Difference of Squares of Two Consecutive Even Numbers: A Mathematical Investigation

Ronald S. Decano, Jernel C. Ondong., Liezel P. Vasay., Greatchen L. Medrano

Holy Cross of Davao College Center for Research Sta. Ana Ave., Davao City

DOI: https://dx.doi.org/10.47772/IJRISS.2025.9020061

Received: 16 January 2025; Accepted: 28 January 2025; Published: 03 March 2025

ABSTRACT

This study investigates the difference of squares between two consecutive even numbers using the formula 4(n+1), derived from the general identity $a^2 - b^2 = (a-b)$ of (a+b). The research aims to uncover more profound insights into this formula's mathematical properties, sequence behavior, and divisibility characteristics. The study adopts theoretical and computational approaches that validate the formula for fifteen examples of consecutive even numbers. The formula 4(n+1) has been validated for a wide range of even numbers, and its results match the manually computed difference of squares $(2n+2)^2-(2n)^2$. For all even 2n, the difference of squares simplifies to 4(n+1), confirming the formula's accuracy and consistency. The formula 4(n+1) provides valuable insights into number patterns, such as divisibility and arithmetic sequences, highlighting its importance in solving mathematical problems. Its simplicity also makes it a powerful tool for teaching foundational concepts and exploring connections across various areas of mathematics.

Keywords: consecutive even numbers, difference of squares

INTRODUCTION

Background of the Study

The difference of squares dates back to ancient mathematics. This identity was formally recorded in the writings of Greek mathematicians, such as Euclid, who described relationships between numbers and geometric representations (Hardy, 1940). Later, Islamic mathematicians, such as Al- Khwarizmi, built upon these principles, which became the basis for modern algebra.

In addition, the focus on even numbers is of great importance in number theory, a field that historically examines the properties and relationships of integers. Consecutive even numbers, a simple and specific case of integer sequences, offer a clear example for understanding algebraic structures and patterns. The study of such sequences builds on centuries of inquiry into integer properties, including their symmetries and predictable behaviors (Smith, 2010).

From the educational viewpoint, the difference in squares' identity represents a keystone in algebra regarding expression simplification, solving equations, and showing simple mathematical principles. Its use of consecutive even numbers is of greater value both for theoretical insight and practical teaching methods, focusing on the role of particular cases in giving mathematical insight (Jones, 2015).

Moreover, the difference of squares is a foundational concept in algebra, often used to simplify expressions, solve equations, and uncover hidden patterns in mathematical sequences. This research proposal investigates the difference of squares between two consecutive even numbers utilizing a formula, deriving from the general identity for the difference of squares. The study focuses on uncovering more profound insights into this formula, exploring its mathematical properties, applications, and potential implications in advanced algebra and number theory.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

Research Problem

The proposed study aims to understand the formula's underlying mathematical structure, verify its accuracy, explore its applications, and uncover deeper relationships between consecutive even numbers and their properties. Below are the fundamental research problems that this study seeks to address:

1. What formula can be derived from the difference of squares of two consecutive even numbers?
2. What is the typical difference in the sequence of differences of squares for consecutive even numbers? How does the difference between consecutive terms evolve as n increases?
3. What is the possible divisibility of the result of the difference of squares between two consecutive even numbers?

Motivation and Objectives

This study aims to explore the intriguing relationship between consecutive even numbers and the difference of their squares and investigate the formula governing this difference. Understanding the difference of squares is a foundational concept in algebra, and it can have far-reaching applications in number theory, sequence analysis, and divisibility. While the difference of squares for any two numbers can be expressed through the identity $a^2 - b^2 = (a-b)(a+b)$, this study focuses on the particular case of consecutive even numbers, which offers unique patterns and mathematical properties.

The deriving formula which represents the difference of squares of two consecutive even numbers 2n and 2n+2, provides a direct and efficient way to calculate this difference. It reveals essential insights into the nature of even numbers and their algebraic relationships. However, despite its simplicity, this formula is rarely discussed in elementary algebra or number theory textbooks, which makes it an interesting subject for deeper exploration.

Additionally, this study is motivated by the potential applications of the formula in various areas of mathematics, such as:

1. Number Theory: Investigating divisibility properties and modular arithmetic.
2. Combinatorics: Understanding how sequences behave and grow, which can have applications in counting problems and sequence generation
3. Computational Mathematics: Providing a tool for fast computation in algorithms that deal with even integers, differences of squares, or sequence generation

The primary objectives of this study are as follows:

4. To explore that the formula deriving from the difference of squares of two consecutive even numbers, 2n and 2n+2, for all even values of n.
5. To examine the sequence formed by the difference of squares of consecutive even numbers and determine whether it follows a linear (arithmetic) progression.
6. To investigate the divisibility properties of the difference of squares formula.

