On Uncertainty and Vagueness: A Comparative Study

Dr. Kuheli Biswas[1], Mr. Amal Das[2], Mr. Ashim Kumar Choudhury[3], Prof. (Dr.) Sanjib Kumar Datta[4]

[1,2,3]Department of Philosophy

[4]Department of Mathematics

[1,2,4]University of Kalyani, P.O.: Kalyani, Dist: Nadia, Pin: 741235, West Bengal, India.

[3]Ramakrishna Mission Vidyamandira, P.O.: Belur Math, Dist: Howrah, Pin: 711202, West Bengal, India.

DOI: https://dx.doi.org/10.47772/IJRISS.2025.90500089

Received: 14 May 2025; Accepted: 15 May 2025; Published: 31 May 2025

ABSTRACT

This paper delves into a multidimensional exploration of uncertainty through various theoretical lenses, including Probability Theory, Possibility Theory, Plausibility Theory, Belief Theory, Fuzzy Logic, Evidence Theory, and Vague Theory. Each framework offers a distinct perspective: Probability Theory deals with randomness, Possibility and Plausibility address feasibility and belief, Belief Theory integrates multiple sources of information, and Fuzzy Logic captures imprecision through degrees of truth. Vague Theory extends the discourse further by modeling information that is not only imprecise but also ill-defined or linguistically ambiguous. Evidence Theory serves as an overarching framework that synthesizes these perspectives. These theories are not mutually exclusive; instead, they complement each other and are often used in combination to model complex uncertainty in real-world applications. Integrating Vague Theory enhances our ability to reason under deep uncertainty, particularly in contexts involving subjective judgment and natural language. Together, these frameworks foster resilient, flexible, and nuanced decision-making under ambiguity.

Keywords and phrases: Uncertainty, Vagueness, Probability and Possibility theory, Evidence, Belief and Plausibility theory, Fuzzy logic.

INTRODUCTION

In two valued logic we find two extreme points of truth values a proposition is either true or it is false. In Mathematical term, it has two clear-cut positions in one position it is found 1 (one) and in another position it is found 0 (zero). And, according to two valued logics, between these two extremes or clear-cut sides of truth values there is no other truth functional status of a proposition in a standard form logical language.

But in our day-to-day conversation, and even in academic discussion we very often find some words or expressions which lack desirable precision. These are called vague words or vague statement. Since they are not well-set at any extreme points of truth value, that is, they are neither true nor false, they are discarded as meaningless.

But through a in-depth consideration we understand that these vague words or vague expressions are not entirely pointless. When we use, obviously in a pertinent situation, any vague word or statement, they successfully mean something, and we could understand what the speaker wants to say through a vague expression. As an example, we may site a case where vague expression is successfully used to convey certain situation. A person is found absconded since many years; nobody can trace him out after a tireless search. Now we cannot conclude that the man is dead, and even cannot assert that he is still alive. Thus, we cannot put any precise truth value, that is, true or false, to the statement that the man is dead or to the statement that the man is alive. But, at the sometime, we cannot discard his present position. It may be the case that one day he will come back or he might be found dead.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

Now our attempt may serve to give a proper logical status to such vague words or vague statements. Two-valued logic fails to give any truth functional status to these vague words or statements. Now if we extend to three-valued logic or many-valued logic (if required) then we may render a new kind of logical position, a third kind of truth functional status to vagueness. However, in this case, uncertainty is often associated with vagueness. Uncertainty refers to a lack of surety or confidence about something. It arises when there is incomplete, unknown, or unreliable information about a situation or outcome. For example, you are uncertain if it will rain tomorrow because the weather forecast is not clear.

Uncertainty and vagueness both deal with imprecision, but in different ways. Uncertainty is about not knowing (lack of information or clarity about what is true). On the other hand, vagueness is about not defining clearly (blurry or fuzzy boundaries of meaning). But in many cases, they can also coexist. For example, if a doctor says a patient is "somewhat ill", it's both vague (how much is “somewhat”?) and uncertain (we don’t know exactly what illness or stage). Therefore, the aim of our research is to explore various types of uncertainty and vagueness theories in logic and conduct a comparative discussion between them.

