A Formulaic Approach: Building a Different Method for Calculating a Right Triangle's Area Using the Hypotenuse and One Leg

Liera Niña V. Manginsay, Gretchen E. Ondayo, Sheilla May S. Engi, Professor Ronald S. Decano

Institute of Advanced Studies, Davao Del Norte State College, Davao City, Davao del Sur, Philippines

DOI: https://dx.doi.org/10.47772/IJRISS.2025.90500009

Received: 16 January 2025; Accepted: 21 January 2025; Published: 27 May 2025

ABSTRACT

This study aims to address situations where the base or height is unclear by creating an innovative method to calculate the area of a right-angled triangle when only the hypotenuse and one leg are known. A new computational technique is proposed to determine the missing dimension necessary for an accurate area calculation by integrating the Pythagorean theorem with the conventional area formula. The resulting formula, $A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})$, offers a simplified approach to a common geometric problem. This method serves as a valuable tool for math teachers, enhances conceptual understanding of geometric relationships, and establishes a solid foundation for further research aimed at improving geometry instruction in high schools.

INTRODUCTION

A fundamental area of mathematics, geometry fosters analytical abilities, logical reasoning, and spatial reasoning. In disciplines like science, engineering, and architecture, it is essential. Despite its importance, geometric concepts are difficult for many students to understand because they are abstract and are taught using traditional memorization-based methods (Chen et al., 2022; Ludwig & Reiss, 2023).

Teachers across the world are implementing creative methods to streamline geometry instruction in order to overcome these obstacles. These include improving student comprehension using technology, visual aids, and real-world problem contexts (Zhang et al., 2023). Teaching models that put an emphasis on conceptual learning and student engagement have proven successful in nations like Singapore and Finland (Sunzuma, 2023).

The K–12 curriculum in the Philippines includes geometry instruction to help students develop their critical thinking and problem-solving abilities. Nonetheless, students' low math proficiency is still evident in national assessments (Perez & Nakamura, 2020). Formula-based instruction, inadequate teacher preparation, and a dearth of interesting, locally relevant teaching resources are all contributing factors (Singh & Thomas, 2023).

Additional obstacles that local classrooms must overcome include overcrowding, a lack of resources, and a range of learning levels. Because teachers lack the resources to simplify material and students struggle with abstraction, these problems impede both teaching and learning (Davies & Taylor, 2023; Li & Wang, 2021). This study supports the Department of Education's objectives to improve critical thinking and practical problem-solving skills by offering simplified formulas and useful strategies to make geometry more approachable (Yang et al., 2024; Chang, 2022).

Statement of the Problem

The goal of this research is to create simplified geometric formulas that will help high school students better grasp abstract ideas. It also looks for creative ways to help develop these formulas in order to improve the clarity and accessibility of geometry education.

Research Objective

To simplify the formula to calculate a right triangle's area given its hypotenuse and one known leg and to investigate creative approaches that improve the formula's comprehension and use in high school geometry classes.

LITERATURE REVIEW

Traditionally, the formula $A = \frac{1}{2}bh$ is used to calculate the area of a right triangle. Determining the area, however, becomes challenging in situations where only the hypotenuse and one leg are known. Although the missing leg can be calculated using the Pythagorean theorem, students may find this two-step procedure cognitively challenging, particularly if they are not very good with math. This challenge underscores the need for simplified formulas that can streamline problem-solving and enhance comprehension.

Sweller (1988) introduced the Cognitive Load Theory (CLT), which emphasizes working memory's limitations in processing complex information. Tarmizi and Sweller (1988) observed that traditional problem-solving techniques could impose a high cognitive load, hindering effective learning in the context of geometry education. By integrating worked examples and simplifying problem structures, they promoted instructional designs that lessen unnecessary cognitive load. Simplified formulas can reduce cognitive load and free up students' attention to concentrate on fundamental ideas and connections, according to research using CLT in geometry education.

