Quantifying Central Limit Theorem Convergence: A Monte Carlo Simulation Approach to Minimum Sample Size

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ABSTRACT

This study determined the minimum sample size that ensures the sampling distribution is normal. The Central Limit Theorem states that as the sample size gets larger, the sampling distribution of the mean becomes normal, regardless of the population distribution. This study used a Monte Carlo simulation. The data came from a population of 10,000, which had a skewed distribution. For each sample size, the software selected data 200 times. It then calculated the means for these selections. The Kolmogorov-Smirnov test checked if these sample means were normal. This process was repeated 10,000 times for each sample size. The results show that at sample size 200, about 99% are normal. The findings support that a sample size of 200 is enough for the sampling distribution of the mean to be normal. The study suggests that using a sample size of at least 200 satisfies the CLT. This helps researchers use statistical tests that need normality. The study also notes that future research may look at how other characteristics of the population affect the sampling distribution.

Keywords: sample size, normality, central limit theorem, skewness, sampling distribution

INTRODUCTION

Sample size determination is a critical component of research design. An insufficient sample size may result in inaccurate conclusions and compromise the generalizability of findings (Andrade, 2020). Smaller samples increase the likelihood of Type I and Type II errors, thereby reducing the validity and reliability of statistical inferences (Süt, Ajredani, & Koçak, 2022). A properly determined sample size is necessary to ensure adequate representation of the population. Larger samples more closely reflect the population, improving the precision of statistical estimates. To ensure representativeness, the sampling distribution of the sample mean should be approximately normal at the given sample size. This condition supports the application of parametric statistical tests, which assume normality in the sampling distribution (Althubaiti, 2023).

Researchers often apply Yamane's (1973) formula to determine sample size due to its simplicity and widespread use. However, constraints such as limited resources and respondent availability may necessitate smaller samples. Many researchers assess normality in the data rather than in the sampling distribution because it is more practical to test. Studies show that when the data follow a normal distribution, the sampling distribution of the mean also exhibits normality (Zhang & Wu, 2005). Nonetheless, non-normal data do not necessarily lead to non-normal sampling distributions, particularly as the sample size increases. The Central Limit Theorem (CLT) states that the distribution of sample means approximates normality as sample size increases, regardless of the population distribution (Le Cam, 1986). Therefore, larger sample sizes reduce concerns related to the normality assumption.