Uncertainty: The Unknown Territory.

Uncertainty is a fundamental aspect of life, permeating every aspect of our existence. It is the unknown, the unpredictable, and the uncontrollable. Uncertainty can be overwhelming, causing anxiety and fear, but it can also be a catalyst for growth, innovation, and progress. In our pursuit of knowledge and understanding, we often encounter uncertainty.

- **It can manifest in various forms, such as:**
    - Lack of information or data
    - Limited understanding or expertise
    - Unpredictable events or outcomes
    - Ambiguity or vagueness
    - Complexity or chaos.
- **Uncertainty can be classified into different types, including:**
    - **Epistemic uncertainty:** related to our knowledge or understanding
    - **Ontological uncertainty:** related to the nature of reality
    - **Aleatoric uncertainty:** related to randomness or chance
- **Managing uncertainty is crucial in various domains, such as:**
    - Decision-making
    - Risk assessment
    - Problem-solving
    - Planning and forecasting
    - Communication and collaboration.
- **To navigate uncertainty, we can employ various strategies, such as:**
    - Gathering more information
    - Seeking expert advice
    - Diversifying options
    - Building flexibility
    - Embracing ambiguity.

By acknowledging and embracing uncertainty, we can develop a more nuanced understanding of the world and our place within it. We can learn to tolerate ambiguity, adapt to changing circumstances, and cultivate resilience in the face of uncertainty. However, in the following sections, we will correlate different theories of uncertainty in a deeper way under the flavor of the concept, exploring its types, sources, and impacts, as well as strategies for managing and embracing it.

Interrelation: Probability, Possibility and Belief theory.

Probability, possibility, and belief theory are related but distinct concepts in uncertainty modeling. Now here is

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

a brief overview of each:

- **Probability Theory:**
  - Deals with chance events and their likelihood
  - Assigns numerical probabilities (0-1) to events
  - Follows axioms: normalization, non-negativity, and countable additivity
  - Examples: coin tossing, random variables
- **Possibility Theory:**
  - Handles uncertain events with fuzzy boundaries
  - Uses possibility measures (0-1) to represent degrees of possibility
  - Based on fuzzy sets and fuzzy logic
  - Examples: image recognition, natural language processing.
- **Belief Theory (Evidence Theory):**
  - Combines probability and possibility theories
  - Represents uncertainty using belief functions (0-1)
  - Handles both probabilistic and non-probabilistic uncertainty
  - Examples: expert systems, sensor fusion, decision-making.
- **Interrelations:**
  - Probability theory is a special case of possibility theory (crisp sets)
  - Possibility theory is a generalization of probability theory (fuzzy sets)
  - Belief theory encompasses both probability and possibility theories
  - Belief functions can be constructed from probability distributions and possibility measures.

In summary, it is found that Probability theory deals with chance events and their likelihood, Possibility theory handles uncertain events with fuzzy boundaries**,** and Belief theory combines both and represents uncertainty using belief functions. These theories are connected and can be used together to model complex uncertainty in various applications.

Possibility theory versus probability logic.

Possibility theory and probability logic are both mathematical frameworks for dealing with uncertainty, but they have different approaches and interpretations:

- **Possibility Theory:**
  - Deals with possibility and necessity measures
  - Based on fuzzy sets and fuzzy logic
  - Uses possibility and necessity measures to represent uncertainty
  - Focuses on handling imprecise and vague information
  - Used in applications like decision-making, information fusion, and machine learning.
- **Probability Logic:**
  - Deals with probability measures
  - Based on probability theory
  - Uses probability measures to represent uncertainty
  - Focuses on handling randomness and uncertainty
  - Used in applications like statistics, machine learning, and artificial intelligence.
- **Key differences:**
  - **Interpretation:** Possibility theory focuses on possibility and necessity, while probability logic focuses on probability.
  - **Approach:** Possibility theory uses fuzzy sets and fuzzy logic, while probability logic uses probability theory.
  - **Uncertainty handling:** Possibility theory handles imprecise and vague information, while probability logic handles randomness and uncertainty.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

In summary, possibility theory and probability logic are both used to handle uncertainty, but they have different approaches and interpretations. Possibility theory focuses on possibility and necessity, while probability logic focuses on probability.

