ABSTRACT

Malaria is one of the most devastating infectious disease that is transmitted to human through the bite of mosquitoes. In many African countries such as Nigeria, malaria remains a significant threat to life and is impacting negatively on the economy. One area that requires more attention in terms of malaria burden is spreading awareness campaign for people to adopt certain protective measures against the disease. Using ordinary differential equations (ODE), we formulated a modified deterministic mathematical model of malaria transmission dynamics incorporating effects of awareness campaign and bed-net usage. The model was extended to study the effects of environmental variability through incorporation stochastic perturbations into the ODE model. We then formulated and distributed some questionnaires that enabled us to estimate some of the model parameters to suit Hong local government area. We investigated some basic properties of the model to show that the models are physically meaningful. The disease-free equilibrium point of the model is shown to be locally asymptotically stable if a certain threshold quantity, called reproduction number is smaller than unity. The numerical simulation of the models confirmed some of the theoretical analyses conducted. We studied the global sensitivity analysis of the model parameters to identify the parameters that have high impact on the reproduction number. Our results show that level of awareness of malaria in Hong local government area is above average, and about 64% of the people in the area are using bed-nets. We also find that the results of simulations for the deterministic model generally underestimates the corresponding stochastic results. Our sensitivity analysis results show that increasing mosquitoes death rates have more significants impacts on the reproduction number and hence, the disease burden.

Keywords: Awareness level, Environmental, fluctuations, Partial, Rank, Correlation Coefficients, Hong, local, government and Perturbations.

INTRODUCTION

Malaria remains one of the most devastating vector-borne diseases worldwide, claiming hundreds of thousands of lives annually. In Nigeria, malaria is a significant public health concern, responsible for significant morbidity, mortality, and economic burdens. [2, 15, 9, 32, 1]. About 231 million cases of malaria were reported in the year 2017 alone by World Health Organization (WHO), occurring in 106 countries, of which, around 81% occurred in African regions resulting in about 91% death of the infected persons [9]. In 2018, there were 228 million reported cases of malaria globally, of which about 91% were in the African continent [9]. In 2021, WHO reported that African region accounted for almost 95% (234 million) of all malaria cases and 96% (593,000) of all deaths due to the disease. In the same year, Nigeria was reported to have recorded about 68 million cases of malaria with 194 000 deaths, [5, 27, 20]. Over the years, various interventions, such as insecticide-treated bed nets (ITNs) and antimalarial medications, have contributed to reducing malaria transmissions. However, a number of factors complicate malaria control efforts in the malaria endemic regions such as Nigeria. Typical among them are lack of awareness of the disease burden, and attitudes towards the use of ITNs. As reported in [9, 18, 14, 13] awareness plays a critical role in preventing and controlling the transmission of the disease. To this end, raising awareness about malaria transmission, prevention, and treatment is essential for promoting behavioral change, improving health outcomes, and reducing the disease's socioeconomic impacts. When individuals and communities are informed about malaria, they are more likely to adopt protective measures, seek prompt medical attention when symptoms arise, and support malaria control initiatives. Awareness campaigns have been reported to play a significant role in promoting the use of ITNs, which is one of the most effective ways to prevent malaria transmission through reduction in human-mosquitoes contact, see [13] for more details.

Mathematical modeling of infectious diseases has proved to play an important role in understanding the insights of the transmission dynamics and appropriate control strategies of infectious diseases. Sir Ross published the first model in 1911 to demonstrate the development of malaria [33, 26]. Since then, many such models have been developed over the years by many researchers to study disease dynamics. For instance, [26] studies the dynamics of malaria incorporating travelers and uses optimal control in the model analysis, and [19] studies the dynamics of COVID-19 in high and low-risk populations. Some other models of infectious diseases can be found in [29, 3, 6, 8, 25]. Modeling of malaria has helped in understanding the transmission dynamics of the disease, including appropriate control strategies to mitigate it, for instance [25, 12, 11, 17]. There are instances of real-life situations where an awareness campaign was conducted to aid in reducing the menace of malaria. For example, a study conducted by the London School of Hygiene and Tropical Medicine, where it was found that a combination of social media-based awareness campaigns and Optimal control methods represents the most cost-effective approach to managing malaria, a study by the World Health Organization (WHO) found that social media campaigns can increase knowledge of malaria prevention and control measures by up to 40%, see [23] and the references therein. Other instances of awareness campaigns in real life can be found in [24, 28]. Mathematical and other models of disease transmission incorporating awareness have been developed by many researchers, see for instance [33, 9, 5, 23, 18, 14]. From the report in [9], the authors formulated a mathematical model of malaria transmission incorporating awareness assuming the disease transmission rates from vector to human and from human to vector, are decreasing functions of level of awareness. The authors concluded that awareness and proportion of bed-net usage and residual spray should be priorities and be increased to at least 75% for the possible elimination of malaria.

Seasonal variations in climate and mosquito abundance significantly influence malaria transmission patterns. To this end, several studies have focused on incorporating seasonality into mathematical models through the introduction of some key time-dependent parameters or through the formulation of stochastic models, see [10, 4, 30, 23] as examples. These model attributes help in identifying periods of high transmission risk and explaining, environmental variability in disease transmission. Thus, enabling more targeted intervention strategies. In this report, we propose a deterministic and stochastic model of malaria transmissions incorporating awareness and ITN usage. We incorporate a biting function that depends on the proportion of ITN users into the model's biting rate. We developed a questionnaire that was administered to some residents of Hong local government area to assess their level of awareness of malaria transmission and to use the responses from the questionnaire to estimate some parameter values in our model.

MATERIAL AND METHODS

Study Area

Hong is a town and a Local Government Area in Adamawa State, Nigeria. Hong Local Government Area was created in 1991 and is bounded by Mubi to the East, Gombi to the West, Song and Maiha Local Government Areas to the South, and Askira-Uba Local Government Area to the North, see Figure 1 for pictorial description. Hong Local Government Area has a land area of about 117,240 km^2 with a projected population of 226, 100. The area falls within the Sudan Savannah zone and has a tropical wet and dry climate. The dry season lasts for five months (November to March) while the wet season lasts from April to October (7 months). The major occupation of people in the area is farming, few are traders and civil servants. The major crops grown in the area include; groundnut, sorghum, maize, rice, and millet. The dominant and ruling tribe of Hong is the Kilba (Hoba).

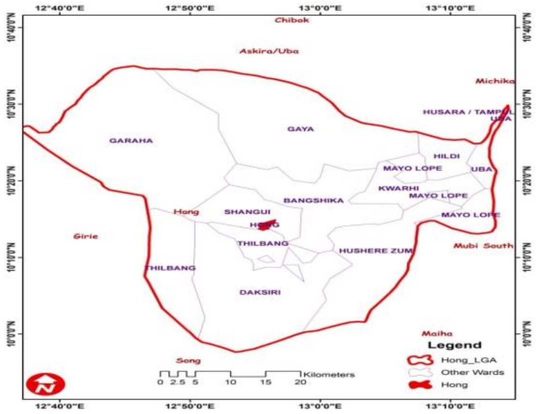

Figure 1: Map showing Hong and its neighbouring towns. Source from

Study Design

This study consist of two parts

1. designing and administering questionnaire to examine the level of awareness and utilization of insecticide-treated bed nets among the people of Hong's local government area as measures for reducing malaria episodes in the area.

2. Formulating a deterministic and a stochastic malaria transmission model of malaria transmission, and using the information obtained to estimate some of the model parameters.

