Investigating the Role of Algebra in Solving Equations

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DOI: https://dx.doi.org/10.47772/IJRISS.2025.907000462

Received: 15 July 2025; Accepted: 22 July 2025; Published: 23 August 2025

ABSTRACT

This case study examines the use of algebra in solving equations by Senior High School Students at Winneba Secondary School. More specifically, it looks at the effects of students' grasp of algebraic concepts on their capabilities to solve both linear and quadratic equations and seeks to establish which algebraic skills most strongly correlate with achievement in solving multi-step equations. In this instance, we conducted an experiment with an experimental research design. Data were collected using a stratified random sample of 100 students through uniquely designed tests measuring algebraic understanding and proficiency in solving equations. 68.5% and 64.2% were the mean scores recorded for algebraic concepts and solving equations respectively. These results illustrate that the average score obtained on the Algebraic Concepts Test ($M = 68.5$, $SD = 12.3$) was greater than the average with regard to Linear and Quadratic Equations Test ($M = 64.2$, $SD = 14.1$). The mean difference between the two tests was 4.30, a difference which was significant, $t(99) = 3.24$, $p < .001$. In the context of solving multi-step equations, simplification ($\beta = 0.42$, $p < 0.001$) and factorization ($\beta = 0.31$, $p < 0.001$) emerged as the strongest predictors, accounting for 65% of the variance in the performance model ($R^2 = 0.65$). The results from this study underscore the need to cultivate both conceptual understanding and procedural fluency in algebra to enhance students' overall problem-solving capabilities. Suggested actions advocate for conceptual-focused teaching frameworks, stronger reinforcement of intermediate algebraic concepts, improved teacher preparation programs aimed at blending theory with practice, and more effective teaching strategies. This research contributes significantly to the enhancement of mathematics instruction not only at Winneba Secondary School but also in comparable educational environments.

Keywords: Algebra, Equation Solving, Algebraic Concepts, Procedural Fluency, Conceptual Understanding, Multi-step Equations.

INTRODUCTION

Algebra is often considered a fundamental part of the learning of mathematics, serving as a basis for the development of logical reasoning, flexible thinking, and systematic problem-solving. It is an entry point to higher level mathematics and it is extensively used in science, technology, economics, and daily life (Kieran, 2007; Kaput, 2008). In the Ghanaian Senior High School curriculum, algebraic concepts including expressions, equations, and inequalities are taught early on with the expectation that students master them before advancing on higher level mathematics. In spite of this focus, a great number of students continue to struggle applying their algebraic knowledge to solve equations (Amoako & Oppong, 2020).

At Winneba Secondary School, there has been an ongoing problem with students not being able to perform well in parts of algebra, specifically in linear or quadratic equations. Understanding of concepts like variables and manipulation of algebraic expressions as well as the rules of equality which are basic to solving equations is difficult for students (Osafo-Affum, 2016). These problems lead to low levels of mathematical ability, low self-esteem, and low motivation to engage with mathematics as a subject. Teachers at the school note that many

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue VII July 2025

students approach equations purely as simplistic calculation exercises rather than deeply algebraic problems which demonstrates a gap in the relation between the teaching of algebra and its practical use.

This situation brings to light the possible gaps of applying effective pedagogical approaches as well as the level of students' understanding in terms of concepts. As Booth et al. (2014) and Nathan & Koedinger (2000) have noted, mastery of algebraic concepts strongly correlates with effective problem-solving skills in students. On the other hand, poorly structured algebraic concepts tend to render students unable to cope with academic and professional challenges in STEM fields.

It is, therefore, important to examine how algebra influences equation solving among students of Winneba Secondary School. The study aims to determine the algebraic components which are the most influential in the equation-solving process in terms of the learners' understanding, and also how much their understanding impacts their performance. Such results would be useful to the mathematics teachers, curriculum developers, and education policy makers. The study strives to provide actionable recommendations designed to address students' gaps in mathematical skills and overall academic performance by exposing major learning barriers and teaching hurdles.

In the long run, enhancing learners' competence in using algebraic principles to solve equations positively impacts their preparation for higher education as well as their readiness for 21st century careers. With the growing emphasis on STEM education as a tool for national development, addressing core areas such as algebra is not only an academic concern, but also a national imperative (Ministry of Education, 2019).

