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Formulating Mathematical Constants: Streamlining Shaded Area Calculations in Circle-Square Geometric Configurations

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ABSTRACT

This research solves the difficulty in computing shaded areas of configurations of circles and squares using the derivation of two constants: c_i=0.2146 for circles that inscribe and c_c=0.5708 for circumscribed circles. The article makes these calculations easier by giving general shortcut equations that avoid the process steps traditionally associated with such problems. With verification against the standard techniques, the new constants provided exact and efficient solutions, considerably reducing the computational effort and the possibility of mistakes. The research contributes to geometry by filling a gap in the literature, namely that no streamlined approach previously existed for calculating shaded area. Practical applications of such findings are evident in school settings, mathematical competitions, and professional work in architecture and engineering. By simplifying these geometric computations, the study enhances accessibility and problem-solving efficiency.

Beyond its immediate applications, the work shows the potential of these constants to inspire further developments in mathematics and related fields. Relevant to Euclidean geometry, optimization, and computational algorithms, the findings have interdisciplinary implications, extending to fields such as physics, design, and data visualization. This research establishes a foundation for future studies, including applications to irregular shapes and three-dimensional geometries and the development of computational tools to integrate these formulas into broader contexts.

Key Words: area, inscribed, circumscribed, shaded region, circle, square

INTRODUCTION

Background

Geometry has been a foundational pillar of mathematics for centuries, and it is critical to understanding the spatial relationships between shapes and structures. Among the vast array of geometric challenges, the study of shaded regions—those areas that lie between or overlap geometric figures—has garnered significant interest due to its elegance and practical applications. Ancient mathematicians such as Euclid and Archimedes laid the groundwork for understanding geometric shapes. Still, exploring relationships between circles and squares added an extra layer of mathematical beauty and complexity to the field.

The interplay between squares and circles is particularly compelling. Squares represent structured symmetry with straight edges and sharp angles, while circles symbolize infinite symmetry with continuous curvature. Their combination often results in intriguing problems, mainly when calculating the shaded regions formed by inscribing a circle within a square or circumscribing a square within a circle. These configurations are aesthetically pleasing and foundational in numerous fields, including architecture, engineering, and design. However, solving these shaded region problems typically involves multiple steps: calculating the areas of individual shapes, subtracting areas, and frequently involving the mathematical constant π, Which adds an extra layer of complexity.

Although traditional methods provide accurate solutions, they are often time-intensive, prone to computational errors, and require repeated application of lengthy formulas. This inefficiency poses a challenge in educational settings, competitive exams, and practical applications where quick, yet accurate calculations are essential. Thus, developing a simplified formula or constant to streamline these computations would significantly contribute to geometry and its related disciplines.

Research Problem

While traditional formulas for calculating shaded regions between a circle and a square are well-established, they are not optimized for efficiency. The process typically involves determining the areas of both shapes, subtracting the appropriate areas, and incorporating the constant π, which can make calculations cumbersome, especially in scenarios requiring precision and speed.

This issue comes especially to the forefront in two of the typical configurations:

1. The area within a square illuminated by a circle is determined by subtracting the area of the circle from the area of the square.
2. When a square is circumscribed by a circle, the shaded region represents the difference between the area of the circle and the area of the square.

Across the board, there are no universal shortcuts or easy paths toward something — you're forced to work through step-by-step processes. Although the math has improved, no general constant or formula has yet been established to resolve these problems quickly. This gap in mathematical tools highlights the need for a novel approach to address the inefficiency in solving shaded region problems.

The research problem is: How to create a mathematical constant for determining the shaded regions between squares and circles in inscribed and circumscribed configurations.

Motivation and Objectives

The motivation for this research stems from both mathematical curiosity and practical necessity. Shaded region problems are a common feature in mathematics curricula and appear frequently in mathematical competitions, problem-solving exercises, and real-world design scenarios. The ability to quickly compute these areas with high accuracy can benefit students, educators, and professionals alike. For educators, it provides a more efficient teaching tool; for students, it enhances understanding and reduces cognitive load; and for professionals, it simplifies tasks in architecture and engineering.