Thus, we concentrated in some selected localities (survey site) in both the northern and southern parts of the local government area. In the northern zone, the survey site includes Munga Kwarhi, and Mijili, while in the southern zone, the survey site includes Hong town, Kala,a, and Pella. Questionnaire was designed to help estimate some of the model parameters. A sample of 357 male and female respondents resident in the Hong Local Government area was selected using a random sampling technique. The designed questionnaire is broken into two for the purpose of this report and presented on Tables 1 and 2. The outputs from the questionnaires will be used to estimate some model parameters that will help in providing a better understanding on malaria transmission in the study area.

Table 1: KEY: SA = Strongly agree, A = Agree, N = Neutral, D = Disagree, SD = Strongly disagree.

Question	SA	A	N	D	SD
1.Malaria is transmitted to human through mosquitoes bite.					
2. Stagnant water are common breeding sites of mosquito.					
3. Headache are some symptoms of malaria.					
4. There is adequate level of awareness and sensitization on the importance of ITN use.					
5. There is adequate level of awareness and sensitization on malaria prevention.					
6.Reducing human-mosquito contact reduces malaria infection.					
7. Use of ITN reduces contacts between human and mosquito.					
8. ITN need to be replaced after sometime.					
9. I believe ITNs are safe to use for everyone.					

10. The government and healthcare providers promote ITNs effectively in my area.					
11. I trust information about malaria prevention provided by health authorities.					
12. Mosquito nets that are not treated with insecticide are equally effective as ITNs in preventing malaria.					
13. ITNs are more effective than using mosquito coils or sprays.					
14. Malaria transmission is a significant health issue in my community.					
15. I would recommend the use of ITNs to friends and family.					
16. I believe more awareness about ITNs and malaria prevention is needed in my area.					

Table 2: Second part of the questionnaire used in estimating some parameters.

Question			
Q17: Do you have ITN now?	Yes	No	
Q18: If answer to 17 is yes, how often do you use it?	Always	Sometimes	Never
Q19: When did you start using it in the last two years?	More than a year ago	Less than a year ago	
Q20: Did you have in the last two years?	Yes	No	
Q21: Why did you stop using it?	I do not think it will prevent malaria	It is not distributed by Government/other organizations	Other reasons

Formulation of the Models

We formulate a deterministic mathematical model to study the impact of awareness on malaria transmission dynamics. We feel that real-world dynamics of diseases are not governed by fixed parameter values due to random variability in the environment. Thus, the deterministic model was later extended into a stochastic model to study the impacts of environmental perturbations through the introduction of white-noise.

Formulation of the deterministic model

In the modeling development, it is assumed that the media campaigns increase the level of awareness regarding self-protection and the methods for reducing the mosquito population. This is captured by the parameter M. Other assumptions includes;

- The dynamics of the transmission is fast enough so that the exposed classes are omitted.
- No vertical transmission in both humans and mosquitoes population.
- All infected individuals who survive, must move to the recovery class.
- Recruitment into both humans and mosquitoes populations are at constant rates.

We used compartmental modelling approach to drive the model equations. The flow diagram, given on Figure 2 is used as an aid to explain model derivation.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue III March 2025

The proposed deterministic model consist of two interacting populations, the human population as hosts and the mosquitoes as vectors. The host population has a total population size $N = S_a + S_u + H_i + R$ grouped into four compartments (classes) namely; susceptible aware class (S_a), which contains susceptible humans who are aware of malaria infection and use preventive measures against the disease, the unaware susceptible humans (S_u), consist of susceptible humans who do not take preventive measures against malaria, the infectious class (H_i), consist of humans who are infected and can infect susceptible mosquitoes and the recovered class (R), containing people that have recovered from malaria and enjoying immunity for some time. The vector population with a total population size given by $N_v = S_v + V_i$, is grouped into two compartments namely; susceptible mosquitoes (S_v,) and infectious mosquitos (V_i.).

It is assumed that all recruitment into the human population either by birth or immigration is through the susceptible unaware class at a rate Λ_h. The human populations experiences natural mortality at a rate μ_h. We assume that individuals in the susceptible aware class experiences reduction in malaria transmission as a result of ITN usage due to awareness. We modelled this using the human-mosquito contact rate $p_1 = \beta_{max} - b(\beta_{max} - \beta_{min}$. This biting rate was formulated by [2], where b is the proportion of ITNs usage, β_{max} and β_{min} represent the maximum and minimum mosquitoes biting rates respectively. The susceptible unaware population have contact rate $f_2 = \beta_{max}$. The force of infection for susceptible unaware and aware humans are modeled as $\lambda_u = \frac{q_1 f_2 V_i}{N}$, $\lambda_a = \frac{q_1 p_1 V_i}{N}$. Here q_1 is the probability of infection from infected mosquitoes to suscebtible humans. We modelled the force of infection from infected human to susceptible mosquitoes as $\lambda_v = \frac{(p_1+f_2)q_2 H_i}{N}$, where q_2 is the probability of infection from infected humans to susceptible mosquitoes. The class S_u increases with the constant growth rate Λ_h, a fraction η of recovered people that loss immunity and join the unaware population at a rate $\sigma\eta R$, and a rate $\theta_2 = \frac{\omega_a}{1+M}$ at which aware people become unaware due to negligience. This class S_u decreases through natural death, rate $\theta_1 = \frac{\omega_u M}{1+M}$ at which unaware people become aware, and through becoming infected at a rate λ_u.

The class S_a increases with a fraction $\xi = 1 - \eta$ of recovered people that loss immunity and join the aware population at a rate $\xi\sigma$, and a rate θ_1 at which unaware people become aware. It decreases through natural death, also a rate θ_2 at which aware people become unaware, and through becoming infected at a rate λ_a. The class H_i increases with the transmission rates λ_u and λ_a. It decreases through natural death, disease induced death rate δ, and recovery at a rate γ. The population of the recovered class R, increases with the number of infectious people that recover at a rate γ. It decreases through natural death, and the number of individuals that looses immunity at a rate σ.

We assume all recruitments into the mosquitoes populations are through the susceptible class S_v at a rate Λ_v. The population of the susceptible mosquitoes decreases through natural death at a rate μ_v, through infectious contacts with infected humans at a rate λ_v, and also throguh death due to destruction of breeding sites/insecticides spray at a rate τM. The infectious mosquitoes population V_i increases at a rate λ_v and decreases through natural death rate and death through insecticide spray and other forms of protections. The flow diagram of the model is given in Figure 2, the description of the state variables and model parameters are given in Tables 2 and 4 respectively. The model equations are given by (1).