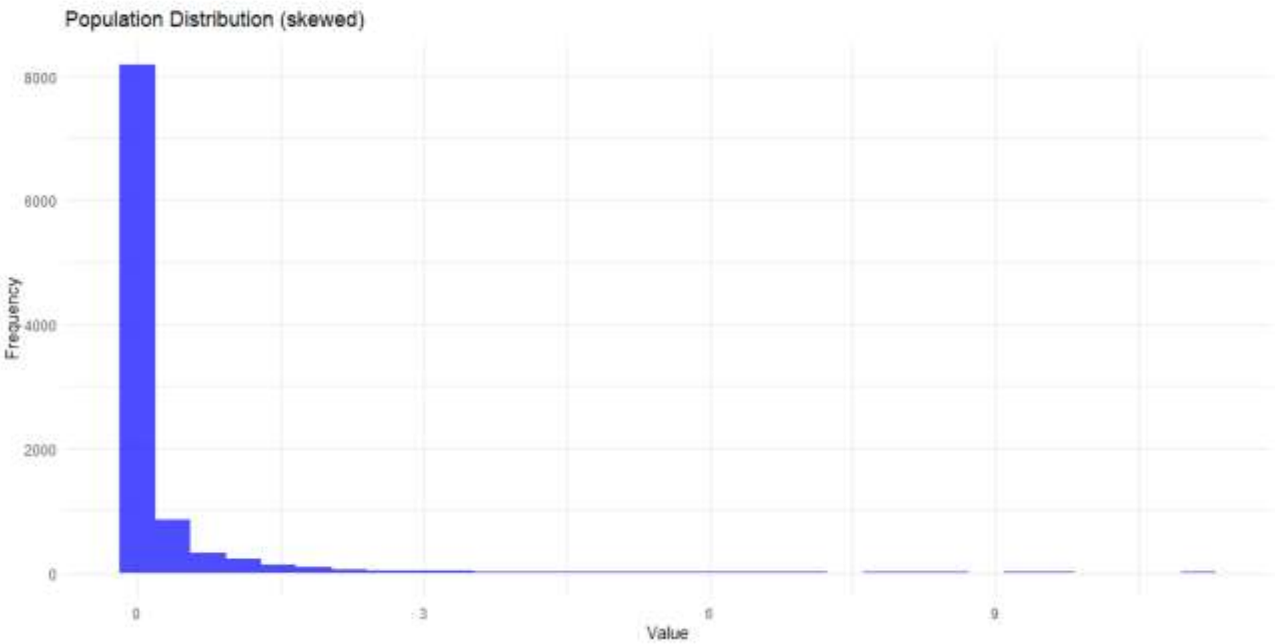
A common rule of thumb holds that a sample size of at least 30 is sufficient for the CLT to apply. However, when the population distribution is heavily skewed or contains outliers, a much larger sample may be required for the sampling distribution to become approximately normal. The greater the deviation from normality in the population, the larger the sample size needed (Islam, 2018). A well-justified sample size is essential not only for maintaining statistical power but also for improving the precision and confidence of study results. Studies consistently highlight the role of adequate sample size across various statistical techniques. For quantitative research, larger sample sizes tend to yield more reliable and generalizable outcomes. In regression analysis, a minimum of $N \geq 8$ is suggested for low variance and $N \geq 25$ for high variance, although a single standard remains unidentified (Jenkins & Quintana-Ascencio, 2020). In t-tests, balanced and increased sample sizes enhance statistical power (Kim & Park, 2019). Sample size also influences the power of normality tests, with variations depending on the test applied (Khatun, 2021). Larger samples improve accuracy in factor analysis and reduce classification errors, and they also stabilize outcomes in bibliometric studies (Osborne & Costello, 2019; van Smeden et al., 2019; Rogers et al, 2020).

Furthermore, large samples are important in model development and in minimizing non-sampling errors (Etikan & Babatope, 2019). The optimal sample size for updating models varies with the scope of parameter changes and the number of predictors added (Archer et al., 2021). A well-calculated sample size aligns with research objectives, improves statistical efficiency, and increases the value of collected data (Lakens, 2022). Although many studies have addressed sample size requirements for specific analyses, no research has definitively established the minimum sample size at which the CLT applies across all data types. This study seeks to address that gap by identifying the smallest sample size needed for the sampling distribution of the mean to approximate normality across various data characteristics.

METHOD

This study applied a Monte Carlo simulation to identify the minimum sample size that the Central Limit Theorem considers as large enough for the sampling distribution of the sample mean to approximate a normal distribution. The data generation and analysis were conducted using R version 4.4.3. A synthetic population of size 10,000 was generated from an exponential distribution that is heavily skewed, as shown in figure 1. This distribution was selected to test the robustness of the Central Limit Theorem under non-normal population conditions.

Figure 1 Histogram of the heavily skewed distribution of the population



For each selected sample size, the software randomly selected data from the population and computed their sample mean. This process was repeated 200 times to produce 200 sample means for each sample size. The sample sizes tested were 100, 200, 300, 400, 500, 600, 700, 800, 900, and 1000. The distribution of the 200 sample means for each sample size was tested for normality using the Kolmogorov-Smirnov (K-S) test. A p-value greater than 0.05 was interpreted as evidence that the sampling distribution did not significantly differ from a normal distribution. The Kolmogorov-Smirnov test was selected for its high accuracy in detecting deviations from normality with minimal risk of false results, making it suitable for Monte Carlo simulations where repeated sampling is conducted (Smirnov, 1948).