- **Detailed differences:**
  - **Interpretation:**
    - **Possibility theory:** Focuses on possibility and necessity measures, which represent the degree of possibility or necessity of an event.
    - **Probability logic:** Focuses on probability measures, which represent the likelihood of an event.
  - **Approach:**
    - **Possibility theory:** Based on fuzzy sets and fuzzy logic, which allow for the representation of imprecise and vague information.
    - **Probability logic:** Based on probability theory, which is a well-established mathematical framework for representing and analyzing randomness.
  - **Uncertainty handling:**
    - **Possibility theory:** Handles imprecise and vague information, which is often encountered in real-world applications.
    - **Probability logic:** Handles randomness and uncertainty, which is also common in real-world applications.
  - **Measures:**
    - **Possibility theory:** Uses possibility and necessity measures, which are defined as:
      - **Possibility measure:** μ (A) = sup {x ∈ A}
      - **Necessity measure:** ν (A) = inf {x ∈ A}
    - **Probability logic**: Uses probability measures, which are defined as:
      - **Probability measure:** $P(A) = \int A\ f(x)\ dx$
  - **Applications:**
    - **Possibility theory:** Used in applications like decision-making, information fusion, and machine learning, where imprecise and vague information is common.
    - **Probability logic:** Used in applications like statistics, machine learning, and artificial intelligence, where randomness and uncertainty are common.
  - **Relationship:**
    - Possibility theory and probability logic are related, but distinct frameworks.
    - Possibility theory can be seen as a generalization of probability logic, as it can handle both precise and imprecise information.
  - **Fuzzy sets:** Possibility theory relies heavily on fuzzy sets, which allow for the representation of imprecise and vague information. Probability logic does not use fuzzy sets.
  - **Probability theory:** Probability logic is based on probability theory, which provides a well-established mathematical framework for representing and analyzing randomness. Possibility theory does not rely on probability theory.
  - **Imprecision:** Possibility theory is designed to handle imprecision and vagueness, while probability logic focuses on randomness and likelihoods.
  - **Uncertainty representation:** Possibility theory represents uncertainty using possibility and necessity measures, while probability logic uses probability measures.

Relationship: Probability, Possibility and Plausibility.

Probability, possibility, and plausibility are related concepts in the field of uncertainty modeling and reasoning. Now we will try to give here's a brief overview of each concept and their relationships:

- **Probability:**
  - Quantifies the likelihood of an event occurring
  - **Range:** [0, 1]
  - **Interpretation:** Probability of an event A is 0.7, means that the chance of A occurring is 70%

- **Possibility:**
  - Measures the degree to which an event is possible
  - **Range:** [0, 1]
  - **Interpretation:** Possibility of an event A is 0.7, means that the evidence supports A to some extent, but does not guarantee its occurrence
- **Plausibility:**
  - Measures the degree of belief in an event
  - **Range:** [0, 1]
  - **Interpretation:** Plausibility of an event A is 0.7, means that the evidence suggests A is plausible, but does not quantify the likelihood
- **Relationships:**
  - **Probability → Possibility:** Probability implies possibility, but not vice versa. If an event is probable, it is also possible.
  - **Possibility → Plausibility:** Possibility implies plausibility, but not vice versa. If an event is possible, it is also plausible to some extent.
  - **Plausibility → Probability:** Plausibility does not directly imply probability, but can be used to inform probability assignments.

In summary, Probability focuses on the likelihood of an event**,** Possibility focuses on the degree of support for an event, and Plausibility focuses on the degree of belief in an event. These concepts are related but distinct, and are used in different contexts to model and reason about uncertainty.

Boolean logic versus Probabilistic logic.

Boolean logic and probabilistic logic are both systems for reasoning and decision-making, but they differ in their approach to handling uncertainty.