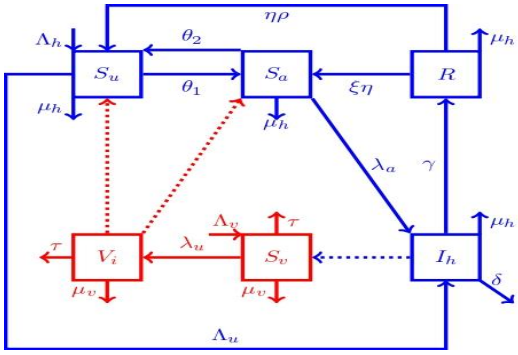

Figure 2: Flow diagram of the model

Table 3: State Variables of the model and their descriptions

Variable	Description
S_u, S_a	Susceptible unaware and aware humans respectively
H_i	Infected humans
R	Recovered humans
S_v	Susceptible mosquitoes
V_i	Infected mosquitoes

Table 4: Model parameters and their descriptions

Parameter	Description
Λ_h, Λ_v	Recruitment rates susceptible unaware human and mosquito populations respectively
ω_a	Rate at which susceptible aware humans become unaware due to negligence
ω_u	Rate at which susceptible unaware humans become aware due to awareness campaign
q_1	Probability of infection from infected mosquitoes the susceptible humans
q_2	Probability of infection from infected human to susceptible mosquitoes
μ_v, μ_h	Natural death rates of mosquito and human populations respectively
σ	Recovery rate for human
η	Proportion of humans that recover into susceptible unaware human
γ	Recovery rate for infected humans
τ	Rate at which mosquitoes are eliminated due to awareness
δ	Disease induced death rate
β_{max}, β_{min}	Maximum and minimum mosquitoes biting rates
M	Level of awareness

$$\dot{S}_u = \eta \;\; \sigma \;\; R + \frac{S_a \omega_a}{1+M} - \frac{S_u q_1 f_2 V_i}{N} - \mu_h S_u - \frac{S_u \omega_u M}{1+M} + \Lambda_h,$$

$$\dot{S}_a = \xi \;\; \sigma \;\; R - \frac{S_a q_1 p_1 V_i}{N} - \mu_h S_a - \frac{S_a \omega_a}{1+M} + \frac{S_u \omega_u M}{1+M},$$

$$\dot{H}_i = \frac{S_a q_1 p_1 V_i}{N} + \frac{S_u q_1 f_2 V_i}{N} - H_i(\gamma + \mu_h + \delta),$$

$$\dot{R} = -R(\sigma + \mu_h) + \gamma \;\; H_i,, \qquad (1)$$

$$\dot{S}_v = -\tau \;\; S_v M - \frac{S_v (p_1 + f_2) q_2 H_i}{N} - \mu_v S_v + \Lambda_v,$$

$$\dot{V}_i = \frac{S_v(p_1+f_2)q_2H_i}{N} - V_i(M\tau + \mu_v)$$

Formulation of the Stochastic Model

Stochastic models could be a more suitable for modeling epidemics in many circumstances. This is because they capture environmental fluctuations which inherent in real-life scenario, [10]. There are many approaches for formulating stochastic models. We follow the approach in [10] by assuming that the environmental stochastic perturbation in our model is a white noise type that is directly proportional to some random pertubations and to each state variable given by a 6-dimensional standard Brownian motion

$B = (\epsilon_1 dW_1, \epsilon_2 dW_2, \epsilon_3 dW_3, \epsilon_4 dW_4, \epsilon_5 dW_5, \epsilon_6 dW_6)^T$ where $\epsilon_i, i = 1 \cdots 6$ are the intensities of the pertubations. We let the stochastic variables be given by $(X_1, X_2, X_3, X_4, X_5, X_6) = (S_u, S_a, H_i, R, S_v, V_i)$. Thus, a stochastic system arising from model (1) is given by (2).

$$\begin{aligned}
dX_1 &= \left(\eta \quad \sigma \quad X_4 + \frac{X_2\omega_a}{1+M} - \frac{X_1q_1f_2X_6}{N_h} - \mu_h X_1 - \frac{X_1\omega_u M}{1+M} + \Lambda_h\right)dt \ +\epsilon_1 X_1 dW_1(t),\\
dX_2 &= \left(\xi \quad \sigma \quad X_4 - \frac{X_2q_1p_1X_6}{N_h} - \mu_h X_2 - \frac{X_2\omega_a}{1+M} + \frac{X_1\omega_u M}{1+M}\right)dt \ +\epsilon_2 X_2 dW_2(t),\\
dX_3 &= \left(\frac{X_2q_1p_1X_6}{N_h} + \frac{X_1q_1f_2X_6}{N_h} - X_3(\gamma + \mu_h + \delta)\right)dt + \epsilon_3 X_3 dW_3(t),\\
dX_4 &= (-X_4(\sigma + \mu_h) + \gamma \quad X_3)dt + \epsilon_4 X_4 dW_4(t), \qquad (2)\\
dX_5 &= \left(-\tau \quad X_5 M - \frac{X_5(p_1+f_2)q_2X_3}{N_h} - \mu_v X_5 + \Lambda_v\right)dt + \epsilon_5 X_5 dW_5(t),\\
dX_6 &= \left(\frac{X_5(p_1+f_2)q_2X_3}{N_h} - X_6(M\tau + \mu_v)\right)dt + \epsilon_6 X_6 dW_6(t).
\end{aligned}$$

It is worthwhile to mention that model (2) differ from model (1) only through the stochastic pertubations. That is, if $\epsilon_i = 0, for \ \ i = 1 \cdots 6$, the two models are the same. Note that model (2) will be used to study the dynamics of environmental variability on malaria transmission dynamics via numerical simulation only.

Non-negativity and boundedness of solution of the dterministic model

In this section, some basic properties of model (1) will be explored.

Theorem 1 *The solution of the equations in model (1) with non-negative initial conditions will remain non-negative for all* $t > 0$.

Proof.

$$t_1 = sup\{t > 0: S_u(t) > 0, S_a(t) \geq 0, H_i(t) \geq 0, R(t) \geq 0, S_v(t) > 0, V_i(t) \geq 0\}.$$

From the first equation of model (1) we have

$$\frac{S_u}{dt} \geq \Lambda_h - \omega_u \quad S_u - \frac{q_1f_2H_vS_u}{N} - \mu_h S_u = \Lambda_h - S_u y(t),$$

where $y(t) = -\left(\omega_u + \frac{q_1f_2H_v}{N} + \mu_h\right)$. By using integrating factor, it can be shown that

$$S_u(t_1) \geq S_u(0)\exp\left(-\int_0^{t_1} y(u)du\right)$$

$$+\exp\left(-\int_0^{t_1} y(u)du\right) \times \int_0^{t_1} \Lambda_h \exp\left(-\int_0^{x} y(u)du\right)dx > 0.$$

It also follows from the 5th equation of model (1) that

$$S_v(t_1) \geq S_v(0)\exp\left(-\int_0^{t_1} y_1(u)du\right)$$

$$+\exp\left(-\int_0^{t_1} y_1(u)du\right) \times \int_0^{t_1} \Lambda_v \exp\left(-\int_0^{x} y_1(u)du\right)dx > 0,$$

where $y_1(t) = -\left(\frac{(q_1+f_2)q_2H_i}{N} + \mu_v + \tau M\right)$. Using similar arguments it can be shown that the solutions for all other state variables are non-negative for all non-negative initial conditions, as required.

Theorem 2 *The closed set*

$$\Omega = \left\{(S_u, S_a, H_i, R, S_v, V_i) \in \mathbb{R}_+^6 : S_u + S_a + H_i + R \leq \frac{\Lambda_h}{\mu_h}, S_v + V_i \leq \frac{\lambda_v}{\mu_v}\right\}$$

is positively-invariant and attracting for the model (1).

Proof. By adding the first 4 equations of model (1) and the last two equations respectively have we

$$\dot{N} \leq \Lambda_h - \mu_h N,$$

$$\dot{N}_v \leq \Lambda_v - \mu_v N_v.$$

It then follows that $N \leq \frac{\Lambda_h}{\mu_h}$ *and* $N_v \leq \frac{\Lambda_v}{\mu_v}$. Hence, the total human and mosquitoes populations are bounded. The rest of the proof follows from [16]

In what follows, we consider the following parameter groupings; $p_2 = \gamma + \mu_h + \delta$, $p_3 = \sigma + \mu_h$, and $p_4 = M\tau + \mu_v$.