According to Piaget and Vygotsky's constructivist learning theory, students actively engage and interact with their surroundings to build knowledge. With the right scaffolding, learners can reach higher levels of understanding, according to Vygotsky's theory of the Zone of Proximal Development (ZPD) (Vygotsky, 1978). This translates into providing tools and techniques that close the knowledge gap between present understanding and future learning in geometry education. Such tools include simplified formulas, which help students understand difficult ideas by expanding on what they already know.

Lave and Wenger's (1991) Situated Learning Theory highlights the significance of context in the learning process. In the Philippine classroom, integrating real-world applications and culturally relevant examples into geometry lessons can improve student comprehension and engagement. For example, learning can be made more meaningful by relating geometric concepts to indigenous patterns or local architecture. The importance and significance of geometry in students' lives can be strengthened further by simplified formulas that apply to these situations.

Deci and Ryan's Self-Determination Theory (1985) states that motivation is essential to learning. According to this theory, relatedness, competence, and autonomy are important factors that influence intrinsic motivation. Students frequently struggle with anxiety and lack of confidence when learning mathematics, which can hinder their progress. Teachers can help students feel more competent, feel less anxious, and have a more positive attitude toward geometry by introducing simplified formulas. This strategy supports the objective of developing an environment for learning that promotes creativity and determination.

Research Gap

The literature on geometry education in the Philippines is expanding, but little research has been done on creating simple computational techniques that directly aid students in solving problems. Although research has looked at cooperative strategies (Baldo & Cruz, 2019), visual learning tools (Bautista & Soriano, 2022), and manipulative use (Rivera & Padilla, 2023), little focus has been placed on rethinking the formulas themselves, especially when students are expected to apply multi-step procedures like the Pythagorean theorem. This two-step procedure may put unnecessary stress on the mind, particularly for students whose mathematical foundations are weaker. While grounded theories like Vygotsky's ZPD (1978) and Sweller's Cognitive Load Theory (1988) stress simplification and scaffolding to improve learning, few empirical studies examine the effects of creating alternate, simplified formulas for simple geometric computations.

In addition, research investigations that present or assess innovative geometric formulas designed for high school students in real-world contexts are evidently lacking. Existing works frequently overlook real-world situations where only a limited amount of information is available, considering that students have access to complete data (such as both legs of a triangle). For students who have trouble with abstraction or multi-step derivations, the

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

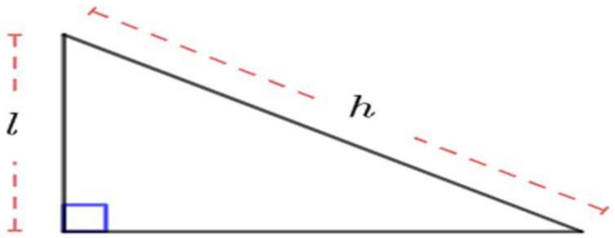
absence of simplified formulas that can function under such restrictions creates a challenge. In order to close this gap, this study recommends and validates an innovative method that is consistent with constructivist principles and cognitive efficiency for determining the area of a right triangle using only the hypotenuse and one leg. In doing so, it contributes to a more inclusive and accessible geometry curriculum, particularly in Filipino classrooms where such innovations are underrepresented in current research.

METHODOLOGY

Derivation of the modified formula

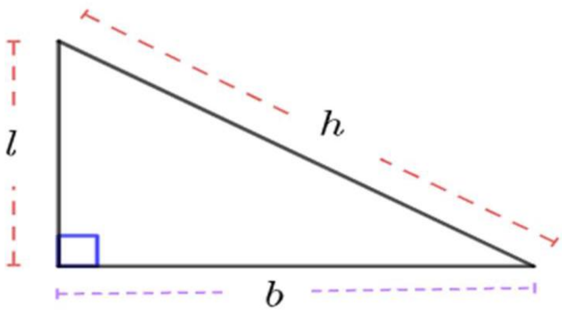
$$\boldsymbol{A = \frac{1}{2}(l)(\sqrt{h^2 - l^2}}$$

Consider a right triangle whose lengths of the hypotenuse (h) and one of the legs (l) are known.