- **Boolean logic:**
  - Uses binary values (true or false, 0 or 1)
  - Based on classical logic and set theory
  - Uses operators like AND, OR, and NOT
  - Does not handle uncertainty explicitly.
- **Probabilistic logic:**
  - Uses probability values (between 0 and 1)
  - Based on probability theory
  - Uses operators like probability conjunction and probability disjunction
  - Handles uncertainty explicitly.
- **Key differences:**
  - **Truth values:** Boolean logic uses binary values, while probabilistic logic uses probability values.
  - **Uncertainty handling:** Boolean logic does not handle uncertainty explicitly, while probabilistic logic does.
  - **Operators:** Boolean logic uses crisp operators, while probabilistic logic uses probabilistic operators.

Probabilistic logic is more suited to handling uncertainty and imprecision, while Boolean logic is more suited to handling crisp, binary values.

- **Detail differences:**
  Here is a more detailed write-up on the differences between Boolean logic and probabilistic logic:
  - **Truth Values:**
    - **Boolean logic:** Uses binary values (true or false, 0 or 1).
    - **Probabilistic logic:** Uses probability values (between 0 and 1).

- **Uncertainty Handling:**
  - **Boolean logic:** Does not handle uncertainty explicitly.
  - **Probabilistic logic:** Handles uncertainty explicitly using probability values.
- **Operators:**
  - **Boolean logic:** Uses crisp operators (AND, OR, NOT).
  - **Probabilistic logic:** Uses probabilistic operators (probability conjunction, probability disjunction).
- **Approach:**
  - **Boolean logic:** Based on classical logic and set theory.
  - **Probabilistic logic**: Based on probability theory.
- **Suitability:**
  - **Boolean logic:** More suited to handling crisp, binary values.
  - **Probabilistic logic:** More suited to handling uncertainty and imprecision.
- **Applications:**
  - **Boolean logic:** Commonly used in computer science, electronics, and mathematics.
  - **Probabilistic logic:** Commonly used in artificial intelligence, machine learning, and data analysis.
- **Reasoning:**
  - **Boolean logic:** Supports deductive reasoning.
  - **Probabilistic logic:** Supports probabilistic reasoning and decision-making under uncertainty.
- **Complexity:**
  - **Boolean logic:** Relatively simple and computationally efficient.
  - **Probabilistic logic:** More complex and computationally intensive due to the use of probability values and operators.

Relationship: Boolean logic and Fuzzy logic.

Boolean logic and fuzzy logic are both systems for reasoning and decision-making, but they differ in their approach to handling uncertainty and imprecision. Boolean logic is a crisp logic, meaning it uses binary values (true or false, 0 or 1) to represent truth values. It is based on the principles of classical logic and uses operators like AND, OR, and NOT to combine statements. Fuzzy logic, on the other hand, is a soft logic that uses fuzzy sets and fuzzy operators to represent and manipulate uncertain or imprecise information. It allows for degrees of truth rather than binary true or false values.

- **Relationship:**
  - Fuzzy logic is an extension of Boolean logic.
  - Fuzzy logic generalizes Boolean logic by allowing for degrees of truth.
  - Boolean logic is a special case of fuzzy logic where truth values are restricted to 0 and 1.
  - Fuzzy logic provides a more nuanced and flexible approach to handling uncertainty and imprecision than Boolean logic.
- **Key differences:**
  - **Truth values:** Boolean logic uses binary values, while fuzzy logic uses degrees of truth.
  - **Operators:** Boolean logic uses crisp operators, while fuzzy logic uses fuzzy operators.
  - **Uncertainty handling:** Boolean logic does not handle uncertainty explicitly, while fuzzy logic does.

In summary, fuzzy logic builds upon Boolean logic by introducing degrees of truth and fuzzy operators to handle uncertainty and imprecision. Boolean logic is a special case of fuzzy logic where truth values are restricted to binary values.

- **Detail differences:** Here is a more detailed write-up on the relationship between Boolean logic and fuzzy logic:
  - **Fuzzy logic is an extension of Boolean logic:** Fuzzy logic generalizes Boolean logic by allowing

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

for degrees of truth rather than binary true or false values. This means that fuzzy logic can handle uncertain or imprecise information, whereas Boolean logic is limited to crisp, binary values.