Theorem 3 *Model (1) has a unique solution in Ω.*

Proof. To prove Theorem 3, we evaluated the norm of the Jacobian matrix of model (1) as

$$\| J \|_\infty = \max\Bigg(\gamma + |p_3|, \left|\frac{S_v(f_2+p_1)q_2}{N}\right| + \left|M\tau + \frac{(f_2+p_1)q_2H_i}{N} + \mu_v\right|,$$

$$\left|\frac{S_v(f_2+p_1)q_2}{N}\right| + \left|\frac{(f_2+p_1)q_2H_i}{N}\right| + |p_4|, |\omega_u| + \left|\frac{q_1p_1V_i}{N} + \mu_h + \omega_a\right|$$

$$+|\xi \quad \sigma| + \left|\frac{S_aq_1p_1}{N}\right|, \left|\frac{q_1f_2V_i}{N}\right| + \left|\frac{q_1p_1V_i}{N}\right| + |p_2| + \left|\frac{S_aq_1p_1}{N} + \frac{S_uq_1f_2}{N}\right|,$$

$$\left|\frac{q_1f_2V_i}{N} + \mu_h + \omega_u\right| + |\omega_a| + |\eta \quad \sigma| + \left|\frac{S_uq_1f_2}{N}\right|\Bigg)$$

Since the state variables are bounded, we let $|\frac{S_v}{N}| \leq L_1$ and $|\frac{V_i}{N}| \leq L_2$, then

$$\| J \| \leq \max(\gamma + |p_3|, |L_1(f_2 + p_1)q_2| + |M\tau + (f_2 + p_1)q_2 + \mu_v|,$$

$$|L_1(f_2 + p_1)q_2| + |(f_2 + p_1)q_2| + |p_4|, |\omega_u| + |q_1p_1L_2 + \mu_h + \omega_a|$$

$$+|\xi \quad \sigma| + |q_1p_1|, |q_1f_2L_2| + |q_1p_1L_2| + |p_2| + |q_1f_2 + q_1p_1|,$$

$$|q_1f_2L_2 + \mu_h + \omega_u| + |\omega_a| + |\eta \quad \sigma| + |q_1f_2|) = L,$$

where L is a Lipschitz constant. Thus, by Lemma 2.2 and Theorem 2.4 of [22], model (1) has a unique solution in Ω.

Disease Free Equilibrium Point and its Stability

Model (1)has a Disease Free Equilibrium Point (DFE) given by

$$E_0 = (S_a^0, S_u^0, H_i^0, R^0, S_v^0, V_i^0) = \left(\frac{\Lambda_h \omega_u}{\mu_h(\omega_u+\omega_a+\mu_h)}, \frac{(\omega_a+\mu_h)\Lambda_h}{\mu_h(\omega_u+\omega_a+\mu_h)}, 0, 0, \frac{\Lambda_v}{M\tau+\mu_v}, 0\right).$$

Reproduction Number

The disease classes are H_i, and V_i, it then follows that F and V matrices for the new infection and transfer terms are given by

$$F = \begin{bmatrix} 0 & \frac{S_a^0\ \ q_1 p_1 \mu_h}{\Lambda_h} + \frac{S_u^0\ \ q_1 f_2 \mu_h}{\Lambda_h} \\ \frac{S_v^0\ \ (f_2+p_1) q_2 \mu_h}{\Lambda_h} & 0 \end{bmatrix}, \tag{3}$$

$$V = \begin{bmatrix} p_2 & 0 \\ 0 & p_4 \end{bmatrix} \tag{4}$$

respectively. The eigenvalues of FV^{-1} are

$$\rho(FV^{-1}) = \begin{bmatrix} \sqrt{\frac{S_v^0\ \ q_1 q_2 (f_2+p_1)(S_a^0\ \ p_1 + S_u^0\ \ f_2)\mu_h{}^2}{p_2 p_4 \Lambda_h{}^2}} \\ -\sqrt{\frac{S_v^0\ \ q_1 q_2 (f_2+p_1)(S_a^0\ \ p_1 + S_u^0\ \ f_2)\mu_h{}^2}{p_2 p_4 \Lambda_h{}^2}} \end{bmatrix}. \tag{5}$$

We define the reproduction number as

$$R_0 = \sqrt{\frac{S_v^0\ \ q_1 q_2 (f_2+p_1)(S_a^0\ \ p_1 + S_u^0\ \ f_2)\mu_h{}^2}{p_2 p_4 \Lambda_h{}^2}}. \tag{6}$$

Theorem 4 *The disease free equilibrium point of model (1) is locally asymptotically stable if* $R_0 < 1$ *and unstable otherwise.*

Proof. We begin the proof by evaluating the Jacobian of model (1) at the DFE to get

$$J = \begin{bmatrix} -\mu_h - \omega_u & \omega_a & 0 & \eta\ \ \sigma & 0 & -\frac{su\ \ q_1 f_2 \mu_h}{\Lambda_h} \\ \omega_u & -\mu_h - \omega_a & 0 & \xi\ \ \sigma & 0 & -\frac{sa\ \ q_1 p_1 \mu_h}{\Lambda_h} \\ 0 & 0 & -p_2 & 0 & 0 & \frac{sa\ \ q_1 p_1 \mu_h}{\Lambda_h} + \frac{su\ \ q_1 f_2 \mu_h}{\Lambda_h} \\ 0 & 0 & \gamma & -p_3 & 0 & 0 \\ 0 & 0 & -\frac{sv\ \ (f_2+p_1) q_2 \mu_h}{\Lambda_h} & 0 & -M\tau - \mu_v & 0 \\ 0 & 0 & \frac{sv\ \ (f_2+p_1) q_2 \mu_h}{\Lambda_h} & 0 & 0 & -p_4 \end{bmatrix}. \tag{7}$$

Four eigenvalues of the Jacobian matrix (7) are $Y = -p_3$, $Y = -\mu_h$, $Y = -(\omega_u + \omega_a + \mu_h)$, $Y = -(\tau M + \mu_v)$. The remaining eigenvalues are the roots of the polynomial

$$B = \Lambda_h{}^2\left(Y^2 + (p_2 + p_4)\right)Y - (f_2 + p_1)(S_a^0\ \ p_1 + S_u^0\ \ f_2)S_v^0\ \ \mu_h{}^2 q_2 q_1 + \Lambda_h{}^2 p_2 p_4. \tag{8}$$

We simplify the terms in the polynomial (8) to get

$$B = {\Lambda_h}^2\left(Y^2 + (p_2 + p_4)\right)Y + {\Lambda_h}^2 p_2 p_4 (1 - R_0^2). \qquad (9)$$

By Descarte's rule of sign, the roots of the polynomial in (9) have negative real parts if $R_0 < 1$. Hence, the proof.

Parameters estimation and Numerical Simulations

In view of the importance of sample size in research, we used the Fisher's formula $sample\ size = \frac{(z-score)^2\sigma_1(1-\sigma_1)}{cf^2} = 369$, at 95% confidence interval ($z - score = 1.6,$) standard deviation $\sigma_1 = 0.6$, confidence interval $cf = 0.05$ see [31] and the references therein for more details. In the work reported in [28], the authors conducted their research using sample population size of 400 in a much larger setting than we are considering here. Thus, a sample population size of 357 is quite reasonable for our work.

How do we quantify level of awareness, rate at which aware people become unaware using questionnaire responses? In this work, we adopt the following simple approach. The responses to the questions (supplied by the respondents) in Table 1 are assigned some weights as shown and summarised on Table 5, the frequency of the options and their weighted values are also shown. Using likert-scale analysis, see [21], the mean weight of the options (sentiment score) is 3.32. Thus, we approximated the parameter $M = 3.32$, and this value will be used in both the deterministic and the stochastic model simulations.