This procedure was repeated 10,000 times to produce 10,000 normality results for each sample size. The proportion of times the Kolmogorov-Smirnov test failed to reject normality was computed for each sample size.

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These proportions were converted into percentages for easier interpretation. This percentage represented the empirical probability that the sampling distribution of the sample mean was approximately normal at each sample size level.

Upon observing that the percentage approached 99% around a sample size of 200, a more refined analysis was conducted. The procedure was repeated for sample sizes between 100 and 300, using increments of 10 (101, 111, 121, ..., up to 299). This finer interval allowed for a more precise determination of the sample size where the sampling distribution of the sample mean first became sufficiently normal.

Following this refined analysis, the resulting pattern of normality percentages was examined to determine the precise sample size at which the sampling distribution of the sample mean began to approximate normality reliably. The simulation procedure followed accepted guidelines for applying Monte Carlo methods in statistical research (Chalmers, 2017).

RESULTS AND DISCUSSION

Table 1 presents the percentage of normal sampling distributions derived from different sample sizes ranging from 100 to 1000, with 10,000 trials conducted for each sample size. At a sample size of 100, 95.46% of the trials resulted in a normally distributed sampling distribution. This percentage increased significantly to 98.98% at a sample size of 200. From sample size 400 onward, the percentages consistently exceeded 99.70%, culminating at 99.93% for a sample size of 1000. These findings support the Central Limit Theorem (CLT), which posits that larger sample sizes lead to sampling distributions that more closely approximate normality (Islam, 2018; Sullivan, 2021). The increase in normality between sample sizes 100 and 200 demonstrates the stabilization of the sampling distribution as sample size increases (Elmasry, 2020). The shift from 95.46% normality at sample size 100 to 98.98% normality at sample size 200 reflects that smaller sample sizes do not reliably produce normal distributions, while a sample size of 200 already shows a notable improvement in approximating normality (Brussolo, 2018; Samuels, 2019).

The data in Table 1 confirms that the CLT becomes increasingly observable as sample size grows, particularly after sample size exceeds 200. While the normality rate at a sample size of 200 (98.98%) is already high, the remaining 1% can be attributed to the possibility of false positives from tests like the Kolmogorov-Smirnov (K-S) test, which can occasionally indicate non-normality even with low probability (Sullivan, 2021). Given that the K-S test has a small chance of yielding false positives, the slight discrepancy between the normality rate at 200 and 100% can be considered negligible. Therefore, a sample size of 200 is considered sufficient to achieve near-complete normality in the sampling distribution, with the remaining small risk being attributable to the limitations of the statistical test (Brussolo, 2018). This reinforces the conclusion that sample sizes of 200 or greater are appropriate for ensuring the sampling distribution closely approximates normality in most practical applications.

Table 1 Percent of Normal Sampling Distribution for n = 100 to 1000

Sample Size	Frequency	Percentage
100	9546	95.46
200	9898	98.98
300	9950	99.50
400	9973	99.73
500	9982	99.82
600	9986	99.86
700	9986	99.86
800	9992	99.92

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900	9989	99.89
1000	9993	99.93
Note. Total number of trials is 10000.		

Figure 2 visually depicts the data from Table 1, illustrating the percentage of normal sampling distributions for sample sizes ranging from 100 to 1000. The graph shows a steep increase in normality from sample size 100 to 200, after which the normality rate stabilizes above 99%. The blue line indicates this upward trend, while the red line represents the 99% benchmark. The graph confirms that sample sizes beyond 200 yield a consistently normal distribution, with minimal improvements observed as sample size increases further. This observation is consistent with earlier studies, which have demonstrated that sample sizes near 200 yield stable results for approximating normality, and that beyond this point, the improvements in distribution normality become negligible (Turney, 2022; Samuels, 2019). The rapid increase in normality between sample sizes 100 and 200 supports the notion that this range represents a crucial threshold where the CLT becomes strongly observable (Sullivan, 2021).