- **Fuzzy logic generalizes Boolean logic by allowing for degrees of truth:** In Boolean logic, a statement is either true or false. In fuzzy logic, a statement can have a degree of truth that ranges between 0 and 1. This allows for more nuanced and flexible reasoning.
- Boolean logic can be seen as a special case of fuzzy logic where the truth values are restricted to 0 and 1. This means that Boolean logic is a subset of fuzzy logic.
- Fuzzy logic provides a more nuanced and flexible approach to handling uncertainty and imprecision than Boolean logic. Fuzzy logic allows for the representation of uncertainty and imprecision using fuzzy sets and fuzzy operators. This provides a more nuanced and flexible approach to handling uncertainty and imprecision than Boolean logic.
- **Truth values:** Boolean logic uses binary values, while fuzzy logic uses degrees of truth. Boolean logic uses binary values (true or false, 0 or 1) to represent truth values. Fuzzy logic uses degrees of truth that range between 0 and 1.
- **Operators:** Boolean logic uses crisp operators, while fuzzy logic uses fuzzy operators. Boolean logic uses crisp operators like AND, OR, and NOT. Fuzzy logic uses fuzzy operators like fuzzy AND, fuzzy OR, and fuzzy NOT.
- **Uncertainty handling:** Boolean logic does not handle uncertainty explicitly, while fuzzy logic does.

Boolean logic does not have a built-in mechanism for handling uncertainty. Fuzzy logic, on the other hand, has a built-in mechanism for handling uncertainty using fuzzy sets and fuzzy operators.

Relationship: Evidence theory, Probability logic and Fuzzy logic.

Evidence Theory, Probability Logic, and Fuzzy Logic are all related to uncertainty modeling and reasoning. But they differ in their approaches and interpretations.

- **Differences:**
  - **Evidence Theory:**
    - Deals with uncertainty and imprecision
    - Uses belief functions and plausibility measures
    - Handles both epistemic and aleatoric uncertainty.
  - **Probability Logic:**
    - Deals with randomness and uncertainty
    - Uses probability measures and Bayesian inference
    - Focuses on aleatoric uncertainty.
  - **Fuzzy Logic:**
    - Deals with vagueness and imprecision
    - Uses fuzzy sets and fuzzy reasoning
    - Handles epistemic uncertainty.
- **Relationships:**
  - Evidence Theory generalizes Probability Logic by allowing for imprecision and uncertainty in probability assignments.
  - Fuzzy Logic is a specific interpretation of Evidence Theory, using fuzzy sets to represent uncertainty.
  - Probability Logic is a special case of Evidence Theory, where the belief function is a probability measure.

In summary, Evidence Theory is a broad framework for uncertainty modeling, encompassing both Probability Logic and Fuzzy Logic. On the other hand, Probability Logic focuses on randomness and aleatoric uncertainty, while Fuzzy Logic focuses on vagueness and epistemic uncertainty. Whereas Fuzzy Logic can be seen as a specific implementation of Evidence Theory, using fuzzy sets to represent uncertainty.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue V May 2025

Relationship: Evidence theory, Probability theory, Possibility theory, Plausibility theory and Belief theory.

- **Here is a brief overview of each theory and their relationships:**
  - **Evidence Theory:**
    - Deals with uncertainty and imprecision
    - Uses belief functions and plausibility measures
    - Handles both epistemic and aleatoric uncertainty.
  - **Probability Theory:**
    - Deals with randomness and uncertainty
    - Uses probability measures and Bayesian inference
    - Focuses on aleatoric uncertainty.
  - **Possibility Theory:**
    - Deals with uncertainty and imprecision
    - Uses possibility measures and necessity measures
    - Handles epistemic uncertainty.
  - **Plausibility Theory:**
    - Deals with uncertainty and imprecision
    - Uses plausibility measures
    - Handles both epistemic and aleatoric uncertainty.
  - **Belief Theory:**
    - Deals with uncertainty and imprecision
    - Uses belief functions
    - Handles both epistemic and aleatoric uncertainty.
- **Relationships:**
  - Evidence Theory generalizes Probability Theory by allowing for imprecision and uncertainty in probability assignments.
  - Possibility Theory is a specific interpretation of Evidence Theory, using possibility measures to represent uncertainty.
  - Plausibility Theory is a component of Evidence Theory, using plausibility measures to represent uncertainty.
  - Belief Theory is a component of Evidence Theory, using belief functions to represent uncertainty.
  - Probability Theory is a special case of Evidence Theory, where the belief function is a probability measure.