Table 5: Summary of the responses to the questions in Table 1 and the weight assigned to the options.

Response option	Weight	Frequency	Weighted value
Strongly disagree	1	531	531
Disagree	2	571	1142
Neutral	3	371	1113
Agree	4	989	3956
Strongly agree	5	868	4340
Sum		3330	11082

From the responses to the questions on Table 2, we find that 234 people possess ITN and 129 of them are using it always and the remaining are using it sometimes. To estimate b, the proportion of ITN users, we assigned weight 1 to those using ITN always and 0.5 for others. Thus we estimated $b = \frac{129+0.5\times105}{357} = 0.51$. We also find that the number of people that have started using ITN recently (less than a year ago) are 56. We use this information to estimate the rate at which unaware individual become aware per day as $\omega_u = \frac{56}{357\times365} = 0.00043\ day^{-1}$. In similar manner, we estimated the rate at which aware become unaware using the fact that 129 of the people who are previously using ITNs have stopped using it completely and 105 are using it partially. Thus we estimated $\omega_a = \frac{129+0.5\times105}{357\times365} = 0.00119\ day^{-1}$.

We obtained the values of the other parameters from the literature to get the baseline values given on Table 6. We then used the baseline values to estimate the parameter ranges by reducing and increasing the baseline values by 65% to get the lower and the upper bounds respectively as shown on the Table, see [8] for more details. These values are then used in the our simulations and sensitivity analysis.

Table 6: Parameter values used in the simulations and sensitivity analysis

Parameter	Baseline Value	Source	Parameter Range
Λ_h	90	assumed	[45,135]
Λ_v	1000	[15]	[650,1350]

ω_a	0.001186	estimated	[0.000593,0.001779]
ω_u	0.00043		
	estimated	[0.00021,0.00064]	
q_1	0.01	[5]	[0.005,0.015]
q_2	0.83	[7]	[0.415,1]
μ_h	4.63×10^{-5}	estimated	$[2.31481,6.9444] \times 10^{-5}$
μ_v	0.01	[23]	[0.005556,0.01667]
σ	0.037	assumed	[0.1873,0.5618]
η	0.7	assumed	[0.35,1]
γ	0.012	[5]	[0.0006,0.0018]
τ	0.003	[23]	[0.0015,0.0045]
δ	0.00035	[23]	[0.00415,0.01245]
β_{max}	0.6334	[8]	[0.3167,0.9501]
β_{min}	0.0696	[8]	[0.0348,0.1044]
M	3.33	estimated	[1.665,4.995]
b	0.645	estimated	[0.255,0.765]

RESULTS AND DISCUSSION

Results

We conducted numerical simulations using the parameter values shown on Table 6. The results of such simulations are now described.

Figure 3 depicts the simulation results for the deterministic and stochastic models for susceptible unaware humans using 4 different initial conditions and the intensities of fluctuations as shown. It is clear that the population continue to grow with time for both the stochastic and the deterministic simulations. It can also be observed that there is no much difference between the stochastic and the deterministic simulations.

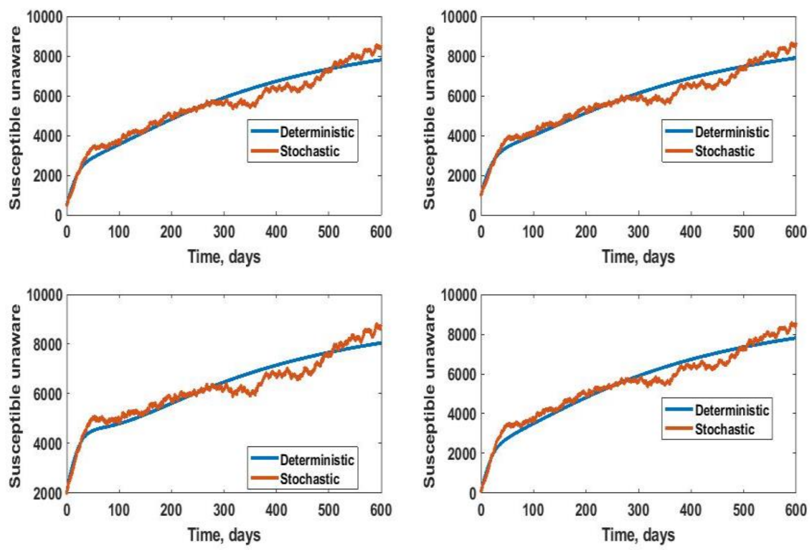

Figure 3: Simulation showing unaware humans population using 4 different initial conditions

$(S_u, S_a, H_i, R, S_v, V_i) = (500,350,200,20,1200,1350), (1000,650,600,100,2000,1800), (2000,900,480,200,3000,500), (100,700,100,78,1400,2000)$ and stochastic intensities $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_5, \epsilon_6 = 50, \epsilon_4 = 0.5$,

Figure 4 depicts both results for deterministic and stochastic simulations for susceptible aware humans, using the four different initial conditions and the same values of stochastic perturbations used in Figure 3. The results show variations between the number of individuals in the stochastic and the deterministic simulations. Since the stochastic fluctuations are introduced to mimic environmental variability, one can say that environmental forcing play key role in population dynamics.

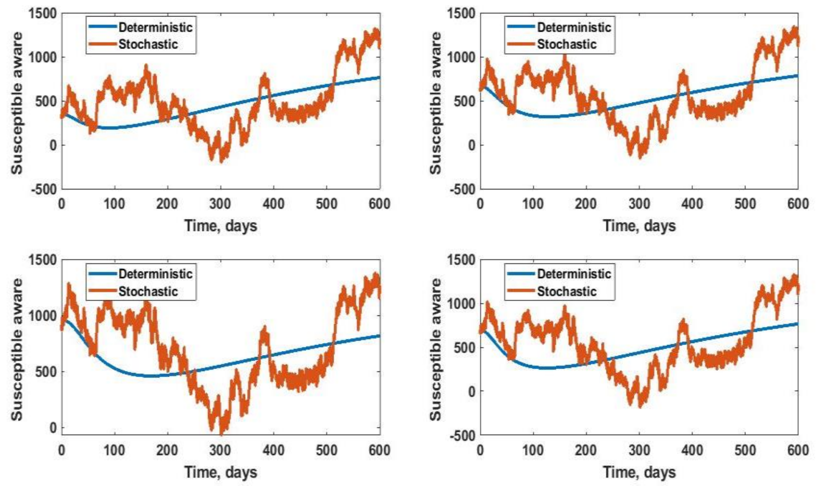

Figure 4: Simulation results for models (1) and (2) for susceptible aware humans.

Figure 5 depicts both results for deterministic and stochastic simulations for infected humans, using 4 different initial conditions and the same values of stochastic perturbations used in Figure 3. One may say that the results on this Figure show similar trends to those observed in Figure 3.