Preliminary Definitions and Notations

The Difference of Squares Formula

The difference of squares formula is one of algebra's most straightforward and valuable identities. It states that:

$$a^2 - b^2 = (a - b)(a + b)$$

This identity allows us to express the difference between the squares of two numbers as the product of their sum and their difference. For two general numbers, *a* and *b*, this formula is a powerful tool in simplifying algebraic expressions, factoring polynomials, and solving equations.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

Consecutive Even Numbers

Even numbers divisible by 2, i.e., can be written as 2n, where n is a number. Consecutive even numbers are pairs of even numbers that differ by exactly 2. If 2n is an even number, the next consecutive even number is 2n+2. For example, 4 and 6, 10 and 12, and 18 and 20 are consecutive even numbers.

The investigation centers on the difference of squares of two consecutive even integers 2n and 2n+2. Thus, the researchers want to explore the algebraic expression:

$(2n+2)^2 - (2n)^2$

Derivation of the Formula

To explore the difference of squares between two consecutive even numbers, we begin by expanding the expression $(2n+2)^2 - (2n)^2$:

$(2n+2)^2 - (2n)^2 = (4n^2+4n+4) - 4n^2 = 4n+4 = 4(n+1)$

Thus, the difference between the squares of two consecutive even numbers 2n and 2n+2 simplifies to:

$(2n+2)^2 - (2n)^2 = 4n + 4 = 4(n + 1)$

This formula 4(n+1) expresses the difference of squares of two consecutive even numbers as a multiple of 4, where n+2 is the first number in the pair. This simple result highlights a critical relationship between the difference of squares and the structure of even numbers.

Mathematical Implications

The formula provides several interesting properties and implications:

- Divisibility by 4: Since 4(n+1) is a multiple of 4, the difference of squares of any two consecutive even numbers is always divisible by 4. This property is crucial in understanding how the set of even numbers behaves under squaring and subtraction.
- Arithmetic Sequence: The formula 4(n+1) generates an arithmetic sequence of multiples of 4 as n increases. The sequence starts at 12 and increases by 8 for each subsequent pair of consecutive even numbers. For example, for n=2,4,6,8n = 2, 4, 6, 8n=2,4,6,8, the corresponding differences of squares are, 12,20,28,36, and so on.
- Linear Growth: The difference of squares grows linearly with n, with a constant rate of increase of 8 for each increase in n. This property can be helpful when investigating sequences of square differences or exploring relationships between square numbers and their consecutive differences.

Applications and Significance

The difference of squares formula has significant applications in various areas of mathematics:

- **Algebraic Simplification**: The difference of squares formula is frequently used in algebra to simplify expressions and factor polynomials. The formula provides a convenient method for expressing the difference between the squares of two consecutive even numbers in a simplified form, which can be used to solve equations or factoring expressions.
- **Arithmetic Sequences**: Since the difference between the squares of consecutive even numbers follow a linear pattern, it can help generate or analyze arithmetic sequences. This is particularly relevant in problems involving a sum of squares, series, or integer sequences.
- **Divisibility and Number Theory**: The result is always divisible by a certain numbe, which ties into studies of divisibility in number theory. It also connects to the behavior of even numbers under squaring and subtraction, which is essential for understanding modular arithmetic, integer partitions,

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

and other number-theoretic concepts.

- **Polynomial Factorization**: The formula for the difference of squares is vital in factoring higher-degree polynomials. While this research focuses on consecutive even integers, understanding the properties of the difference of squares in this context could lead to insights into factoring polynomials with integer or even-numbered roots.
- **Mathematical Problem Solving**: The formula could be applied to solve problems in algebraic problem-solving contexts, such as competitions or mathematical modeling, where recognizing patterns in squares and their differences is critical to finding solutions.

LITERATURE REVIEW

Relevant Theories Difference of Squares Identity

The fundamental identity for the difference of squares states that $a^2-b^2=(a-b)(a+b)$. This theory was first formalized in Euclid's elements. This theorem is fundamental to algebra and is the basis for analyzing the difference of squares of two consecutive even numbers. It explains how the difference between the squares of two numbers can be expressed as a product of their sum and difference. For two consecutive even numbers 2n and 2n+2, the difference can be expressed as:

$$(2n+2)^2 - (2n)^2 = 4n^2+4n+4-4n^2 = 4n+4 = 4(n+1)$$

This investigates how this identity can be generalized or applied to other sets of numbers or polynomial expressions.

