

An Investigation into the Effect of Concept-Based Instruction on Senior High School Students' Geometric Thinking and Achievement in Circle Theorem

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Abstract: This study was grounded on Van Hiele's geometric levels of thinking as a conceptual framework to assess and analyse senior high school students' geometric understanding. A non-equivalent quasi experimental design was used to compare the geometrical achievements of students taught with concept-based method and those instructed with traditional method. The study employed purposive sampling technique to select two schools; experimental group (41) and control group (37). Quantitative analysis was carried out using a two-way mixed factorial analysis of variance (ANOVA). The findings of the study revealed a significant difference in the mathematics achievement of learners taught circle theorem with concept-based instruction as compared to those taught with traditional method. The study reveals that students achieved higher geometric thinking when taught with concept-based instruction.

Keywords: Concept-based instruction, Circle Theorem, Van Hiele's levels, Geometric thinking

I. INTRODUCTION

A strong foundation in mathematics is a vital tool and an important ingredient, not only for research but also for optimum development and total economic growth and the social life of people. As captured in the Ghanaian Senior High Schools' Core Mathematics syllabus "there simply cannot be any meaningful development in virtually any area of life without knowledge of science and mathematics" (Ministry of Education, 2010). Because of the utility value of mathematics, the government of Ghana has made mathematics a compulsory subject at the pre-tertiary levels of the educational system. However, the overall achievement of students in mathematics especially in the area of geometry is unsatisfactory (Anamuah-Mensah, Mereku & Asabere-Ameyaw, 2008). Evidence from the results of Ghanaian students' participation in the Trends in International Mathematics and Science Study (TIMSS) 2011 report shows an abysmal achievement of Ghanaian students in mathematics most especially in the application of geometric concepts and expressions (Mullis, Martin, Foy & Aron, 2011). According to the TIMSS report, most Ghanaian students have conceptual difficulties in the area of geometrical concepts and relations. Most of these difficulties in geometry topics are related to the

concepts of circle theorem (West African Examination Council (WAEC)). Several researchers including those by Dogwi (2014) and Lim (2006) have also highlighted students' difficulties in the context of circle geometry. Moreover, despite all the efforts and reforms to improve geometry classroom instructions and students' performance in geometry, circle theorem is still considered by Ghanaian students as the most difficult concept in the SHS core mathematics curriculum in Ghana (Bosson-Amedenu, 2017).

In the context of mathematics teaching, one of the reasons why students have difficulties in studying geometry is the lack of conceptual understanding of the concepts involved. This has led to their inability to reason at the highest levels of geometric thinking Dogwi (2014). The failure to understand mathematical concepts and hence poor achievement might have stemmed from the teaching methods. In teaching mathematics, teachers' instructional approach is a major determinant to students' level of understanding and development of the topic. According to Dogwi (2014), the instructional method and the choice of exercise a teacher adopts play a key role in any meaningful learning process. For example, Han (1986), (as cited in Thompson, 1992), argues that students' attitude towards geometry may be as a result of the method of instruction. This fact has led to the discussions of mathematics learning and how mathematics should be taught for an in-depth understanding of both basic concepts and correct procedures in educational literature by many researchers in the area of mathematics (e.g. Hiebert & Lefevre, 1986; Riddle-Johnson & Schneider, 2015). Consequently, students' learning difficulties coupled with problems of instructional methods have necessitated the research community to focus on research studies that could improve students' performance in geometry.

A range of theories and models on learners' thinking and on how learners should learn mathematics have been proposed and continuously researched by mathematics educators in order to improve learners' performance and improve mathematics classroom instruction and these include: Van Hiele's phase-based instruction, Bruner's discovery learning,

Gagne's guided learning, Piaget's constructivism, Hiebert and Lefevre's conceptual and procedural knowledge and Chappell and Killpatrick's conceptual and procedural teaching. In addition, recent developments in literature on how students learn mathematics particularly in the light of increased use of advanced state of technology has now mainly focused towards a great interest on conceptual or procedural distinctions and approaches to mathematics (e.g. Bergsten, Engelbrecht & Kagesten, 2017; Borji, Radmehr & Font, 2019; Joffrion, 2005; Mahir, 2009). Among these studies, Bergsten and his colleagues (2017) reported that the interest in exploring conceptual and procedural distinctions to mathematics has taken a greater space in literature in recent research in mathematics education unlike an earlier main focus on mathematical content knowledge.