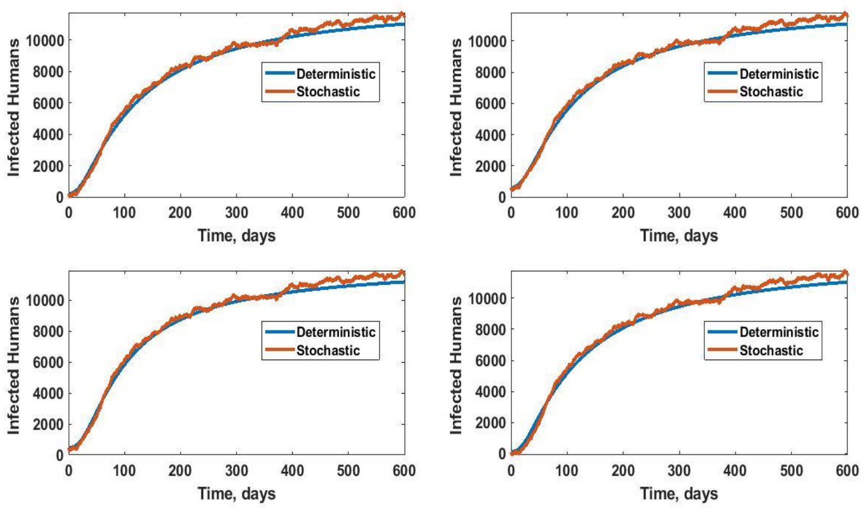

Figure 5: Simulation of the models showing the dynamics of the infected humans under different initial conditions.

Figure 6 depicts both results for deterministic and stochastic simulations for recovered humans, using 4 different initial conditions and the same values of stochastic perturbations used in Figure 3. One noticeable feature of the results is that the number of individuals in the recovered class will reaches zero at some time, then rises again. The time in which the number of individuals reaches zero probably corresponds to the time in which the number of susceptible mosquitoes is at peak as shown on Figure 7.

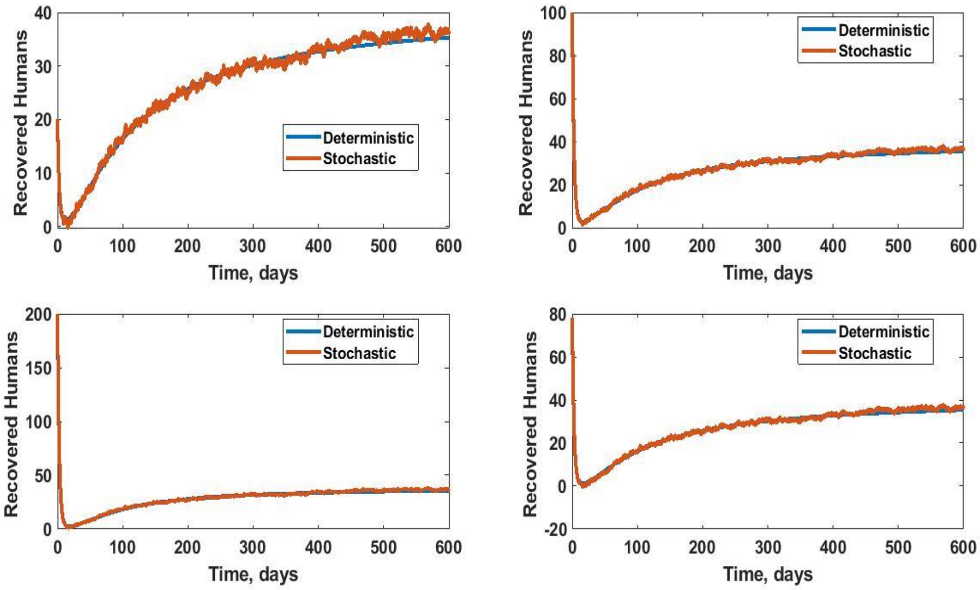

Figure 6: Simulation of the models showing the dynamics of the recovered humans under different initial conditions.

Figure 7 depicts both results for deterministic and stochastic simulations for susceptible mosquitoes using 4 different initial conditions and the same values of stochastic perturbations used in Figure 3. The results on this Figure shows that under the given conditions, the number of susceptible mosquitoes reaches a peak value then falls and stabilize after sometime. Good agreement is also exhibited between the stochastic and the deterministic results.

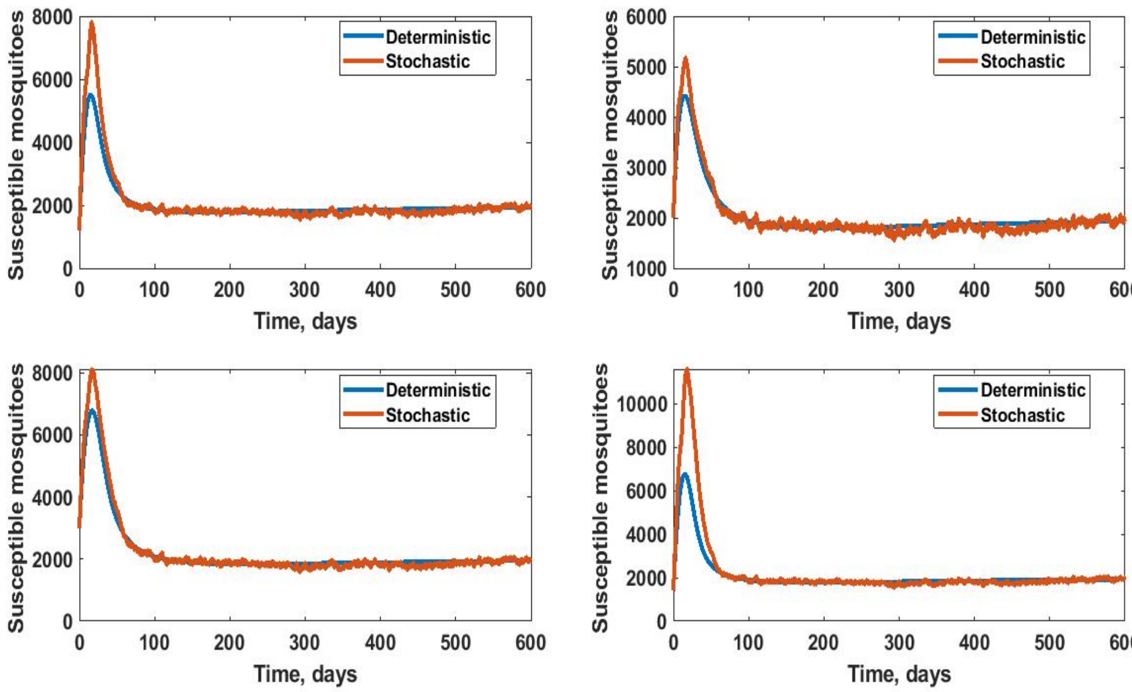

Figure 7: Simulation of the models showing the dynamics of the susceptible mosquitoes under different initial conditions.

Figure 8 depicts both results for deterministic and stochastic simulations for susceptible infected mosquitoes, using 4 different initial conditions and the same values of stochastic perturbations used in Figure 3. One may say that the results on this Figure are similar to those depicted for infected humans.

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue III March 2025

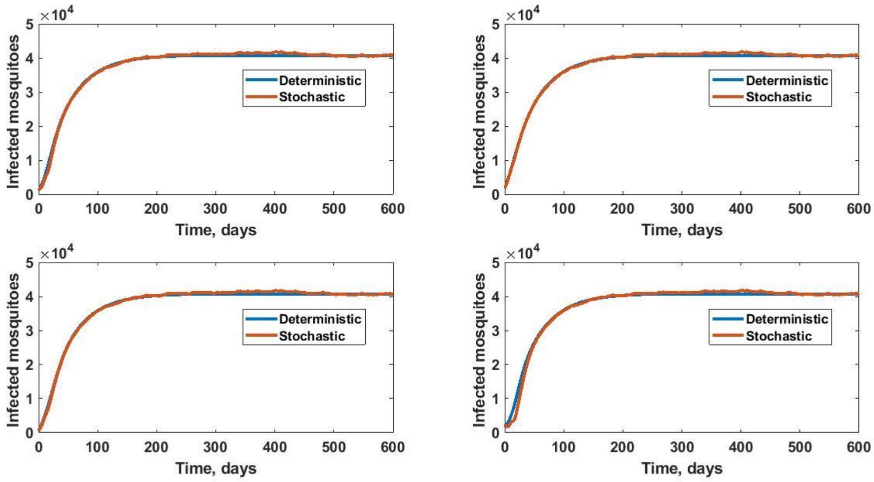

Figure 8: Simulation of the models showing the dynamics of the infected mosquitoes under different initial conditions.