Arithmetic Progressions

- Consecutive even integers form an arithmetic progression. This can lead to a deeper understanding of their properties, such as mean and variance, and provide insights into optimization problems and number theory.

Quadratic Functions and Their Properties

- The study of squares invokes quadratic functions, which have profound geometric interpretations. Analyzing the quadratic nature of the squares of consecutive even numbers can yield insights into vertex forms, roots, and the role of parameters.

Number Theory Applications

It analyzes the implications of the formula on prime factorization and divisibility, exploring even numbers concerning conjectures like Goldbach's conjecture or twin prime theorem**.**

Research Gaps

The proposed study can address several key research gaps:

1. A more focused investigation is needed into algebraic identities for consecutive even numbers and their differences in squares.
2. The application of this formula in educational contexts and problem- solving techniques still needs to be explored.
3. Computational applications of such algebraic identities, particularly in optimization, can provide new directions for research.
4. A deeper exploration of the formula's implications in number theory, combinatorics, and algebraic structures is needed.

The proposed study has the potential to contribute to both theoretical understanding and practical applications in mathematics by filling these gaps.

The study also aims to bridge the gap between abstract mathematical theory and practical computational methods, using tools like Excel to validate the formula and explore its properties numerically. This combination of theoretical and computational investigation provides a solid framework for understanding the structure of consecutive even integers and their algebraic characteristics.

METHODOLOGY/ APPROACH

The study is mathematical research that uses computational tools and numerical methods to devise an alternate formula, precisely the difference of squares of two consecutive numbers.

Theoretical Approach

The study of the difference of squares between two consecutive even numbers using the formula 4(n+1) provides a rich area for exploration. This theoretical approach will focus on understanding the properties of the formula within the context of algebra, number theory, and sequence generation. It will build on foundational concepts in these areas while examining the formula's broader implications in generating sequences, uncovering divisibility properties, and analyzing algebraic structures. Generally, this study intends to use:

Direct/ Deductive Proof. Shows that a proposition is true by applying definitions, existing facts, and sound reasoning. Without making any assumptions about the outcome, it starts with known premises and works its way through the steps to get the intended conclusion.

Computational Approach

The computational approach to this study will leverage numerical experimentation, algorithmic validation, and computational tools to explore and validate the formula 4(n+1) for the difference of squares of two consecutive even numbers. This approach will help uncover further insights into the formula's behavior, efficiency, and potential applications in broader mathematical contexts. It will also provide a more practical way of visualizing the mathematical relationships and testing various hypotheses associated with the study.

The study aims to computationally validate this formula and explore various properties by performing the following tasks:

- **Verifying the formula**: Ensuring that the formula holds for a range of even numbers.
- **Analyzing the growth pattern**: Observing how the difference of squares grows with increasing values of n.
- **Exploring divisibility**: Check that the result is always divisible by four and explore its behavior modulo other numbers (e.g., mod 8, mod 16).
- **Generating arithmetic sequences**: Identifying the sequence of results and exploring the properties of this sequence.
- **Analyzing computational efficiency**: Evaluating the time complexity of calculating the difference of squares for large values of N to assess the scalability of the formula.

RESULTS

Difference of Squares of Two Consecutive Even Integers

This section explores the difference of squares between two consecutive even numbers using the formula 4(n+1), where n is an even number.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

Illustrative Example 1: $4^2 - 2^2$, where n = 2 Applying the formula: **4 (n+1)**

4(n+1) = 4(2+1) by substituting n with 2

= 4(3) by PEMDAS

= **12**

This is the difference of squares between two consecutive numbers, 2 and 4.

Illustrative Example 2: $\mathbf{6^2 - 4^2}$, where n = 4 Applying the formula: 4(n+1)

4(n+1) = 4(4+1) by substituting n with 2

= 4(5) by PEMDAS

= **20**

This is the difference of squares between two consecutive numbers, 4 and 6.

Illustrative Example 3: $\mathbf{8^2 - 6^2}$,where n= 6 Applying the formula: 4(n+1)

4(n+1) = 4(6+1) by substituting n with 2

= 4(7) by PEMDAS

= **28**

This indicates the difference of squares between two consecutive numbers, 6 and 8.

Illustrative Example 4: $\mathbf{10^2 - 8^2}$, where n= 8 Applying the formula: 4(n+1)

4(n+1) = 4(8+1) by substituting n with 2

= 4(9) by PEMDAS

= **36**

This is the difference of squares between two consecutive numbers, 8 and 10.