Meanwhile, Chappell and Killpatrick's (2003) conceptual and procedural teaching framework has also been used and investigated as a theoretical base by many researchers in the area of algebra and calculus studies (Khoule, Bonsu & El Houari, 2017; Borji, et al., 2019; Joffrion, 2005; Mahir, 2009). Many of these studies have suggested that conceptual teaching approach is the best method to teach mathematical concepts while others suggested that geometry instruction should be based on hand-on-activity investigation, critical thinking and problem solving (Crowley 1987; Armah, Cofie & Okpoti, 2018). Other empirical studies have also highlighted similar suggestions that mathematics instruction in general should be based on student-centred pedagogies such as problem-based, inquiry-based and project-based methods (see Asomah, Wilmot & Ntow, 2018; Kandil & Isiksal-Bostan, 2018). Evidence from well-controlled comparative studies related to conceptual and procedural teaching and learning of mathematics have shown that the conceptual approach to mathematics is more essential than the traditional method of procedural approach to mathematics in terms of students thinking and understanding of mathematics (e.g. Bergsten, Engelbrecht & Kagesten, 2017). While emphasizing on students learning fundamental concepts, it is important to decide on an applicable instructional approach to develop students' conceptual understanding of mathematics without compromising procedural skills.

The terms conceptual teaching and procedural teaching used in teaching context means; the teaching for meaning and understanding of concepts and the teaching of rules and procedures respectively. These terms have synonymously been used by many researchers as; conceptual instruction and traditional instruction, concept-based instruction and procedural-based instruction, concept-based learning environment and procedural-based learning environment; conceptual teaching and procedural teaching (e.g. Chappell & Killpatrick, 2003; Borji, et al., 2019; Hiebert & Lefevre, 1986; Valmoria & Tan, 2019; Langton, 1991). Concept-based instruction is a learning environment where students work collaboratively to construct their own knowledge through problem-solving and guided discovery. Joffrion (2005)

described conceptual teaching as: "relationships between numbers, topics, or representations explicitly pointed out; concepts are connected to students' current knowledge and future learning; explanations of the reasons for executing elements of the procedure are emphasized". Conceptual teaching focuses on helping students to understand the concept first, then using the knowledge gained to solve problems. This approach involves the use of discussion, group work, guided discovery and hand-on-activity to develop an understanding of the concept to be learned.

Previous studies (e.g. Langton, 1991 and Borji et al., 2019) have demonstrated that this instructional method can develop students with critical thinking skills and improve their problem solving abilities. It also helps students reach conceptual understanding of a topic and create sustainability in learning (Borji et al., 2019). According to Putnam (1987) (as cited in Langton, 1991), linking entry knowledge and skills to new concepts and procedures, the fundamental principle of concept-based instruction is what is most needed to address the growing needs of the current advancing state of technology. Chambell and Killpatrick (2003) and other studies in mathematics education have recommended the use of the concept-based instructional method in teaching mathematics to enable students to understand mathematical concepts. For instance, Langton (1991) reported that students developed an in-depth understanding of concepts after being taught with a concept-based approach. Borji, Radmehr and Font (2019) also suggested that students taught with conceptual teaching create more sustainable understanding and increase students' procedural abilities and efficiency. For Khoule, Bonsu and El Houari (2017), conceptual teaching reduces, substantially, the mathematics anxiety levels of mathematics students.

With reference to past and recent studies on the advantages of concept-based instruction, many research publications have been emphasizing on this method as an effective instructional approach to teach mathematical concepts. Although several of these researchers have recommended the use of concept-based instruction in teaching mathematics (e.g. Langton, 1991; Chambell and Killpatrick 2003; Borji et al, 2019), we did not come across any study that explored the influence of concept-based instruction on students' achievement in circle theorem. Similarly, we found many research studies (e.g. Crowley 1987; Armah et al., 2018; Chemuka, 2017) in the field of geometry that have tended to concentrate on using Van Hiele's model and Geogebra, a software, to improve geometry classroom instruction and students' performance. However, in Ghana, literature (Armah, et al., 2017; 2018) have pointed at less numerical availability of studies that have been carried out on Van Hiele's theory and geometry in general.