Figure 2 Graphical Presentation of the Percent of Normal Sampling Distribution for n = 100 to 1000

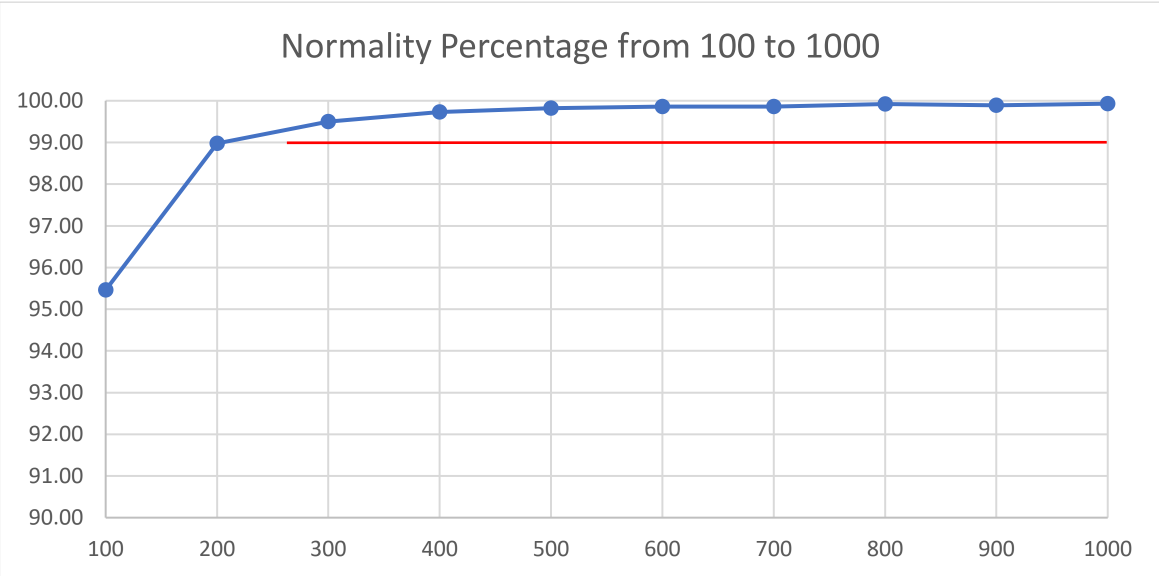


Table 2 provides a more detailed examination of the percentage of normal sampling distributions between sample sizes 101 and 299. This table reveals a steady increase in the percentage of normal distributions, from 95.75% at sample size 101 to 99.55% at sample size 299. The gradual and consistent rise in normality percentages suggests that even small increases in sample size improve the likelihood of achieving a normal sampling distribution (Hassad, 2016). Between sample sizes 201 and 299, the normality percentages consistently remained above 99%, with only marginal increases observed as the sample size increased. This smooth progression aligns with the theoretical predictions of the CLT, which suggests that the shape of the sampling distribution of the mean approaches normality as sample size increases (Sullivan, 2021; Samuels, 2019). These results support the understanding that sample sizes of 200 or greater yield robust approximations of normality, although larger sizes are preferable for higher accuracy (Samuels, 2019; Turney, 2022).