In this case, we cannot find the area right away because the length of base is unknown.

We begin our derivation by letting the length of the base be b as shown.



Let b be the length of the base.

To find b, we use Pythagorean Theorem.

$\boldsymbol{l^2 + b^2 = h^2}$ **(Pythagorean Theorem)**

Changing the subject of the equation to b, we have

$b = \sqrt{h^2 - l^2}$ ---------- (equation 1)

To find the area, we have

$A = \frac{1}{2}(l)(b)$ -----------(equation 2)

Substituting the Eq. 1 to Eq. 2, we have

$$\boldsymbol{A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})}$$

Therefore, the area of a right triangle with known hypotenuse and known one leg is

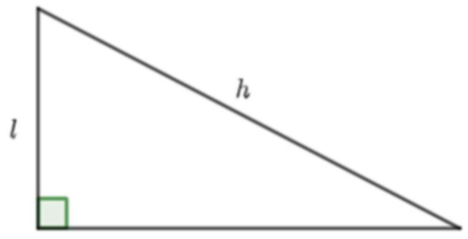
ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

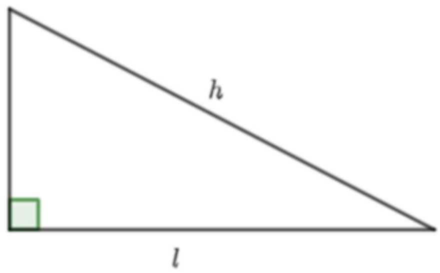
$\boldsymbol{A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})}$

where l is the length of either of the legs and h is the length of the hypotenuse.

Note that this formula works for both cases below.

Consider the area of right triangle with a known leg and a hypotenuse which is shown in either of the figures below. Note that the legs, l, of a right triangle are also the base, b, and height, h, of such triangle.





In the figure shown above, we cannot use the formula, $A = \frac{(base)(height)}{2}$ since in both figures, either the base (left figure) and height (right figure) are unknown. However, by Pythagorean Theorem, we can find the base or height as:

$$base\ or\ height = \sqrt{h^2 - l^2}$$

Therefore, in the cases shown, we can form a new formula.

The area of a right triangle with a known one leg and known hypotenuse is

$A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})$

Where l is either of the legs and h is the hypotenuse.

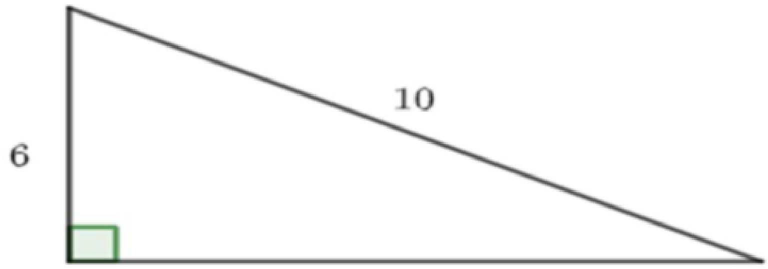
Examples and Applications

The area of a right triangle with known one leg and known hypotenuse

$$\boldsymbol{A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})}$$

Examples 1:

The figure below is a right triangle whose hypotenuse is 10 cm and whose height is 6 cm. Find the area of the triangle.



Solution:

From the figure it is known that l = 6 cm and h = 10 cm. So, the area is

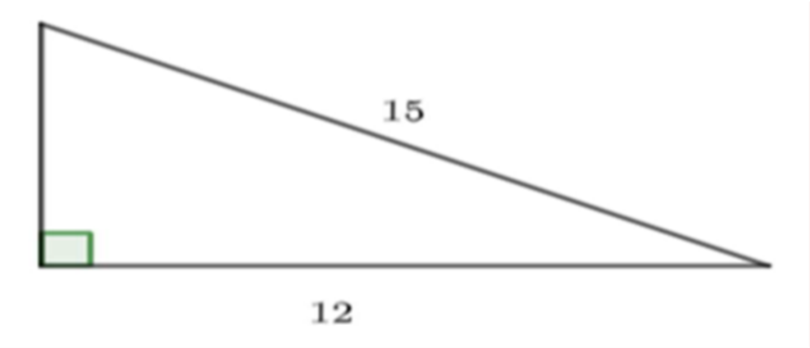
$$A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})$$

$$A = \frac{1}{2}(6)(\sqrt{10^2 - 6^2})$$

$$A = \frac{1}{2}(6)(8)$$

$$A = 24cm^2$$

Example 2: Find the area of the triangle below.