Moreover, although the learning of circle geometry has been investigated by many researchers in mathematics, research studies on circle theorem in Ghana are limited. The only study that was found in the course of literature review on Ghana is the study conducted by Tay and Mensah-Wonkyi (2018) that found a positive impact of using Geogebra on senior high

school (SHS) students' mathematics achievement in circle theorem. However, Chemuka (2017) who also conducted a study on using Geogebra and Van Hiele's theory to improve learners' achievement in circle theorem found out that the use of Geogebra can only improve students' mathematics achievement at the basic levels as proposed by Van Hiele, that is, levels 1 and 2. According to him, the higher levels (abstraction, deduction and rigor) did not show any significant improvement in students' achievement. He suggested that the "use of Geogebra in teaching and learning of circle theorem" was limited at improving students' visualization and abstraction levels of Van Hiele. To him, Geogebra enables students to recognise and name specific circle theorems and properties but unable to carryout logical analysis. A replicated study was conducted by Ogbonnaya and Chemuka (2017) who also found similar results that were consistent with the former study. Once again, the authors suggested that the use of Geogebra in teaching circle theorem "might not help much in attaining correct results". The researchers concluded that although studies regarding technology integration (Geogebra) have yielded positive significant difference when compared with the traditional method, "the significant changes depended on specific Van Hiele Levels".

In light of the foregoing, this study is an attempt to explore the effectiveness of some instructional approaches that have been identified as likely to help develop students' understanding of circle theorem. This is done using Van Hiele levels to analyse learners' geometric thought because of its "relevance to geometry teaching and learning" (Chemuka, 2017). That is, it provides a more comprehensive description of students' geometric reasoning levels. The model provides the best theoretical framework for assessing and analysing students' geometry learning outcomes, while at the same time allows for an in-depth understanding of students' thinking (McAuliffe, 1999). Much empirical researchers have used this theory to analyse learners' mathematical understanding in geometry (e.g. Armah et al, 2017; Ogbonnaya and Chemuka, 2017; Abdullah & Zakariah, 2011). Recent and past studies (e.g. Ogbonnaya and Chemuka, 2017; Burger & Shaughnessy, 1986) have also shown that the Van Hiele's theory is a useful theoretical approach to analyse learners' geometric thought and help to have a better understanding of learners geometric learning outcomes. In summary, this study investigated the effects of two teaching approaches namely, concept-based instruction and traditional method on SHS students' achievement on the concept of circle theorem compared at the different Van Hiele's levels. The following null hypothesis was tested:

1. H_{01} : There is no significant difference between the achievement scores of students taught circle theorem with concept-based method and students taught with traditional method at different Van Hiele's levels.

II. THEORETICAL FRAMEWORK

The theoretical framework underpinning this study is the Van Hiele's theory of geometric thought that describes students' geometric thinking levels. This theory is regarded as most the appropriate theoretical framework for conducting and designing geometric activities for understanding and teaching of geometry as well as describing students learning outcomes (Luneta, 2014). Knowing a student's Van Hiele level in geometry is essential in ascertaining the students' mastery and development in that concept (Burger & Shaughnessy, 1986). Much research has used and supported this theory in analysing learners' mathematical understanding most especially in the field of geometry (e.g. Armah et al. 2017; Ogbonnaya & Chemuka, 2017; Mayberry, 1983).

Furthermore, the study is also aimed at using Van Hiele's description of geometric reasoning as a framework to complement conceptual and procedural teaching framework to determine the effect of a concept-based teaching method on the performance of learners in circle theorem. An important focus of the study is to use the conceptual and procedural teaching framework to teach circle geometry, and Van Hiele's descriptions as a theoretical base to assess and analyse students' levels of geometric reasoning. The Van Hiele's descriptors are based on five differentiated levels of understanding which include recognition, analysis, abstraction, deduction and rigor (Crowley 1987; Burger & Shaughnessy, 1986; Mayberry, 1983). The Van Hiele levels are illustrated diagrammatically below in figure 1.