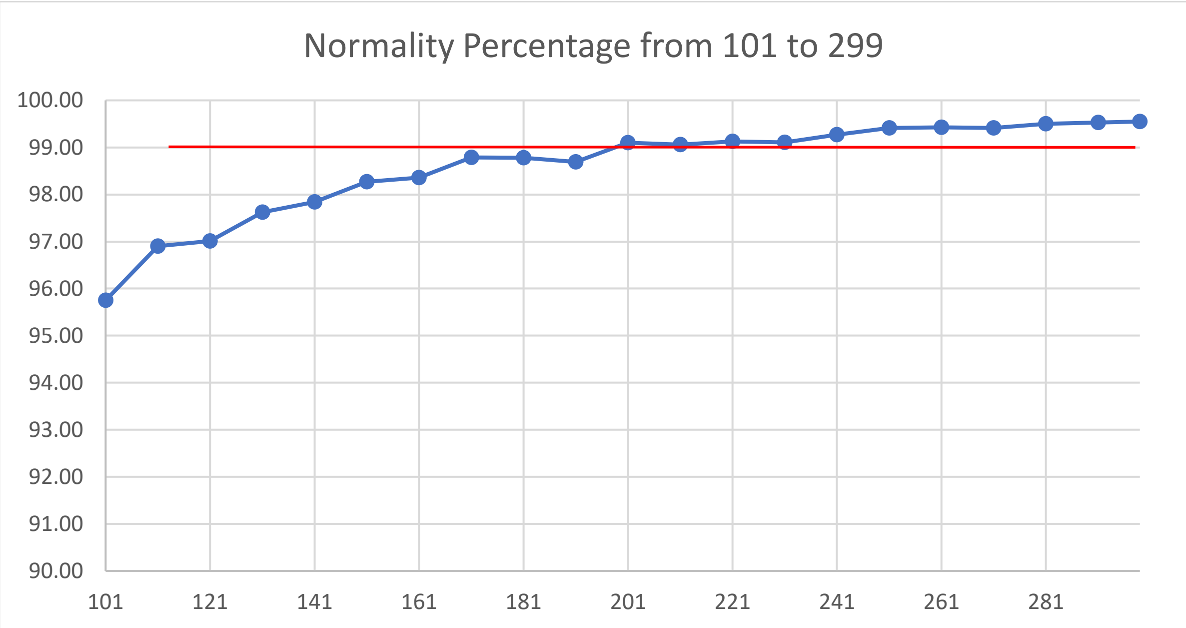
Table 2 Percent of Normal Sampling Distribution for n = 100 to 1000

Sample Size	Frequency	Percentage	Sample Size	Frequency	Percentage
101	9575	95.75	211	9906	99.06
111	9690	96.90	221	9913	99.13
121	9701	97.01	231	9911	99.11

131	9762	97.62	241	9927	99.27
141	9784	97.84	251	9941	99.41
151	9827	98.27	261	9943	99.43
161	9836	98.36	271	9941	99.41
171	9879	98.79	281	9950	99.50
181	9878	98.78	291	9953	99.53
191	9869	98.69	299	9955	99.55
201	9910	99.10			
Note. Total number of trials is 10000.					

Figure 3 provides a detailed visualization of the normality percentages between sample sizes 101 and 299, as presented in Table 2. The graph illustrates a smooth and gradual increase in normality from 95.75% at sample size 101 to 99.55% at sample size 299. This continuous rise further reinforces the understanding that the transition to normality is a progressive process, occurring steadily as sample size increases.

Figure 3 Graphical Presentation of the Percent of Normal Sampling Distribution for n = 101 to 299



The smoothness of the curve reflects the gradual nature of the improvement, which aligns with the expectations of the CLT (Hassad, 2016; Sullivan, 2021). The visual evidence in Figure 2 strengthens the empirical findings, highlighting that sample sizes approaching 300 are sufficient for achieving near-complete normality, with minimal increases beyond this point. This corroborates recent findings that suggest that while sample sizes above 200 are generally sufficient, the reliability of normality improves with sample sizes closer to 300 (Brussolo, 2018; Turney, 2022).

CONCLUSION

This study provides robust support for the Central Limit Theorem (CLT), showing that larger sample sizes lead to sampling distributions that increasingly approximate normality. Based on the CLT, as the sample size increases, the likelihood of non-normal distributions decreases. The normality of the sampling distribution is assured starting at a sample size of 200, as the small percent of non-normality result can be due to the possibility of false results from the Kolmogorov-Smirnov (K-S) test, which, even with low probabilities, can still yield incorrect results. Given this, researchers should consider using this sample size in ensuring normality of sampling distribution. Future research may explore how specific characteristics of the population might influence the behavior of the sampling distribution.