Solution:

$$A = \frac{1}{2}(l)(\sqrt{h^2 - l^2})$$

$$A = \frac{1}{2}(12)(\sqrt{15^2 - 12^2})$$

$$A = \frac{1}{2}(12)(9)$$

$$A = 54$$

DISCUSSION

The results of this study show that the recently created formula for determining a right triangle's area using just the hypotenuse and one leg is mathematically equivalent to the traditional method that requires the base and height. Being able to simplify the problem-solving process, however, is what makes this new approach strong. The new method reduces the number of computational steps by doing away with the necessity to apply the area formula after determining the missing leg using the Pythagorean theorem. As demonstrated by Sweller's (1988) Cognitive Load Theory, this simplified procedure not only reduces cognitive load but also fosters a simpler and easier-to-comprehend conceptual understanding of the relationship between the geometric elements in a right triangle.

This formula's accessibility for students who have trouble with multi-step algebraic procedures is one of its main advantages. It offers an easier method to solve area problems, which can boost one's confidence and mathematical proficiency. This approach facilitates constructivist learning from a pedagogical standpoint by letting students concentrate less on procedural complexity and more on comprehension. The formula's limited applicability, however, is that it only works when the hypotenuse and one leg are known, which isn't always the case in problems in the classroom or on standardized tests. Nevertheless, it has encouraging potential as an additional tactic. For educators, it provides a useful substitute for scaffolding instruction for students who struggle. This study provides researchers with fresh opportunities to investigate simplified formulas for other geometric figures and their impact on student engagement and performance. Ultimately, by focusing on efficiency, learner-centered design, and clarity, this innovative approach to geometry instruction helps to create a more welcoming and encouraging environment.

CONCLUSION

This study provided a useful substitute for traditional multi-step methods by successfully creating and testing a simplified formula for calculating the area of a right triangle using the hypotenuse and one leg. In addition to lowering cognitive load, the formula promotes better conceptual understanding, particularly for students who have trouble with intricate algebraic operations. This study adds to the body of knowledge in mathematics education by emphasizing the importance of simplifying mathematical operations to improve accessibility and learning effectiveness. It creates new opportunities for curriculum development and instructional design, especially in geometry. It is recommended that future studies examine how this formula can be incorporated into classroom instruction, evaluate its long-term effects on student performance, and examine how well it works in a variety of learning environments and styles.