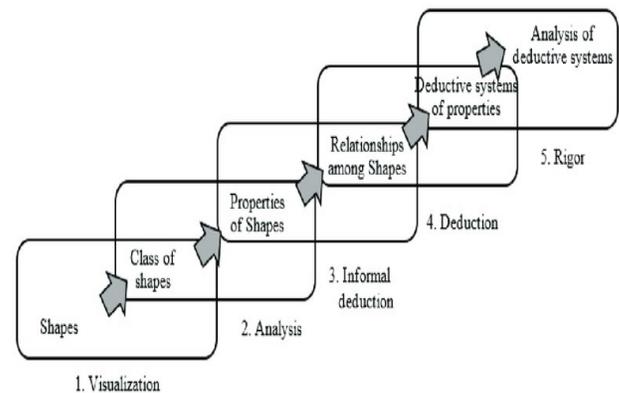


Figure 2-Theoretical framework; adapted from Van de Walle (2004; as cited in Luneta, 2014)

At level 1 (recognition), students can recognize and name geometric figures base on appearance alone, but cannot recognize their properties. At level 2 (analysis), students can identify geometric figures by their properties. At level 3 (abstraction), students can logically order geometric figure and understand the relationship between them. At level 4 (deduction), students can carry out formal deductions and establishes a network of relationship among definitions and theorems. At level 5 (rigor), students can construct geometric proofs and can also see geometry in abstract. According to

Kilpatrick, Swafford and Findell (2001) “the first three levels show the development of procedural fluency in geometry and the last two the development of conceptual understanding” (as cited in Luneta, 2014).

Research design, sample and data collection

The study adopted a quasi-experimental non-equivalent (pre-test post-test experimental and control group) design. Pallant (2001) encourages the use of this design when dealing with intact classes. The design is shown below.

Group	Pre-test	Treatment	Post-test
Treatment group	T1	X	R1
Control group	T2	C	R2

In this study, concept-based teaching approach was applied to the treatment group whereas the control group received regular classroom instruction (teaching procedurally). Consequently, the study used two categorical independent variables and one dependent variable. The independent variables, the variables that cause or influence an outcome, were the two instructional learning environments used in teaching geometry, thus, concept-based approach and procedural-based approach (traditional method). The dependent variable or outcome variable of this study was students’ performance in a geometry achievement test. The pre-test (T1 and T2) and post-test (R1 and R2) were conducted to compare students’ entry-level achievement and treatment effect respectively between the intervention and control groups.

Two intact classes in Senior High School, year two, were randomly identified from two different schools namely, WASEC and WA-SECTEC in the Wa Municipality of the Upper West region of Ghana. The schools were purposively selected to ensure that the schools belonged to the same rank as used by the Ghana Education Service (Ghana Education Service). This provided a baseline for comparison by ensuring that all other factors that could affect the results of the study are held constant except the teaching approaches. Because the study was conducted in a classroom setting, intact classes were used in order to avoid disrupting the school programmes for students. Hence, the sample consisted of 78 students in which 41 students were in the control group who received the traditional lesson and 37 students in the intervention group who received the concept-based instruction. The sample size for each group was dependent on the number of students in a particular class at the time the interventions were carried in the selected class. The researchers gained approval from the Institutional Review Board (IRB) of the University of Cape Coast and all the participating students voluntarily agreed and signed the informed consent form.

Research Instrument

The research instrument consisted of geometry achievement test (GAT) items. The geometry achievement test items were

constructed mainly on circle theorem and based on the learning objectives in the SHS core mathematics teaching syllabus. Some of the items were constructed by the researcher (to ensure that the items are within the Ghanaian context) while some were adapted from previous research studies. The test contained fifteen essay-type questions. In selecting the questions, each item selected had to pass through: expert criticism, item difficulty and item discrimination analyses. Both the pre and post-intervention test items were similar in terms of item type and difficult levels to ensure an accurate comparison. Because of the relevance of Van Hiele’s theory to geometry teaching and learning, the questions were also developed to reflect the five levels of his model of thought and understanding with each question pre-assigned to a specific Van Hiele level. In determining the possible Van Hiele level for each question, Mayberrys’ (1983) level descriptors of Van Hiele’s geometric reasoning were used to verify the suitability of the questions for a given level. A sample item is presented in question 15 which is intended to measure students’ ability to construct proofs at Van Hiele’s level 5. This question assessed students’ ability to construct geometric proofs and also being able to visualize geometry in abstract. The question was:

(15) Proof the theorem; a line drawn from the centre of a circle perpendicular to a chord, bisect the chord.