From a mathematical perspective, pursuing a universal constant for these problems contributes to the broader goal of creating elegant, efficient, and generalized solutions in geometry. Simplifying shaded region calculations reduces dependency on step-by-step approaches and fosters innovation in problem-solving techniques.

The objectives of this research are:

1. To derive a mathematical constant: Identify a unique constant that can simplify the computation of shaded regions in circle-square geometric configurations.

2. To create a shortcut equation, create a formula that works for both inscribed circles within squares and circumscribed circles.
3. Validate the formula by testing it against traditional methods for accuracy, reliability, and efficiency.

4. To demonstrate practical applications: Illustrate the formula's usefulness in educational, competitive, and real-world problem-solving scenarios.

PRELIMINARY DEFINITIONS AND NOTATIONS

To ensure clarity and consistency throughout this study, the following key definitions, notations, and assumptions will be used:

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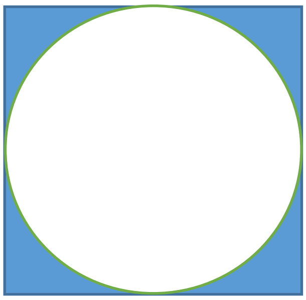
Definitions

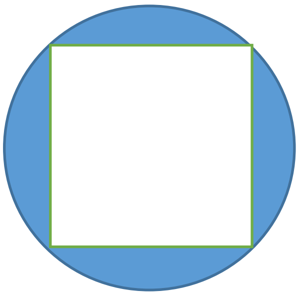
1. Square. A square is a shape with four sides that have equal lengths and angles of 90 degrees. Let s represent the length of a side of the square.

2. Circle. A circle is a set of points in a plane equally far from a fixed point known as the center. The circle's radius will be denoted by r.

3. Inscribed Circle. An inscribed circle fits perfectly inside a square, touching all four sides of the square at precisely one point.



4. Circumscribed Circle. A circumscribed circle encloses a square, with the square's corners touching the circle's perimeter.



5. The shaded area indicates the space found between a square and an inscribed circle, or between the square and a circumscribed circle.

6. c_i, c_c; Constants c_i and c_c are used to simplify shaded area calculations, with c_i determining the shaded area in the inscribed circle and c_c determining the shaded area of the circumscribed circle, respectively

Notations

The following notations will be used:

1. s: The square's side length.

2. r: The radius of the circle. For inscribed circles, $r = \frac{s}{2}$. For circumscribed circles, $r = \frac{s\sqrt{2}}{2}$.

3. A_{square} : The area of the square, given by s^2.

4. A_{circle}: The area of the circle, given by πr^2.

5. A *shaded:* The shaded region.

a. For an inscribed circle:

$$A_{shaded} = A_{square} - A_{circle}$$

b. For circumscribed circle:

$$A_{shaded} = A_{circle} - A_{square}$$

6. c_i, c_c; Constants c_i and c_c are used to simplify shaded area calculations.

Assumptions

1. The square is a perfect geometric figure with no distortion.
2. The circle is perfectly centered within the square in the case of the inscribed circle and is perfectly circumscribed around the square.
3. The mathematical constants, such as π, are approximated to a reasonable degree of precision, usually $\pi = 3.1416$.
4. The study uses Euclidean geometry throughout, where lines are straight, and angles are based on the Euclidean postulates.
5. All measurements (length, area) are expressed in consistent units to avoid conversion errors.

6. The areas of squares and circles are calculated using standard geometric formulas:

   - $A_{square} = s^2$
   - $A_{circle} = \pi r^2$

LITERATURE REVIEW

Relevant Theories

The relation between circles and squares has been an interesting issue in geometry for centuries, and a few essential theorems and results form the basis of this study.