Uncertainties in model parameters always exists. These might be due to wrong inputs/outputs, inaccurate measurements, environmental fluctuations and so on. To quantify the effects of uncertainties on the parameters in R_0, we conducted a global sensitivity analysis by taking 1000 samples from each parameter range shown on Table 6 that appear in R_0 using Latin Hypercube Sampling (LHS). This corresponds to sample size of 1000×15 matrix where, each row corresponds to unique parameter set. A partial Rank Correlation Coefficients (PRCC) of the parameters was calculated and tornado plot of the results is presented on Figure 9.

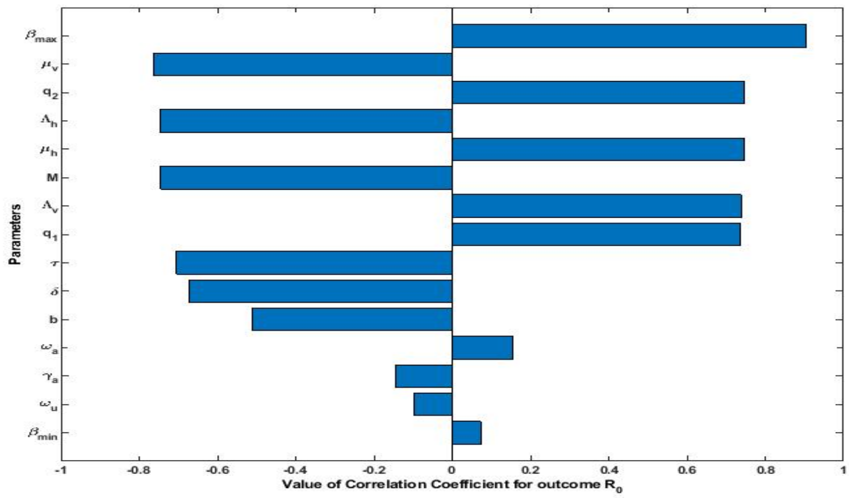

Figure 9: Sensitivity analysis of the model parameters, showing the results for partial rank correlation coefficient

The top 5 parameters in terms of sensitivities are β_{max}, μ_v. q_2, Λ_h, μ_h, and M. The implication of the PRCC results is that increasing the level of awareness, death of mosquitoes, using ITNs, as well as increasing the rate at which susceptible unaware becomes aware will reduce the malaria burden through reduction in R_0. On the other hand, decreasing the recruitment rate into mosquito population, decreasing probabilities of infections given that there are contacts, will reduce malaria transmission in the area.

DISCUSSION

In this work, we formulated a deterministic and stochastic model of malaria models to study the impacts of awareness and bed-nest usage in Hong. We estimated some parameter values for the simulation of the models. The results for the aware humans clearly indicate some differences between the stochastic and the deterministic models. One can say that the outputs from the deterministic model underestimate the corresponding outputs of the stochastic model. In other words, there could be a huge difference between predictions using models with fixed parameter values and those with random parameter values. Fluctuations in output can be used to explain the impacts of environmental factors such as temperature and rainfall.

From the analysis of the questionnaire using a five-point likert scale, we find a weighted mean value on a scale of 5, for Hong local government. This mean value suggests that the inhabitants of the local government agree that an average level of awareness about malaria is available. This finding is important but should be used with caution because it may depend on the structure of the questionnaire and the weights assigned to the options. Nevertheless, our method hashas provided a way in which the level of awareness, which is a non-numerical value, can be quantified. Our sensitivity analysis results have confirmed that increasing the level of awareness and bed-net usage can lead decrease in the menace of malaria through the reduction in the value of the threshold parameter Thus, we suggest that policymakers in Hong local government should strive harder to provide more awareness on malaria to the inhabitants of the area. The local government authorities and other concerned citizens should put emphasis on awareness programs that are vital for malaria prevention, early symptom recognition, timely treatment, and empowering families to take proactive measures. The appearance of the recruitment rate into the human population as one of the parameters with high sensitivity in the model output requires further attention. It will be interesting to devise a means of estimating of estimating the value of this parameter in the area using real-life data.

One potential drawback in our current formulation is that we cannot study the impacts of environmental factors such as temperature and rainfall individually in the current state of the models. This is because the variability introduced into the stochastic model is designed to capture general environmental variabilities rather than a specific one. Another drawback that can be noticed is the constant recruitment used in the models. Although such usage is common in Mathematical models of disease dynamics, it only serves as a trade-off between mathematical tractability and reality. To this end, robust stochastic modeling of malaria transmission dynamics in Hong local government and beyond is required. The model may be formulated to incorporate;

1. Demographic data of the area to provide an enhanced estimate of the recruitment rate into the population of the area.

2. Malaria drug treatment.

3. Impacts of stagnant water on malaria dynamics in the area.

CONCLUSION

In this study, we formulated a deterministic and a stochastic mathematical models to study the impacts of awareness and bed-net usage to mitigate the impacts of malaria transmission in Hong local government of Adamawa state, Nigeria. We administered 357 questionnaires to some inhabitants of Hong in other to assess the level of awareness of malaria as a public heath problem, and ownership and usage of bed-nets. We used five point likert scale to analyse the responses from the questionnaire. We estimated the values of some of the model parameters from the questionnaire responses. We find that the level of awareness in the area is good but requires improvement and that about 51% of the people in the area are using bed-nets in the night to mitigate the menace of malaria transmission. Numerical simulations of our indicates that rising awareness level can lead to increase in destruction of mosquitoes breeding sites or killing them leading to significant reduction in the number of mosquitoes and hence, reduction in malaria transmission. We believe that estimating model parameters through administration of questionnaires is crucial for generating accurate predictions and designing effective control strategies for malaria transmission. Our study confirms that the dynamics of environmental variability play a significant role in shaping malaria dynamics and that malaria transmission is largely influenced by the basic reproduction number R_0. Our investigations further reveal that for malaria transmission to be effectively managed in Hong, awareness level through social media and other channels should be raised to higher level than it is now. This might lead to more people to use bed-nets, and to reduce the availability of mosquito breeding sites in Hong local government. Finally we hope that our methodology for parameter estimation can be adopted/improved by other researchers in their various area of research.

Funding

This research work is funded by TETFUND, Grant number

TETF/DR&D/COE/HONG/IBR/2024/VOL.I

Declaration of competing interest

The author affirms that there are no conflicts of interest concerning the research, authorship, or publication of this paper.