Research Objectives

How does students' understanding of algebraic concepts at Winneba Secondary School influence their ability to solve linear and quadratic equations?

Which specific algebraic skills most significantly contribute to students' success in solving multi-step equations at Winneba Secondary School?

Research Questions

What is the mean difference of algebraic concepts test and linear and quadratic equation test among students of Winneba Secondary School?

What Specific algebraic skills most significantly contribute to students' success in solving multi-step equations at Winneba Secondary School?

Hypotheses

H_{01}: There is no statistically significant difference between students' understanding of algebraic concepts and linear and quadratic equations at Winneba Secondary School.

Theoretical Review

The theoretical framework of this study is based on a number of different theories related to learning and mathematical thinking that look at how scholars obtain, comprehend, and apply concepts to solve algebraic equations. This includes Constructivist Learning Theory, Cognitive Load Theory, as well as Conceptual and Procedural Knowledge Theory. These theories collectively enhance understanding of the challenges and the steps involved in students' problem-solving processes in algebra.

Constructivist theory suggests that learners construct their own understanding and knowledge of the world by experiencing it and reflecting on those experiences (Piaget, 1972; Vygotsky, 1978). While learning algebra, students should understand that this theory means they must construct a personal understanding of variables, symbols, and operations through meaningful mathematical engagement.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue VII July 2025

Jean Piaget (1972) focused on the developmental stages that learners go through, and he described formal operational thinking, which is often characterized by abstract reasoning and usually develops in teenage years. This stage is critical for working with algebra and solving equations as symbols are used and abstract thinking is essential.

Lev Vygotsky (1978), in contrast, emphasized the social aspect of learning and developed the concept of the Zone of Proximal Development (ZPD). Guided teaching and peer teaching, as he explained, enable learners to attain more advanced levels of comprehension. In algebra instruction, scaffolding by teachers, such as step-by-step demonstrations of solving equations, aids students in internalizing processes to eventually achieve independent functioning.

As observed with students at Winneba Secondary School, constructivist theory supports the need for active involvement, meaningful interaction, as well as appropriate scaffolding to facilitate the formulation of their understanding of algebraic concepts and problem solving.

Cognitive Load Theory (Sweller, 1988) focuses on the instructional design and the supports for effective learning, considering the capacity limitations of working memory. Sweller (1988) posits that students working with or being presented information that is overly complex and/or poorly organized will exhaust their working memory, thus hindering learning.

In algebra, the solving of equations entails pattern recognition, operation applying, rule enforcing, and many other processes occurring simultaneously. This holistic approach could be challenging, especially for learners who have yet to internalize rote skills. For students at Winneba Secondary School, cognitive overload severely limits the strategies they apply when solving equations—even when they grasp the fundamental ideas.

Appropriate instructional techniques that focus on specific visual aids, worked examples, or step-by-step construction of complex problems from simpler ones can diminish extraneous cognitive burden and improve learning outcomes (Sweller, 1988). As a result, algebra instruction informed by Cognitive Load Theory highlights the importance of appropriate pedagogical practices that address cognitive overload with efficient teaching aimed at enhancing understanding.

This theory differentiates two forms of knowledge within mathematics (Hiebert & Lefevre, 1986). Conceptual knowledge is grasping various components of mathematics and its principles, including its relations and structures, such as understanding why operations are performed in equations.

Procedural knowledge is the ability to perform specific steps as in operational definitions of mathematics, such as applying the distributive property or isolating variables.

Both types of knowledge are essential in algebra, but they must be balanced. Research indicates that an imbalance in favor of overly focusing on procedural knowledge can lead to mechanistic problem-solving strategies devoid of understanding (Rittle-Johnson & Schneider, 2015).

This explains why students in Winneba Secondary School exhibit a gap between performing the stepwise rote algebraic procedures and their ability to explain understanding of the logic behind the steps performed, making application far less versatile. It explains the need to analyze which algebra skills conceptual or procedural determine success in solving equations.