One significant result is **Pythagoras' Theorem**, which is essential in understanding the relationship between the side length of a square and the diagonal that connects opposite corners. In the context of the circumscribed circle, this diagonal represents the circle's diameter. According to the Pythagorean Theorem, for a square with side length s, the diagonal d can be calculated as:

$$d = s\sqrt{2}$$

This diagonal serves as the diameter of the circumscribed circle, and the radius is half of this value:

$$r_{circumscribed} = \frac{s\sqrt{2}}{2}$$

Another significant concept is the area of an inscribed circle. The inscribed circle fits precisely within a square, with the circle's diameter equivalent to the square's side length. Thus, the inscribed circle has a radius of $\frac{s}{2}$ and the area of the circle is given as:

$$A_{circle,inscribed} = \pi \left(\frac{s}{2}\right)^2 = \frac{\pi s^2}{4}$$

The shaded region's area in this configuration is the difference between Area of the square and area of the inscribed circle:

$$A_{shaded\ ,inscribed} = A_{square} - A_{circle,inscribed} = s^2 - \frac{\pi s^2}{4} = s^2\left(1 - \frac{\pi}{4}\right)$$

For the circumscribed circle, Euler's Formula provides a better insight of the geometric connection. The formula for the radius of a circumscribed circle derived before, is vital for estimating the area of the circle.

$$A_{circle,circumscribed} = \pi\left(\frac{s\sqrt{2}}{2}\right)^2 = \frac{\pi s^2}{2}$$

The shaded region outside the square but inside the circumscribed circle is then:

$$A_{shaded\ ,circumscribed} = A_{circle,circumscribed} - A_{square} = \frac{\pi s^2}{2} - s^2 = s^2\left(\frac{\pi}{2} - 1\right)$$

Research Gap

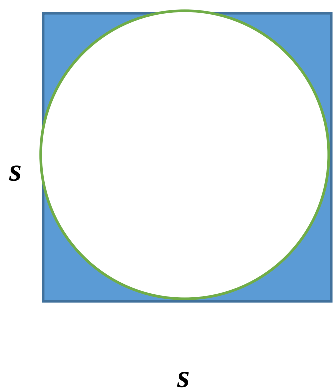
While the mathematical relationships between squares and circles in inscribed and circumscribed configurations are pretty well documented, a straightforward, simple approach has still not been used to compute the areas of the shaded regions in such setups. Most resources that deal with these topics present stepwise procedures, and often, they turn out to be very long series of formulas that one must interpret very carefully to avoid making mistakes, more so when trying to apply what is learned to complex geometric problems or real-life scenarios.

Further, even though numerous ideas of theoretical concepts used in these very calculations-per those by Euler's and Pythagoras' Theorems-have created a basis to support these computations, there is no such crucial point in deriving a general simple formula that would be able to directly and positively compute all the shaded areas for both contexts. No formula can be forwarded in the present literature that will apply in both inscribed and circumscribed, regardless of whether it is to be read from an educational perspective or a helpful application.

This research will bridge this gap by formulating consistent, simplified formulas for areas of the shaded regions in such configurations so that students and professionals can approach such geometric problems with high efficiency and better accessibility. This research also evaluates the possible benefits of such simplified formulas for use in educational contexts, where the students need to understand geometric relations to further scrutinize and apply them in architecture, engineering, and design.

METHODOLOGY

Derivation of c_i , the constant value in finding the shaded area in the inscribed circle:



Let ***s*** - length of the edge of the square: For an inscribed circle,

$$A_{shaded} = A_{square} - A_{circle}$$

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$$A_{shaded}= s^2-\pi r^2$$

Considering that the radius r of the circle is equal to half the length of a side of the square, where $r = \frac{1}{2}s$ and π is approximately 3.1416.

Substituting,

$$A_{shaded} = s^2 - \pi \left(\frac{1}{2}s\right)^2$$

$$A_{shaded} = s^2 - \pi \left(\frac{s}{2}\right)^2$$

$$A_{shaded} = s^2 - \pi \left(\frac{s^2}{4}\right)$$

$$A_{shaded} = s^2 - \frac{\pi s^2}{4}$$

Simplify the equation:

$$A_{shaded} = \frac{4 s^2}{4} - \frac{\pi s^2}{4}$$

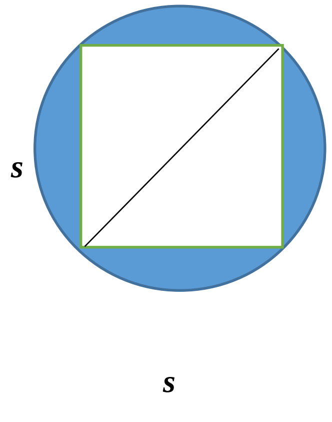
$$A_{shaded} = \frac{(4-\pi)s^2}{4}$$

$$A_{shaded} = \left(1 - \frac{\pi}{4}\right) s^2$$

$$A_{shaded} = (0.2416)\, s^2$$

Thus, the simplified formula is: $A_{shaded} = (c_i)\, s^2$, where c_i is equal to **0.2146** or the constant value.

Derivation of c_c , the constant value in finding the shaded area in the circumscribed circle:



Let $r = radius$ and $s = edge\ of\ the\ square$:In Circumscribed circle, $A_{shaded} = \pi r^2 - s^2$

Considering the radius r of a circle is half the edge of a square, $r = \frac{1}{2}s\ and\ \pi = 3.1416.$

To determine the area of the shaded region, we begin by finding the diameter of the circle, which corresponds to the diagonal of the square. Utilizing the Pythagorean Theorem, $c^2 = a^2 + b^2$, where $\boldsymbol{c}$ represents the hypotenuse, while $\boldsymbol{a}$ and $\boldsymbol{b}$ stand for the square's sides. Given that the hypotenuse is the same as the diagonal and the diameter of the circle, and that $\boldsymbol{a}$ and $\boldsymbol{b}$ are the square’s sides, we can proceed with the calculations.

$$c^2 = a^2 + b^2 \quad \_ \quad d^2 = s^2 + s^2$$

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$$d^2 = 2s^2$$

$$\sqrt{d^2} = \sqrt{2s^2}$$

$$d = s\sqrt{2}$$

The shaded area can be calculated using $d = s\sqrt{2}$ or $r = \frac{s\sqrt{2}}{2}$, given that the radius is half the diameter. Therefore,

$$A_{shaded} = \pi r^2 - s^2$$

$$A_{shaded} = \pi \left(\frac{s\sqrt{2}}{2}\right)^2 - s^2$$

$$A_{shaded} = \pi \left(\frac{s^2 \cdot 2}{4}\right) - s^2$$

$$A_{shaded} = \pi \left(\frac{2s^2}{4}\right) - s^2$$

$$A_{shaded} = \pi \left(\frac{s^2}{2}\right) - s^2$$

$$A_{shaded} = \frac{\pi s^2}{2} - s^2$$

$$A_{shaded} = s^2 \left(\frac{\pi}{2} - 1\right)$$

$$A_{shaded} = s^2\,(0.5708)$$

Thus, the simplified formula is $A_{shaded} = s^2\,(c_c)$, where c_c is equal to **0.5708** or the constant value.

RESULTS

To evaluate and determine the effectiveness of the shortcut equation using the established constants applicable to both scenarios—where a circle is contained within a square and where a square is confined within a circle—various square side lengths, from 1 cm to 15 cm, were analyzed through the conventional method, the formulated equation, and the constants.

TABLE 1. Inscribed Circle in a Square

The Length of the Edge/Side of the Square (cm)	The Area of the Square using s^2. (cm^2)	The Area of the Circle using πr^2 (cm^2)	The Area of the Shaded using $A_{square} - A_{circle}$	Derived Constant	Shaded Area using the Derived Formula $A_{shaded} = (0.2146)\,s^2$
1	1	0.7854	0.2146	0.2146	0.2146
2	4	3.1416	0.8584	0.2146	0.8584
3	9	7.0686	1.9314	0.2146	1.9314
4	16	12.5664	3.4336	0.2146	3.4336

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5	25	19.635	5.365	0.2146	5.365
6	36	28.2744	7.7256	0.2146	7.7256
7	49	38.4846	10.5154	0.2146	10.5154
8	64	50.2656	13.7344	0.2146	13.7344
9	81	63.6174	17.3826	0.2146	17.3826
10	100	78.54	21.46	0.2146	21.46
11	121	95.0334	25.9666	0.2146	25.9666
12	132	113.0976	30.9024	0.2146	30.9024
13	169	132.7326	36.2674	0.2146	36.2674
14	196	153.9384	42.0616	0.2146	42.0616
15	225	176.715	48.285	0.2146	48.285

In Table 1, we display the results for calculating the shaded area between a square and its inscribed circle, utilizing both the standard formula and the approximation derived from shortcuts. It shows that multiplying the constant value of 0.2146 by the square's area yields a result that is equivalent to that found using the traditional method.