Since the items were open-ended, short answer questions and mathematical proofs, the students’ answers were rated by two different scorers. This was done to minimize subjectivity of results and check the consistency in scoring. However, Pearson-Product-Moment correlation was used to check interrater reliability of the scorers. The reliability coefficient obtained between scorer 1 and scorer 2 was 0.996 hence, it was inferred that the two ratings were consistent, and that the test scores were reliable.

Treatments

Throughout the study, concept-based teaching was implemented to the intervention group whereas the control group received conventional method of instruction (procedural-based instruction). Lessons of both the control and treatment groups were held in a typical mathematics classroom. While the main focus in the control group was on teacher demonstrating how to solve as many problems as possible through explaining the procedures and /or tricks required so that students can solve similar problems in future consistent with what usually happens in procedure-based lessons, the concept-based instruction class for the experimental group focused on verifying and justifying each step of procedure and circle theorem property used in solving questions. Each lesson lasted for a period of two hours as stipulated in the school time table, and each group was met three times. However, most of the class time in the experimental group was devoted to developing students’ understanding of concepts.

The two instructional lesson plans were reviewed by two experienced mathematics instructors to ensure that the two instructional methods were compatible with the learning environments defined and in line with Joffrion (2005) extracted conceptual and procedural teaching indicators. The two groups, the control and the treatment group were facilitated by the same instructor, and lessons taught were strictly in accordance to the lesson plans designed for each group thereby minimizing any possible researcher interest or teacher bias. However, each lesson segment was observed and evaluated by two independent observers using Hiebert and Lefevre (1986) conceptual and procedural teaching indicators (adapted from Joffrion, 2005). This was done to ensure the reliability and validity of the findings. The lessons segments were examined and coded as conceptual (C), procedural (P) or neither (N) depending on the type of knowledge emphasized. The independent observers compared their evaluations and discussed for consistency and there was an agreement between them for each lesson segment and thus reliability was assured.

Data Collection and Analysis

To analyse the data collected, the post-test scores of students in the control and experimental groups obtained from the achievement test were analysed and presented using tables and descriptive statistics. Also, the post-test scores were further analysed inferentially using a two-way, mixed between-within subject analysis of variance (mixed factorial ANOVA) to evaluate the effect of the two instructional interventions (concept-based method and traditional method) on students’ achievement scores at different Van Hiele levels. A two-way mixed analysis of variance (ANOVA) is an omnibus test used to compare and analyse means between groups as well as within groups. This statistic allows for concurrent multiple testing and also enable the testing for any interaction between the independent variables. All preliminary checks were carried out and satisfied to ensure that there was no violation of the assumption of normality, homogeneity of variance and covariance.

III. RESULTS AND DISCUSSIONS

Table 1-Descriptive Statistics of Post-test Scores at Different Van Hiele’s Levels for Control and Experimental Groups

Level	Name	Marks %	Control		Mark %	Experimental	
			Mean	SD		Mean	SD
Level 11	recognition	79.7	4.78	0.91	90.2	5.41	0.70
Level 12	analysis	79.7	4.78	1.05	86.6	4.90	1.13
Level 13	abstraction	44.5	5.40	2.12	60.6	7.26	2.24
Level 14	deduction	28.6	3.81	1.91	51.6	6.19	2.33
Level 15	rigor	12.8	0.51	0.73	24.5	0.97	1.03

Table 1 reports the descriptive statistics of post-test scores at each of the Van Hiele’s levels in the control and intervention groups. As Table 1 shows, the treatment group obtained higher means scores than the control group at all the different Van Hiele levels of geometric thought. At Van Hiele’s Levels 1 and 2, the control group ($M = 4.78$, $M = 4.78$, respectively) was closely matched with the experimental group ($M = 5.41$, $M = 4.90$, respectively) with both groups achieving a remarkable performance [(79.9% and 90.2%, the average percentage score in level 1 respectively); (79.9% and 86.6%, the average percentage score in level 2 respectively)]. This is an indication that most of the students in both groups have attained the recognition and analysis levels of Van Hiele’s geometric thinking. That is, most students in these groups could visualise or name and identify properties of geometric figures correctly.