The combination of the Constructivist Learning Theory (Piaget, 1972; Vygotsky, 1978), Cognitive Load Theory (Sweller, 1988), and Conceptual-Procedural Knowledge Theory (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015) demonstrates a strong capacity for explaining the role of algebra in solving equations. Students' success in algebra seems to hinge on their educational support, cognitive preparedness, and their understanding, both at the conceptual and the procedural levels. This is the framework which was underpin the design, analysis, and interpretation of the study conducted at Winneba Secondary School.

METHODOLOGY

To examine the relation between the comprehension of algebraic concepts and the ability to solve equations among students at Winneba Secondary School, an experimental approach was utilized for this study. This approach is appropriate because it is possible to observe the relationships between algebraic skills and students' performance in solving linear, quadratic, and multi-step equations without any form of intervention.

The study sample comprises all students in the Senior High School at Winneba Secondary School who take Mathematics for all levels (SHS 1 to SHS 3) and for all streams. Using a stratified random sampling approach, a sample size of 100 students was obtained. Stratification helps in obtaining adequate representation from different grade levels and academic tracks which minimizes sampling bias and enhances generalizability.

For the study, an Algebraic Concepts Understanding Test (ACUT) to measure students understanding of fundamental algebraic concepts like variables, expressions and the properties of equality, and an Equation Solving Test (EST) which checks students' ability to solve linear, quadratic and multi-step equations were used. Both tests went through a validation process where they were piloted to test for validity and reliability.

As long as the school administrators permitted, the tests were given within the normal school hours in a controlled classroom setting. The students are given step by step instructions and enough time to finish the test. All students were free to take the test, and all participants were promised confidentiality and could withdraw at any time without consequences.

The data was analyzed using the SPSS (Statistical Package for the Social Sciences) application. For measuring the students' performance, descriptive statistics in the form of means and standard deviations were computed. The gaps in understanding algebra and the solving of equations were assessed using a paired sample t-test. Also, multiple regression analysis was performed to ascertain which specific algebraic competencies most strongly predict the successful completion of multi-step equations. All statistical evaluations were performed with a significance threshold of $p < 0.05$.

With regard to ethical considerations, informed consent, confidentiality, and voluntary participation were maintained throughout the study in order to protect the rights and welfare of the participants.

RESULTS AND DISCUSSION

Research Question 1: What is the mean difference of algebraic concepts test and linear and quadratic equation test among students of Winneba Secondary School?

Table 1: shows the descriptive statistics for students' scores on the Algebraic Concepts Understanding Test (ACUT) and their performance on solving linear and quadratic equations.

Table: Descriptive Statistics

Test	N	Mean	Std. Dev.
Algebraic Concepts Test	100	68.5	12.3
Linear & Quadratic Equations Test	100	64.2	14.1

Table 1 shows the averages and the standard deviations for two mathematics assessments the students undertook: the Algebraic Concepts Test and the Linear & Quadratic Equations Test. All the tests were taken by a sample of 100 students. It is evident from the data that students did better on the Algebraic Concepts Test than on the Linear & Quadratic Equations Test (68.5/12.3, 64.2/14.1). This difference in means suggests that students have a better understanding of the basic of algebraic concepts relative to other topics covered in the curriculum. Moreover, the lower standard deviation of the test scores suggests that the students were achieving at a similar level to each other as the results were less dispersed from the average.

As opposed to the above conclusion, the lower mean score on the Linear & Quadratic Equations Test suggests that the content was more difficult to master for the learners. The greater SD of 14.1 suggests that there is much

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue VII July 2025

difference among learners in how they scored which indicates the presence of unaddressed comprehension problems.

The two mathematics tests show variation in both the test performance and consistency across students. Most students seem to understand the foundational algebra topics, as indicated by the stronger average performance and lower variability on the Algebraic Concepts Test. This may be due to effective teaching, simplicity of the content, or repeated exposure at prior learning stages.

There is a clear need for improvement in instruction on the Linear & Quadratic Equations Test, as indicated by the Test's lower mean score and higher variability. The abstract and procedural character of the content may account for a wide spread of scores, where a significant number of students failed to keep up with the material, despite some students mastering the material. Consistent with prior research, these findings support the idea that there is an increased difficulty for students to transition from algebraic concepts to solving equations (Kieran, 2007; Booth et al., 2014). Visual, differentiated teaching strategies along with scaffolding problem solving are examples of instructional adjustments that could help address the challenges posed by equations for students.