TABLE 2. Circumscribed Circle to a Square

The Length of the Edge/Side of the Square (cm)	The Area of the Square using s^2. (cm^2)	The Area of the Circle using πr^2 (cm^2)	The Area of the Shaded using $A_{circle} - A_{square}$	Derived Constant	Shaded Area using the Derived Formula $A_{shaded} = (0.5708)\ s^2$
1	1	1.5708	0.5708	0.5708	0.5708
2	4	6.2832	2.2832	0.5708	2.2832
3	9	14.1372	5.1372	0.5708	5.1372
4	16	25.1328	9.1328	0.5708	9.1328
5	25	39.2700	14.2700	0.5708	14.2700
6	36	56.5488	20.5488	0.5708	20.5488
7	49	76.9692	27.9692	0.5708	27.9692
8	64	100.5312	36.5312	0.5708	36.5312
9	81	127.2348	46.2348	0.5708	46.2348

10	100	157.0800	57.0800	0.5708	57.0800
11	121	190.0668	69.0668	0.5708	69.0668
12	132	226.1952	82.1952	0.5708	82.1952
13	169	265.4652	96.4652	0.5708	96.4652
14	196	307.8768	111.8768	0.5708	111.8768
15	225	353.4300	128.4300	0.5708	128.4300

Finding the shaded area when there is a circle inscribed to a square using the traditional formula (see Table 2). However, the moment the derived constant is used the new shortcut algorithm: applying the derived constant the computation becomes simplified. That is when you multiply the areas of the square depicted plus constant 0.5708 you get the same area as you would have used the traditional way.

DISCUSSION

Analysis and Interpretation

The results of this work make significant contributions toward geometric relationships, specially making the computation of shaded areas of circle-square configurations easier. Since the constants for an inscribed circle c_i = 0.2146 and c_c = 0.5708 for a circumscribed circle streamline calculations by avoiding multi-step procedures. This innovation saves time and computational errors, so it is a practical application for educational and professional usage. The work highlights how elegant mathematical proportionality is, demonstrating the simplicity of fixed relationships between areas in solving complex problems.

The developed work fills a significant gap compared to the previously published literature. The conventional methods were essentially stepwise calculations involving the constant π along with algebraic manipulations coupled with geometric theorems of Pythagoras. Although accurate, these methods are computationally less efficient in practical calculations. In contrast, derived constants provide a consistent and validated shortcut- a new contribution not found in previous literature. This fits the overall mathematical objective of developing general and efficient solutions and, thus, the possibility of these constants helping solve problems.

The implications of these results are not restricted to geometry. In mathematics, the findings strengthen the principles of Euclidean geometry, especially those concerning the relation between a square's side and the diagonal or radius of the inscribed circle. This work parallels the construction of universal mathematical constants and has implications beyond optimization and computational algorithms. The connections to other sciences are also remarkable. In engineering and design, where area calculations to a high degree of accuracy are crucial, these constants simplify analyses of materials and structures. In physics, they could also help in optics and mechanics, where the geometrical measures of distributions of light or forces are crucial. Besides, the method can spark data visualization and computational graphics to seek more efficient rendering and analysis techniques for geometric shapes.

Overall, this research delivers a meaningful contribution to theory and practice in mathematics, theoretically and practically. The constants and formulas developed here have the potential to influence curricula, enhance problem-solving techniques, and promote further derivations for other geometric configurations. With cross-disciplinary relevance, results promise applicability in architecture, engineering, and robotics, among other fields.