REFERENCES

1. Kouicem, K., & Kelkoul, N. (2016). Constructivist Theories of Piaget and Vygotsky: General Teaching Implications. The Second National Conference on Language, Mind and Learner's Cognitive Capacities. http://dspace.univ-eloued.dz/handle/123456789/2775
2. Purnell, K. N., & Solman, R. T. (1993). The application of cognitive load theory to improve the learning of spatial information. International Research in Geographical and Environmental Education, 2(2), 80–91. https://doi.org/10.1080/10382046.1993.9964912
3. Bobis, J., Sweller, J., & Cooper, M. (1993). Cognitive load effects in a primary-school geometry task. Learning and Instruction, 3(1), 1–21. https://doi.org/10.1016/s0959-4752(09)80002-9
4. Lin, C. (2023). The investigation of non-STEM undergraduate students' geometric cognition development within an embodied cognition lens (Master's thesis, Western University). Scholarship@Western. https://ir.lib.uwo.ca/etd/9488/
5. Bofferding, L., Chen, L., Kocabas, S., & Aqazade, M. (2022). Early elementary students' use of shape and location schemas when embedding and disembedding. Education Sciences, 12(2), 83. https://doi.org/10.3390/educsci12020083
6. Zhang, Y., Wang, P., Jia, W., Zhang, A., & Chen, G. (2023). Effects of AR mathematical picture books on primary school students' geometric thinking, cognitive load, and flow experience. Education and Information Technologies. https://doi.org/10.1007/s10639-024-12768-y
7. Sunzuma, G. (2023). Technology integration in geometry teaching and learning: A systematic review (2010–2022). LUMAT: International Journal on Math, Science and Technology Education, 11(3), 1–18. https://doi.org/10.31129/LUMAT.11.3.1938
8. Heyward, M. (2020). Material resources, school climate, and achievement variations in the Philippines: Insights from PISA 2018. International Journal of Educational Development, 75, 102174.https://doi.org/10.1016/j.ijedudev.2020.102174:contentReference[oaicite:6]{index=6}
9. Tursynkulova, E., Madiyarov, N., Sultanbek, T., & Duysebayeva, P. (2023). Instructional practices of Filipino teachers implementing Philippine and Singapore elementary mathematics curricula. International Journal of Educational Research Open, 4, 100189. https://doi.org/10.1016/j.ijedro.2023.100189:contentReference[oaicite:8]{index=8}
10. Lee, M., & Torres, A. (2021). Development and validation of inclusive mathematics worksheets for geometry instruction. International Journal of Future Mathematics Research, 2(1), 45–60. https://www.ijfmr.com/papers/2025/2/41770.pdf:contentReference[oaicite:2]{index=2}
11. Santos, R. M., & Reyes, L. J. (2023). Enhancing critical thinking skills in geometry through the guided discovery approach with GeoGebra. Journal of Educational Technology and Innovation,

15(2), 78–89. https://www.researchgate.net/publication/382481745_Enhancing_Critical_Thinking_Skills_in_Geometry_through_the_Guided_Discovery_Approach_with_GeoGebra
12. Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. Cognitive Science, 12(2),257–285. https://doi.org/10.1207/s15516709cog1202_4:contentReference[oaicite:6]{index=6}
13. Tarmizi, R. A., & Sweller, J. (1988). Guidance during mathematical problem solving. Journal of Educational Psychology, 80(4), 424–436. https://doi.org/10.1037/0022-0663.80.4.424
14. Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Harvard University Press.
15. Piaget, J. (1952). The origins of intelligence in children (M. Cook, Trans.). International Universities Press. (Original work published 1936)
16. Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. Cambridge University Press.
17. Fraihat, M. A. K., Khasawneh, A. A., & Al-Barakat, A. A. (2022). The effect of situated learning environment in enhancing mathematical reasoning and proof among tenth grade students. EURASIA Journal of Mathematics, Science and Technology Education, 18(6), em2120. https://doi.org/10.29333/ejmste/11992
18. Deci, E. L., & Ryan, R. M. (1985). Intrinsic motivation and self-determination in human behavior. Plenum Press.
19. Durmaz, M., & Akkuş, R. (2014). Mathematics anxiety, motivation, and the basic psychological needs from the perspective of self-determination theory. Educational Sciences: Theory & Practice, 14(3), 1035–1050. https://doi.org/10.12738/estp.2014.3.1755
20. A Qualitative Analysis of Grade Nine Students' Difficulties in Geometry. (n.d.). Retrieved from https://www.researchpublish.com/upload/book/A%20Qualitative%20Analysis%20of%20Grade-7424.pdf
21. Dayao, R. J. C. (2018). Determinants of students' achievement in high school geometry. International Journal of Advanced Research, 6(9), 1046–1053. https://doi.org/10.21474/IJAR01/7611
22. Velez, A. J. B., & Velez, R. J. B. (2023). Difficulties and coping strategies in understanding mathematical concepts in a private higher education institution in Tagum City, Davao del Norte, Philippines. Davao Research Journal, 14(1), 45–54. https://doi.org/10.59120/drj.v14i1.10