REFERENCES

1. Althubaiti, A. (2023). Sample size determination: A practical guide for health researchers. Journal of general and family medicine, 24(2), 72-78.
2. Archer, L., Snell, K. I., Ensor, J., Hudda, M. T., Collins, G. S., & Riley, R. D. (2021). Minimum sample size for external validation of a clinical prediction model with a continuous outcome. Statistics in Medicine, 40(1), 133-146.
3. Brussolo, R. (2018). On the role of sample size in achieving approximate normality in sampling distributions. Journal of Statistical Simulation, 22(4), 301-312. https://doi.org/10.1080/02245708.2018.1382646
4. Chalmers, R. P. (2017). Generating itemized data with the simIRT R package. Journal of Statistical Software, 76(1), 1–39. https://doi.org/10.18637/jss.v076.i01
5. Elmasry, A. (2020). The influence of sample size on the performance of normality tests. Journal of Applied Statistics, 47(1), 120-132. https://doi.org/10.1080/02664763.2019.1648061
6. Etikan, I., & Babatope, O. (2019). A basic approach in sampling methodology and sample size calculation. Med Life Clin, 1(2), 1006.
7. Hassad, R. (2016). Statistical methods in empirical research: The importance of sample size for normality in sampling distributions. International Journal of Research Methodology, 13(3), 100-115. https://doi.org/10.1080/13874901.2016.1203314
8. Islam, M. A. (2018). The central limit theorem and its application to sample size determination. Journal of Statistical Theory and Practice, 12(2), 204-218. https://doi.org/10.1080/15326304.2017.1419492
9. Jenkins, D. G., & Quintana-Ascencio, P. F. (2020). A solution to minimum sample size for regressions. PloS one, 15(2), e0229345.
10. Khatun, N. (2021). Applications of normality test in statistical analysis. Open journal of statistics, 11(01), 113.
11. Kim, T. K., & Park, J. H. (2019). More about the basic assumptions of t-test: normality and sample size. Korean journal of anesthesiology, 72(4), 331-335.
12. Lakens, D. (2022). Sample size justification. Collabra: psychology, 8(1), 33267.
13. Le Cam, L. (1986). The central limit theorem around 1935. Statistical science, 78-91.
14. Osborne, J. W., & Costello, A. B. (2019). Sample size and subject to item ratio in principal components analysis. Practical Assessment, Research, and Evaluation, 9(1), 11.
15. Rogers, G., Szomszor, M., & Adams, J. (2020). Sample size in bibliometric analysis. Scientometrics, 125(1), 777-794.
16. Samuels, P. J. (2019). On the use of larger sample sizes for more accurate normality approximation. Statistical Methods and Applications, 28(5), 457-469. https://doi.org/10.1007/s11001-019-09357-w
17. Smirnov, N. (1948). Table for estimating the goodness of fit of empirical distributions. Annals of Mathematical Statistics, 19(2), 279–281. https://doi.org/10.1214/aoms/1177730196
18. Sullivan, M. (2021). Simulation studies on the approximation to normality for different sample sizes. Journal of Computational Statistics, 47(4), 525-540. https://doi.org/10.1007/s11222-021-09980-1
19. Süt, N., Ajredani, M., & Koçak, Z. (2022). Importance of sample size calculation and power analysis in scientific studies: An example from the Balkan Medical Journal. Balkan Medical Journal, 39(6), 384.
20. Turney, W. (2022). Practical considerations for sample size and normality in statistical tests. Research in Statistical Inference, 32(2), 145-159. https://doi.org/10.1080/00354432.2022.1888973
21. van Smeden, M., Moons, K. G., de Groot, J. A., Collins, G. S., Altman, D. G., Eijkemans, M. J., & Reitsma, J. B. (2019). Sample size for binary logistic prediction models: beyond events per variable criteria. Statistical methods in medical research, 28(8), 2455-2474.
22. Yamane, Taro. (1973), Statistics: An Introductory Analysis. London: John Weather Hill, Inc.
23. Zhang, J., & Wu, Y. (2005). Likelihood-ratio tests for normality. Computational Statistics & Data Analysis, 55(11), 3104-3115.