Limitations and Challenges

Although the derived constants and formulas offer a convenient means of calculating shaded regions in circle

-square configurations, there are some significant limitations: First, the results will depend upon idealized geometrical situations and will thus rely upon perfect squares and circles defined within Euclidean geometry. Any distortion in the shapes or irregularities in measurement or from these ideal conditions could affect the accuracy of any calculations. For example, in the real world, where there are tolerance or imperfections in manufacturing or design, these constants can have a bad result if applied without additional adjustments.

Another limitation in the application's scope is that the formulas are tailored for configurations involving a square and a single circle, either inscribed or circumscribed. They do not directly extend to more complex geometric arrangements, such as overlapping shapes, multiple figures, or three-dimensional analogs. As such, the derived constants may have limited utility in broader contexts where these configurations are part of more intricate designs or systems.

In addition, the constants assume a large amount of numerical precision for the values of π and all other fundamental computations. When the rounding errors or approximations are large, this will become significant in either very small or large shapes. This may become a problem in applications like computational modeling or simulations, where high numerical stability and precision are essential.

Finally, while the research simplifies computations, it may not entirely replace traditional methods in educational contexts where understanding the step-by-step process of geometric derivations is essential for learning foundational concepts. Simplified formulas could be seen as a tool for practical applications rather than a substitute for comprehensive mathematical education.

The findings do suggest avenues for continued work, such as fitting it to imperfect shapes, extending the methodology to greater geometric complexity, or considering computational tools that keep real-world constraints in check. Still, the derivation itself is a useful advance, most notably in applications when either efficiency or simplicity is in important demand.

CONCLUSION AND FUTURE WORK

Summary of Findings

This research introduces a novel approach to simplifying the calculation of shaded areas in circle-square configurations by deriving two constants: $\boldsymbol{C_i}$=0.2146 for inscribed circles and $\boldsymbol{C_c}$=0.5708 for circumscribed circles. These constants streamline the computation process, eliminating the need for multi-step procedures and reducing computational errors. The study validates these constants against traditional methods, demonstrating their accuracy and efficiency.

The contributions lie in developing general shortcut formulas that improve problem-solving skills in geometry, filling a lacuna in the existing literature wherein no such simplified approach was previously available. The work also explains some practical applications of the constants in education, competitive problem-solving, and professional fields such as architecture and engineering. Beyond practical utility, the results contribute toward the overall mathematical goal of creating beautiful and efficient solutions, but with potential interdisciplinary applications in physics, computational graphics, and optimization.

Future Research

Building on the research presented here, several directions deserve further work to increase its reach and influence. One interesting area is applying the constants that are derived in irregular or imperfect geometric forms, like rectangles or ellipses, which happen much more often than perfect ones. The work can also be extended into three dimensions to include sphere-inscribed or circumscribing cubes.

The second area of application could be to incorporate these formulas into computational tools or applications that would make these accessible for use in engineering, architecture, and design applications. Building algorithms or calculators automatically that use these constants would add more practical value, especially in handling complex geometric issues for professionals.

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Further research studies may be conducted on the educational significance of these results. Investigations could be done on how the inclusion of these constants in mathematical curricula may enhance problem-solving efficiency and students' understanding of geometric relations. Their use in competitions may also be studied to measure their effects on performance and accuracy under time constraints.

The final direction is to extend the study and observe how these constants behave under different precision levels or approximations, which could shed light on their robustness in computational settings. Working with numerical analysis or optimization could make the formulas more precise for high-accuracy applications, such as simulations or data visualization.

These avenues of future work not only enhance the current research but also open opportunities for its application in interdisciplinary and practical domains.

REFERENCES

1. Euclid, The Elements, translated by Sir Thomas L. Heath, Dover Publications, 1956.
2. Archimedes, The Works of Archimedes, edited by T. L. Heath, Dover Publications, 2002.
3. Leonhard Euler, Introduction to Analysis of the Infinite, translated by John D. Blanton, Springer, 1988.
4. Howard Eves, An Introduction to the History of Mathematics, 6th ed., Saunders College Publishing, 1990.