In summary, Evidence Theory is a broad framework that encompasses Probability Theory, Possibility Theory, Plausibility Theory, and Belief Theory. On the other hand, Possibility Theory and Plausibility Theory are specific interpretations of Evidence Theory. Whereas Belief Theory is a component of Evidence Theory, and Probability Theory is a special case of Evidence Theory. However, we should note here that these theories are not mutually exclusive, and many researchers use combinations of these theories to model and reason about uncertainty.

CONCLUSION

Effectively understanding and managing uncertainty necessitates a comprehensive and multidimensional theoretical approach. Probability Theory offers tools for dealing with randomness and quantifiable risk, while Possibility and Plausibility Theories assess the feasibility and belief in outcomes that may not be strictly numerical. Belief Theory aggregates evidence from multiple sources, allowing for flexible reasoning under partial knowledge. Fuzzy Logic enhances classical logic by introducing degrees of truth, enabling the modeling of imprecise or gradational concepts that are common in real-world reasoning.

Vague Theory extends this even further by addressing not just imprecision, but fundamental vagueness and linguistic ambiguity—situations where concepts lack clear boundaries or formal definitions. For example, expressions like "a tall person" or "very soon" cannot be easily quantified, yet they are essential to human communication and judgment. Vague Theory equips us to model and interpret such inherently fuzzy expressions, enhancing our reasoning in domains where subjective or language-based assessments are dominant.

Evidence Theory serves as an integrative framework that encompasses and connects all of these perspectives, providing a robust structure for managing diverse forms of uncertainty. Together, these theories form a toolkit that can be tailored to different types of uncertainty—whether aleatoric (due to randomness), epistemic (due to lack of knowledge), or vague (due to linguistic or conceptual ambiguity).

This combined theoretical framework is particularly powerful in complex and dynamic fields such as artificial intelligence, medical diagnostics, financial forecasting, climate modeling, risk assessment, and human decision-making. While uncertainty can never be fully eliminated, these theories provide the means to understand, navigate, and act wisely in the face of it. Ultimately, embracing uncertainty through structured reasoning is not a limitation but a strength—it deepens our insight, enhances our adaptability, and expands the boundaries of What We Can Know and Achieve.

ACKNOWLEDGEMENT

The fourth author sincerely acknowledges the financial support rendered by DST-FIST 2024-2025 running at the Department of Mathematics, University of Kalyani, P.O.: Kalyani, Dist: Nadia, Pin: 741235, West Bengal, India.

BIBLIOGRAPHY

1. Alexeyev, A. V. and Borisov, A. N., Et al. “A linguistic approach to decision-making problems.” Fuzzy Sets and Systems, 22(1-2), 1987: pp. 25-41. Web.
2. Atanassov, K. and Gargov, G. “Interval valued intuitionistic fuzzy sets.” Fuzzy Sets and Systems, 31(3), 1989: pp. 343-349. Web.
3. Albrecht, M. “Approximation of functional relationships to fuzzy observations.” Fuzzy Sets and Systems, 49(3), 1992: pp. 301-305. Web.
4. Black, M. “Vagueness: an exercise in logical analysis.” Philosophy of Science, 4(4), 1937: pp.427-455. Web. (Reprinted in International Journal of General Systems. 17(2-3), 1990: pp. 107-128. Web).
5. Copi, Irving M. and Cohen, C., Et al. Introduction to Logic. London & New York: Routledge Taylor & Frincis Group, 2016, pp. 539-561. Print.
6. J. Hurley, P. A Concise Introduction to Logic. Boston: Wadsworth Cengage Learning, 2012, pp. 554-570. Print.
7. J. Klir, G. and Yuan, B. Fuzzy Sets and Fuzzy Logic: Theory and Applications. New Jersey: Prentic Hall PTR, 1995, pp. 177-278. Print.