INTERNATIONAL JOURNAL OF RESEARCH AND SCIENTIFIC INNOVATION (IJRSI)
ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue II February 2025

On Propagation Sequence

Leonard Karshima Shilgba

Department of Computing Sciences, Faculty of Science, Admiralty University of Nigeria, Ibusa, Delta State. Nigeria

DOI: https://doi.org/10.51244/IJRSI.2025.12020088

Received: 08 February 2025; Revised: 19 February 2025; Accepted: 21 February 2025; Published: 22 March 2025

ABSTRACT

A sequence is a function whose domain is the set of natural numbers $\mathbb{N}$ (or sometimes $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$) and whose codomain is a given set **Ω**. Formally, a sequence in **Ω** is a function.

$f: \mathbb{N} \to \Omega$

$n \to f_n,$

where f_n denotes the nth term of the sequence.

Fibonacci sequence is known in mathematics as the sequence in which each term after its first two terms is the sum of two preceding terms. The importance of the sequence is underpinned by its appeal from nature, and thus to scientists who are interested in the development and growth patterns in nature.

In this seminal paper, we announce a new sequence called propagation or polygon sequence, of which Fibonacci sequence is only a subsequence, and investigate its golden ratio and properties. Population studies require the development of appropriate models for determining growth trends. In this communication, we have characterized and established a mathematical relationship between two principal subsequences of the propagation sequence (one of the subsequences being Fibonacci sequence), which yields an integral factor. We establish bounds of the integral factor within suitable ranges. Furthermore, we derive a propagation population model and propagation differential equation, both of which suggest potential for future research interests and engagement. This relates to propagation sequence because of the inherent generative nature in population growth of species of interest.

Keywords: Propagation, golden ratio, differential factors.

Subject Classification (2010): 35J55, 35J45

INTRODUCTION

Let us consider some famous categories of sequences:

Sequences of Real Numbers $\mathbb{R}$

Arithmetic Sequence:

$f_n = a + (n-1)d,$

where a is the first term and d is the common difference.

Example: 2,5,8, 11, … (where a = 2, d = 3).

Geometric Sequence:

$f_n = ar^{n-1},$

where a is the first term and r is the common ratio.

Example: 3,6,12, 24, … (where a=3, r = 2).

Harmonic Sequence:

$f_n = \frac{1}{n}$,

which is the basis for the harmonic series.

Fibonacci Sequence:

$f_1 = 1, f_2$=1, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

Sequences of Complex Numbers $\mathbb{C}$:

Geometric Sequence in $\mathbb{C}$:

$f_n = re^{in\theta}$

(describes points on a spiral in the complex plane).

Exponentially Growing Sequence:

$f_n = i^n$

follows a periodic cycle $(i, -1, -i, 1, \ldots)$.

Complex Fibonacci Sequence:

Defined similarly to the real Fibonacci sequence but allowing complex values.

Sequences of Functions:

Pointwise Convergent Sequence:

A sequence $f_n(x)$ of functions converges pointwise to $f(x)$ if:

$$\lim_{n\to\infty} f_n(x) = f(x)$$

for all $x \epsilon D$, some domain.

Uniformly Convergent Sequence:

A sequence $f_n(x)$ of functions converges uniformly to a function f if

$\sup_{x \epsilon D} |f_n(x) - f(x)| \to 0$ as $n \to \infty$.

Fourier Series:

A sequence (of partial sums) of trigonometric functions approximating periodic functions:

$$S_n(x) = \sum_{k=0}^{n} a_k \cos(kx) + b_k \sin(kx).$$

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue II February 2025

Legendre Polynomials $P_n(x)$ and *Hermite Polynomials* $H_n(x)$: Classical sequences of orthogonal polynomials in mathematical physics.

The examples above highlight sequences in different mathematical contexts, with wide applicability in analysis, algebra, and applied mathematics.