REFERENCES

1. Ayodeji M Adebayo, Oluwaseun O Akinyemi, and Eniola O Cadmus. Knowledge of malaria prevention among pregnant women and female caregivers of under-five children in rural southwest nigeria. PeerJ, 3:e792, 2015.
2. Folashade B Agusto, Sara Y Del Valle, Kbenesh W Blayneh, Calistus N Ngonghala, Maria J Goncalves, Nianpeng Li, Ruijun Zhao, and Hongfei Gong. The impact of bed-net use on malaria prevalence. Journal of theoretical biology, 320:58–65, 2013.
3. Idris Ahmed, Goni Umar Modu, Abdullahi Yusuf, Poom Kumam, and Ibrahim Yusuf. A mathematical model of coronavirus disease (covid-19) containing asymptomatic and symptomatic classes. Results in physics, 21:103776, 2021.
4. Sonia Altizer, Andrew Dobson, Parviez Hosseini, Peter Hudson, Mercedes Pascual, and Pejman Rohani. Seasonality and the dynamics of infectious diseases. Ecology letters, 9(4):467–484, 2006.
5. Fahad Al Basir and Teklebirhan Abraha. Mathematical modelling and optimal control of malaria using awareness-based interventions. Mathematics, 11(7):1687, 2023.
6. Isa Abdullahi Baba and Evren Hincal. Global stability analysis of two-strain epidemic model with bilinear and non-monotone incidence rates. The European Physical Journal Plus, 132(5):208, 2017.
7. Saminu Iliyasu Bala and Bello Gimba. Modeling the impacts of income inequality on malaria transmission dynamics. Int. J. Eng. Res. Appl., 12(2):31–46, 2022.
8. Saminu Bala and Bello Gimba. Global sensitivity analysis to study the impacts of bed-nets, drug treatment, and their efficacies on a two-strain malaria model. Mathematical and Computational Applications, 24(1):32, 2019.
9. Fahad Al Basir, Arnab Banerjee, and Santanu Ray. Exploring the effects of awareness and time delay in controlling malaria disease propagation. International Journal of Nonlinear Sciences and Numerical Simulation, 22(6):665–683, 2021.
10. Y Cai, Y Kang, Weiming Wang, and M Zhao. A stochastic differential equation sirs epidemic model with ratio dependent incidence rate. In Abstr. Appl. Anal, pages 1–11, 2013.
11. Nakul Chitnis, Jim M Cushing, and JM Hyman. Bifurcation analysis of a mathematical model for malaria transmission. SIAM Journal on Applied Mathematics, 67(1):24–45, 2006.
12. Stefan Edlund, Matthew Davis, Judith V Douglas, Arik Kershenbaum, Narongrit Waraporn, Justin Lessler, and James H Kaufman. A global model of malaria climate sensitivity: comparing malaria response to historic climate data based on simulation and officially reported malaria incidence. Malaria journal, 11(1):1, 2012.
13. Stanley Eneh, Francisca Onukansi, Ogechi Ikhuoria, and Temitope Ojo. Designing and deploying caller tunes on mobile phones to promote malaria vaccine uptake in africa: can the technology acceptance model (tam) help? Malaria Journal, 23(1):325, 2024.
14. Iffatricia Haura Febiriana, Dipo Aldila, Bevina Desjwiandra Handari, Puji Budi Setia Asih, and Muhamad Hifzhudin Noor Aziz. Exploring the interplay between social awareness and the use of bed nets in a malaria control program. Journal of Biosafety and Biosecurity, 6(3):196–210, 2024.
15. Farinaz Forouzannia and Abba Gumel. Dynamics of an age-structured two-strain model for malaria transmission. Applied Mathematics and Computation, 250:860–886, 2015.
16. Salisu Mohammed Garba, Abba B Gumel, and MR Abu Bakar. Backward bifurcations in dengue transmission dynamics. Mathematical biosciences, 215(1):11–25, 2008.
17. Bello Gimba and Saminu Iliyasu Bala. Modeling the impact of bed-net use and treatment on malaria

transmission dynamics. International Scholarly Research Notices, 2017, 2017.

18. Robertus Dole Guntur, Jonathan Kingsley, and Fakir M Amirul Islam. Malaria awareness of adults in high, moderate and low transmission settings: A cross-sectional study in rural east nusa tenggara province, indonesia. PLoS One, 16(11):e0259950, 2021.
19. Alhassan Ibrahim, Usa Wannasingha Humphries, Amir Khan, Saminu Iliyasu Bala, Isa Abdullahi Baba, and Fathalla A Rihan. Covid-19 model with high-and low-risk susceptible population incorporating the effect of vaccines. Vaccines, 11(1):3, 2022.
20. Isaac Isiko, Simon Nyegenye, Aaron Mwesigwa, Jackson Micheal Asingwire, Haron Olot, Shekina-Rhoda Chioma Amaka, Lenz Nwachinemere Okoro, and Praise Amarachi Amaka Etane. Determinants of malaria spread among under-five children in nigeria: results from a 2021 nigerian malaria indicator cross-sectional survey. BMC pediatrics, 24(1):646, 2024.
21. Ankur Joshi, Saket Kale, Satish Chandel, and D Kumar Pal. Likert scale: Explored and explained. British journal of applied science & technology, 7(4):396, 2015.
22. H.K. Khalil. Nonlinear Systems. Prentice Hall, 2002.
23. Francis Oketch Ochieng. Seirs model for malaria transmission dynamics incorporating seasonality and awareness campaign. Infectious Disease Modelling, 9(1):84–102, 2024.
24. Mary Isioma Ofili and Bartholomew Chukwuebuka Nwogueze. Level of awareness and utilization of insecticide-treated bed nets among medical students as measures for reducing malaria episodes. Scientific Reports, 14(1):10156, 2024.
25. Mayowa M Ojo and Emile Franc Doungmo Goufo. Assessing the impact of control interventions and awareness on malaria: a mathematical modeling approach. Commun. Math. Biol. Neurosci., 2021:Article–ID, 2021.
26. S Olaniyi, KO Okosun, SO Adesanya, and RS Lebelo. Modelling malaria dynamics with partial immunity and protected travellers: optimal control and cost-effectiveness analysis. Journal of Biological Dynamics, 14(1):90–115, 2020.
27. Jeremiah Oluwamayowa Omojuyigbe, Adedoyin John-Joy Owolade, Taiwo Oluwaseun Sokunbi, Habib Ademola Bakenne, Blessing Abraham Ogungbe, Habeebullah Jayeola Oladipo, and Prosper Ifunanya Agughalam. Malaria eradication in nigeria: State of the nation and priorities for action. Journal of Medicine, Surgery, and Public Health, 1:100024, 2023.
28. Adetunji Omonijo and Adejumoke O Omonijo. Assessment of the status of awareness, ownership, and usage of long-lasting insecticide treated nets after mass distribution in ekiti state, nigeria. Journal of parasitology research, 2019(1):1273714, 2019.
29. Muhammad Ozair, Abid Ali Lashari, Il Hyo Jung, Young Il Seo, and Byul Nim Kim. Stability analysis of a vector-borne disease with variable human population. In Abstract and Applied Analysis, volume 2013, page 293293. Wiley Online Library, 2013.
30. Robert C Reiner, Matthew Geary, Peter M Atkinson, David L Smith, and Peter W Gething. Seasonality of plasmodium falciparum transmission: a systematic review. Malaria journal, 14(1):1, 2015.
31. Stanley K Waithaka, ENM Njagi, JJN Ngeranwa, DM Mwangi, BM Chiuri, LJ Njagi, and WK Gatua. Quantitative reference ranges for fasting profiles and oral glucose tolerance test for healthy adults in metropolitan region of nairobi, kenya. International Journal of Health Research, 3(1):13–19, 2010.
32. Switzerland WHO: Geneva. The potential impact of health service disruptions on the burden of malaria: A modelling analysis for countries in sub-saharan africa. 2020.