Illustrative Example 5: $\mathbf{12^2 - 10^2}$ **,** where n = 10 Applying the formula: 4(n+1)

4(n+1) = 4(10+1) by substituting n with 2

= 4(11) by PEMDAS

= **44**

This is now the difference of squares between two consecutive numbers, 10and 12.

Table 1: Difference of Squares of Larger Two Consecutive Even Numbers

n	**n+2**	$\mathbf{a^2}$	$\mathbf{b^2}$	$\mathbf{a^2 - b^2}$	**4(n+1)**
9980	9982	99640324	99600400	35524	35524
9982	9984	99680256	99640324	39932	39932
9984	9986	99720196	99680256	39940	39940

9986	9988	99760144	99720196	39948	39948
9988	9990	99800100	99760144	39956	39956
9990	9992	99840064	99800100	39964	39964
9992	9994	99880036	99840064	39972	39972
9994	9996	99920016	99880036	39980	39980
9996	9998	99960004	99920016	39988	39988
9998	10000	100000000	99960004	39996	39996

The table demonstrates that the formula $4(n+1)$ yields equivalent results to the general difference of squares identity $a^2 - b^2$.

Table 2: Difference in the Sequence of Difference of Squares for Consecutive Even Numbers

N	n+2	$(n+2)^2 - n^2$	4 (n+1)	Difference in the Sequence of Difference Squares of Two Consecutive Even Numbers
2	4	12	12	
4	6	20	20	8
6	8	28	28	8
8	10	36	36	8
10	12	44	44	8
12	14	52	52	8
14	16	60	60	8
16	18	68	68	8
18	20	76	76	8
20	22	84	84	8
22	24	92	92	8
24	26	100	100	8
26	28	108	108	8
28	30	116	116	8
30	32	124	124	8
32	34	132	132	8
34	36	140	140	8
36	38	148	148	8
38	40	156	156	8
40	42	164	164	8

The results indicate that the differences of squares of consecutive even numbers follow a common difference of 8.

This can be further proven by deriving the formula: $D_n = 4(n+1)$

For consecutive even numbers, the difference between consecutive terms in this sequence is:

$D_{n+2} - D_n = 4((n+2) + 1) - 4(n+1)$

Simplifying:

$D_{n+2} - D_n = 4(n+3) - 4(n+1)$

$= 4n+12-4n-4$

$= 8$

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

Table 3.1. Divisibility of Differences of Squares of Two Consecutive Even Numbers

n	n+2	$(n+2)^2 - n^2$	4 (n+1)	Divisible by 4
				Remainder must be 0
2	4	12	12	0
4	6	20	20	0
6	8	28	28	0
8	10	36	36	0
10	12	44	44	0
12	14	52	52	0
14	16	60	60	0
16	18	68	68	0
18	20	76	76	0
20	22	84	84	0
22	24	92	92	0
24	26	100	100	0
26	28	108	108	0
28	30	116	116	0
30	32	124	124	0
32	34	132	132	0
34	36	140	140	0
36	38	148	148	0
38	40	156	156	0
40	42	164	164	0

The results show that differences of the squares of two consecutive even integers divided by 4 obtain a remainder of Zero, Likewise, the formula has a factor of 4, thus it is only divisible by 4.

Table 3.2. Difference of Squares of two Consecutive Even Integers in Modulo 8

n	n+2	$(n+2)^2 - n^2$	4 (n+1)	Modulo 8
2	4	12	12	4
4	6	20	20	4
6	8	28	28	4
8	10	36	36	4
10	12	44	44	4
12	14	52	52	4
14	16	60	60	4
16	18	68	68	4
18	20	76	76	4
20	22	84	84	4
22	24	92	92	4
24	26	100	100	4
26	28	108	108	4
28	30	116	116	4
30	32	124	124	4
32	34	132	132	4
34	36	140	140	4
36	38	148	148	4
38	40	156	156	4
40	42	164	164	4

The table shows that the difference of squares of two consecutive even integers in modulo eight results in 4. In other words, the difference of squares of two consecutive even integers, when divided by 8, obtains a common remainder of 4.

DISCUSSION

Analysis and Interpretation

Difference of Squares of Two Consecutive Even Numbers

The results demonstrate a consistent mathematical relationship regarding the difference of squares of two consecutive even integers. Utilizing the formula 4(n + 1), where n is an even integer, the outcomes from the illustrative examples reinforce the theoretical foundation that the difference of squares can be succinctly expressed in this manner. The calculations in the examples consistently yield valid results aligned with the mathematical expression of $a^2 - b^2 = (a-b)(a+b)$. By applying this to consecutive even integers, such as $2n$ and 2n + 2, it is seen that:

If n =2, then: 4(2 + 1) = 12 aligns with $4^2 - 2^2 = 12$.