The *propagation sequence* or *polygon sequence* is a sequence whose first three terms are arbitrarily selected, and thereafter, a patterned sequence of terms follows. The sequence is also called *polygon sequence* for this reason, that, comparatively, the smallest number of sides any polygon could have is three.

Consider the sequence:

S_k: 1, 1, 3, 5, 4, 12, 7, 23, 11, 41, 18, 70, 29, 117, 47, 193, 76, 316, 123, ... (EX)

It would be a useful exercise to provide the next three terms of the sequence S_k.

The sequence of the odd-positioned terms of the sequence above is the *Fibonacci sequence*:

a_k: 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, ..., where $a_k = a_{k-2} + a_{k-1},\ k \geq 3.$

The sequence S_k above exhibits the following property:

$S_{2k+3} = S_{2k-1} + S_{2k+1},\ k \geq 1,$ and $S_{2k} = \sum_{j=1}^{3} S_{2k-j},\quad k \geq 2.$ (P)

Any sequence $\{S_k\}$ with the properties in (P) is called *propagation (or polygon) sequence*. Every propagation sequence contains a Fibonacci subsequence. And we shall consider relevant features of propagation sequence in this literature. The first communication about this concept is in [1].

GOLDEN RATIO

The golden ratio, denoted by \(\varphi \), is an irrational number defined as:

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887 \ldots$$

The ratio appears in various mathematical and natural contexts, particularly in geometry, algebra, and number theory.

One of the most famous appearances of the golden ratio is in the Fibonacci sequence:

The ratio of consecutive Fibonacci numbers converges to the golden ratio:

$$\lim_{n\to\infty} \frac{f_n}{f_{n-1}} = \varphi.$$

The golden ratio is the positive solution of the quadratic equation:

$x^2 - x - 1 = 0$:

Solving the equation using the quadratic formula yields the solutions:

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue II February 2025

$x = \frac{1+\sqrt{5}}{2}$ (golden ratio) and $x = \frac{1-\sqrt{5}}{2}$.

In geometry, the golden ratio appears in the proportions of a *golden rectangle*, where the ratio of the longer side to the shorter side is Furthermore, cutting a square from a golden rectangle leaves another golden rectangle. Additionally, golden ratio also appears in *pentagons* and *pentagrams*, as the ratio of diagonal to side length in a regular pentagon is φ. For further readings, see [2], [3], [4], and [5].

It is known that the relationship between Fibonacci sequence and the Golden Ratio does not depend on the specific values of its sequence terms, but rather on the ratios of the consecutive terms in the sequence. Being a monotone increasing sequence, the Golden Ratio of Fibonacci sequence derives from the sequence of the ratios of the consecutive terms (obtained by dividing the larger term by its preceding smaller term).

Regarding the above Fibonacci sequence *{ak }*, we obtain the following ratios:

- 3/1=3 (1)
- 4/3=1.333 (2)
- 7/4=1.750 (3)
- 11/7=1.571 (4)
- 18/11=1.636 (5)
- 29/18=1.611 (6)
- 47/29=1.620 (7)
- 76/47=1.617 (8)
- 123/76=1.618 (9)

We have stopped at the ninth ratio. However, it is easy to verify that after the tenth ratio, which is 1.617, all succeeding ratios approximate 1.618. And 1.618 approximates the *Golden Ratio*, which is independent of the value of sequence terms of any Fibonacci sequence, in the same terms as π is the ratio of circumference to diameter of any circle (with any chosen radius).

The sequence $\{S_{2n}\}$ of even-positioned terms of a propagation sequence in (P) forms a subsequence $\{ S^{+}{}_{n}\}$ such that: S_{2n}=$S^{+}{}_{n}$, n $= 1, 2,$ ... We find that

for $n \geq 3$, $S^{+}{}_{n}$=$S^{+}{}_{n-1}$+ $S^{+}{}_{n-2}$ + 6 (1)

Accordingly, $\{ S^{+}{}_{n}\}$ differs from the standard Fibonacci sequence by a *uniform difference* of 6, which we call the *differential factor* of propagation sequence.