Research Question 2: What Specific algebraic skills most significantly contribute to students' success in solving multi-step equations at Winneba Secondary School?

Multiple regression analysis was conducted to identify the algebraic skills that significantly predict students' success in solving multi-step equations. The predictors included simplification, factorization, transposition, and application of inverse operations.

Table 2: ANOVA

Predictor	B (Unstandardized Coefficient)	Standard Error	Beta (Standardized Coefficient)	t-value	p-value
Simplification	0.35	0.07	0.42	5.00	<0.001
Factorization	0.25	0.06	0.31	4.17	<0.001
Transposition	0.15	0.05	0.20	3.00	0.003
Inverse Operations	0.10	0.04	0.14	2.50	0.014

The model explains approximately 65% of the variance in students' scores on multi-step equations ($R^2 = 0.65$, $p < 0.001$).

The regression results indicates that, algebraic skills 1 through 4 are all relevant to a student's success in solving multi-step equations, with simplification and factorization influencing success most strongly. This underscores the importance of foundational skills in algebraic operations long before the complex tasks of equation solving are undertaken. It confirms Cognitive Load Theory (Sweller, 1988), which suggests building fluency in basic skills to enhance problem-solving skill in working memory limited environments. Therefore, this is what should be taught for greater equation-solving ability by students at Winneba Secondary School.

Test of hypothesis

H_{01}: There is no statistically significant difference between students' understanding of algebraic concepts and linear and quadratic equations at Winneba Secondary School

Table 2: Paired Sample t-test

Test Pair	Mean	Std. Dev.	t	df	Sig.
Algebraic Concepts - Linear & Quadratic Equations	4.30	13.29	3.24	99	0.000

A paired sample t-test analysis was performed on students' data to find out if there was a significant difference in their performance on the two assessments Algebraic Concepts Test and Linear and Quadratic Equations Test. The assessments were given to a cohort of 100 students. It was found that the mean score on Algebraic Concepts

Test was significantly greater than mean on Linear and Quadratic Equations Test, 68.5 compared to 64.2 with a standard deviation of 12.3 and 14.1 respectively. The two-test means showed a significant difference of 4.30 with t (99) = 3.24 and p < .001. Following Cohen's (1988) metrics: 0.2 = small effect, 0.5 = medium effect, 0.8 = large effect. A Cohen's d of 0.32 implies a small to medium level of effect size which indicates the performance difference is significant in a statistical sense however it indicates a modest improvement in explaining algebraic concepts compared to quadratic and linear equations. Their performance and understanding suggests mastery of general algebraic concepts was better than their ability to apply it in solving linear and quadratic equations. The small to moderate effect size supports the notion of focus instruction suggesting specific adaptations might be required for students' understanding of the concepts and procedures relative to the equations.

CONCLUSION AND RECOMMENDATIONS

This research assessed the impact of algebra on the ability to solve equations among students at Winneba Secondary School. The results showed a strong positive relationship difference between students' understanding of algebraic concepts and their ability to solve linear and quadratic equations. In addition, specific algebraic skills, especially simplification and factorization, were found to predict success in multi-step equation solving. These findings emphasize the need for both algebraic understanding and computation skill for effective problem solving.

This study also corroborates that the more deeply students grasp the underlying algebraic principles and possess command of fundamental skills, the better they would be able to solve complex equations. This is in line with the views of Constructivist Learning Theory, Cognitive Load Theory, and the Conceptual and Procedural Knowledge framework that substantiate the need for teaching systems that integrate balanced conceptual on insight alongside ample practice in lower-level procedural skills.

RECOMMENDATIONS

Considering the results, the following recommendations were suggested:

Teachers need to apply cognitive and conceptual understanding strategies through visual, real-life, and discussion-based aids to foster better mental modeling of algebra concepts.

Educators should focus especially on the core algebraic skills of simplification and factorization, teaching them thoroughly before moving on to more advanced multi-step equations, to help students avoid conceptual overload.

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ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue VII July 2025

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