As the examples progress through increments of n, the formula outputs reflect the general characteristics defined by this difference of squares identity.

This consistency confirms the existing mathematical literature on square differences and demonstrates a practical way to calculate them without evaluating squares explicitly.

Difference of Squares of Between Larger Two Consecutive Even Numbers in Excel

The first table reflects a detailed breakdown, showing that the results align with the generalized formula and the algebraic difference of squares identity $a^2 - b^2$ for larger even numbers. The second table reinforces this pattern, revealing that the calculated differences consistently yield a common difference of 8. This phenomenon highlights a predictable, sequential relationship.

Difference in the Sequence of Differences of Squares for Consecutive Even Numbers

The common difference of 8 underscores a broader concept in number theory where certain transformations or identities yield linear behaviors among even numbers.

Such a finding may resonate with the patterns discussed in number theory, particularly in works analyzing polynomial sequences and their differences.

Divisibility and Modulo Analysis

The results indicate a strong relationship between the difference of squares and divisibility:

Divisibility by 4. The consistency in yielding a remainder of 0 when the difference of squares is divided by 4 reflects a fundamental trait of even integers. Since all investigated cases result in divisibility, this could be a point of interest when considering the properties of even integers in modular arithmetic, as established in the mathematical literature. For instance, divisibility properties often parallel theories around primes and composites, where congruences define number characterizations.

Modulo 8. Notably, the common remainder of 4, when the differences are assessed under modulo 8, further solidifies the numerical behavior of the integers involved. This suggests that while the distinctions between two consecutive even integers are divisible by 4, their higher congruence distinctions align to yield a remainder of 4 in modulo 8.

This outcome may be compared to existing studies discussing the nature of polynomials modulo on certain bases, as different modular systems can yield unique patterns and properties that can reflect deeper mathematical truths.

ISSN No. 2454-6186 | DOI: 10.47772/IJRISS | Volume IX Issue II February 2025

The implications of this analysis extend beyond individual cases and into a broad understanding of relationships within even integers. The mathematical discourse surrounding sequences, identities, and their properties may benefit from acknowledging these patterns, as they enforce predictable outcomes when discussing quadratic relationships.

Further, the systematic approach to the difference of squares through theoretical formulas and empirical calculations showcases the robustness of established mathematical concepts. As new mathematicians engage with these principles, the findings could serve as a pedagogical tool, encouraging exploration into algebraic identities while emphasizing the utility of modular arithmetic in demonstrating the fundamental properties of numbers. The results reinforce that mathematical identities are not merely abstract concepts but tools that can illuminate patterns within sequences, ultimately enhancing our understanding of the number system.

Limitations and Challenges

While the findings robustly demonstrate patterns concerning the differences of squares of two consecutive even integers:

1. Scope: The results are specific to even numbers and do not extend to odd numbers or mixtures of odd and even integers.
2. Generalization: The analysis is grounded in the defined arithmetic properties of integers. It is limited to the context of numbers and might not fully apply to higher-dimensional algebraic structures or in non-integer domains.
3. Application of Arithmetic: The reliance on the properties of squares and even numbers suggest potential limitations if approached from different mathematical branches, such as geometric interpretations or within advanced number theory.

CONCLUSION AND FUTURE WORK

Summary of Findings

The following findings contribute valuable insight into the algebraic properties of even numbers and their squares, supporting theoretical and practical mathematics applications.

1. The formula 4(n+1) accurately computes the difference of squares for all even values of 2n and their consecutive numbers 2n+2.
2. The differences form a linear (arithmetic) progression with a consistent common difference 8.
3. The sequences formed by 4(n+1) are divisible by 4, with a remainder of zero when divided, indicating their divisibility properties. Further examination reveals that the difference of squares of these numbers yields a consistent remainder of 4 when considered under modulo eight operations.

Future Research

Future areas for investigation may include:

- Investigating the differences of squares using odd numbers, integers, and the patterns that may arise.
- Exploring the applications of this formula in other mathematical concepts or real-life scenarios.
- Extending the research to consider the implications of these findings in number theory or algebra.

REFERENCES

1. Hardy, G. H. (1940). A mathematician's apology. Cambridge University Press.
2. Jones, P. (2015). Teaching algebra through patterns and structures. Educational Mathematics Press.
3. Smith, J. (2010). Number theory and its applications. Springer.