About the example of propagation sequence given in (EX) above, we have:

$\{ S^{+}{}_{n}\}$= 1, 5, 12, 23, 41, 70, 117, 193, 316, …, and the following ratios are obtained:

- 5/1=5 (10)
- 12/5=2.4 (11)
- 23/12=1. 916 (12)

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue II February 2025

- 41/23=1. 782 (13)
- 70/41=1.707 (14)
- 117/70=1.671 (15)
- 193/117=1.649 (16)
- 316/193=1.637 (17)
- 515/316=1.629 (18)

Again, it is easy to verify that these ratios approach 1.618, the golden ratio approximation.

DIFFERENTIAL FACTOR

As suggested by the differential factor in (1) above, we derive a relationship between the subsequences $\{S_n{}^*\}$ and $\{S^+{}_n\}$, where $S_n{}^* = S_{2n-1}$, and $S^+{}_n = S_{2n}$, $n = 1, 2, \ldots$

$$S_{2n+1} = 6k + S_{2(n-2)}, \; n \geq 3, \qquad (2)$$

where the integral factor $k \geq 1$, and is constant within a considerable range of terms. For instance, for the propagation sequence in our example (EX), $k = 1$ for $3 \leq n \leq 18$, while $k = 2$ for at least $19 \leq n \leq 36$. It would be a useful open exercise to determine the limit of k as $n \to +\infty$.

PROPAGATION CLASSES AND DIFFERENTIAL EQUATIONS

In similar terms as equivalence classes of certain relations, from (2) above, it is obvious that any propagation sequence terms are grouped into mutually exclusive propagation classes, which are distinguishable from each other by multiples of the differential factor 6. In population studies, demographic capturing is important, and reveals certain germane properties about the population. Over certain periods of time, changes in a population could give a projection of the outlook for the population. Generally, changes in populations do not depend only on time.

Suppose a given population P depends on both time t and another factor, say, q. That is, $P = P(t, q)$. Then, such a population is called a *propagation population* if it satisfies the differential equation:

$$\frac{\partial P}{\partial t} = 6k + \frac{\partial P}{\partial q}, \; k = 1, 2, \ldots \qquad (3)$$

The equation (3) is called *Propagation Differential Equation*. For each k, $P_k(t, q)$ is a solution of Equation (3), which is solvable:

It is not difficult to see that the solutions of equation (3) are of the form

$P_k(t, q) = 3k(t - q) + c$, where c is an arbitrary constant, and $k = 1, 2, \ldots$. An important property of those solutions is that

$$\frac{\partial P_k}{\partial t} = -\frac{\partial P_k}{\partial q}.$$

ISSN No. 2321-2705 | DOI: 10.51244/IJRSI |Volume XII Issue II February 2025

CONCLUSION

Our prognosis is that the subject of propagation sequence and propagation differential equations shall be investigated in a manner that further enriches our submission in this seminal presentation. Possible vistas of exploration could include probable applications to daily price movement analysis in stock market, boom-and-bust cycle patterns, density-dependent growth patterns, or density-independent growth patterns in populations. Populations grow and change (generating propagation trajectories) over time based on resource availability, environmental factors, and species characteristics.

REFERENCES

1. Shilgba, L. K. (2022). "Communications on Propagation Sequence", *Research Week Presentation, Admiralty University of Nigeria.*
2. Soegaard, M. (2019). "The Golden Ratio-Principles of form and layout*", Interaction Design Foundation.*
3. (1996). "Applications of Fibonacci Numbers", *Proceedings of the Sixth International Conference, Kluwer Press.*
4. Dunlap, R. (1997). **"**The Fibonacci Sequence and the Golden Ratio**"**
5. *Stakhov, A. (2009). "***The Mathematics of Harmony"**