However, the treatment group outperformed the control group in Level 3 ($M_{cont} = 5.40$, $M_{exp} = 7.26$) and also in Level 4 ($M_{cont} = 3.81$, $M_{exp} = 6.19$) while Van Hiele’s level 5 was closely matched ($M_{cont} = 0.51$, $M_{exp} = 0.97$, respectively). In addition, a remarkable performance was obtained in the experimental group for level 3 and 4 with the exception of level 5 (60.6%, 51.6% and 24.5%, the average percentage score at each level respectively) while the performance of students in the control group for level 3, 4 and 5 was abysmally poor (44.5%, 28.6% and 12.8%, the average percentage score at each level respectively). This is an indication that a good number of students have attained the abstraction and deduction levels of geometric reasoning in the experimental group. This implies that most students in the experimental group could give informal arguments and do simple geometric proofs. On the other hand, this indicates that most students in the control group have difficulties in making logical deductions and proofs and hence could not achieve the abstraction and deduction levels of geometric reasoning. Both groups performed poorly at Van Hiele’s Level 5 suggesting that the students were not ready for formal geometric proofs. An example is reported on the analysis of *question (15)* which is intended to measure students’ ability at Van Hiele’s level 5. The question assessed students’ ability to construct geometric proofs and also being able to see geometry in abstract. The question was:

(15) Proof the theorem; a line drawn from the centre of a circle perpendicular to a chord, bisect the chord.

A two-way, mixed between-within subject analysis of variance (mixed factorial ANOVA) carried out to assess the impact of two different instructional methods on students’ achievement scores at different Van Hiele levels showed a significant interaction effect between the type of teaching and Van Hiele’s levels [$F(2.62, 199.49) = 7.96, P = 0.000$, *partial eta square* = 0.095]. This is reported in Table 2. This effect indicates that the type of teaching approach used had a different effect on Van Hiele levels or the differences in scores between controls and experimental is

dependent upon the type of Van Hiele level. In addition, there was a statistically significant main effect (within-subject effect) for Van Hiele levels [$F(2.62, 199.49) = 151.34, P = 0.000 < \alpha(0.001), partial\ eta\ square = 0.66$]. This suggests that students' scores differ across different Van Hiele levels. The main effects (between-subject effect) comparing the two types of interventions were significant [$F(1, 76) = 41.97, P = 0.000 < \alpha(0.001), partial\ eta\ square = 0.36$] suggesting that there was a substantial difference in scores for the two groups (those who received conceptual teaching and those who received traditional method).

Table 2-Summary of Two-way Mixed ANOVA (Test between-within Subject Effect)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Between-subject effect						
Intercept	7548.832	1	7548.832	2733.805	0.000	0.973
Type of teaching	115.909	1	115.909	41.976	0.000	0.356
Error	209.858	76	2.761			
Within-subject effect						
Level	1412.932	2.625	538.283	151.34	0.000	0.66
levels * type teaching	74.317	2.625	28.312	7.960	0.000	0.095
Error	709.545	199.4	3.557			

The significant interaction effect between the two different teaching approaches and Van Hiele levels was examined by testing the simple main effect of type of teaching at each Van Hiele level, that is, the differences between control and experimental group for each of the five Van Hiele levels using Bonferroni adjustment. Bonferroni comparisons were selected for use to minimise the risk of committing a Type I error rate across the five pairwise comparisons (Gregory & Daniel, 2015). The results of the pairwise post hoc comparisons using Bonferroni correction are reported in Table 3.

Table 3-Bonferroni Post hoc Comparison from Mixed ANOVA of Post-Test between Control and Experimental Groups at different Van Hiele levels

Level	Comparison	Control		Experimental		M_{diff}	p-value
		Mean	SD	Mean	SD		
Level 1	Control vs. exp.	4.784	0.916	5.415	0.706	0.63	0.074
Level 2	Control vs. exp.	4.784	1.057	4.902	1.135	0.12	0.737
Level 3	Control vs. exp.	5.405	2.127	7.268	2.247	1.86	0.000
Level 4	Control vs. exp.	3.700	1.912	6.195	2.336	2.38	0.000

Level 5	Control vs. exp.	0.510	0.731	0.980	1.036	0.5	0.192
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NB: M_{diff} represents the mean difference between control and experimental groups.

The results (see Table 3) indicated that learners' scores in the treatment group were statistically significantly better than learners' scores in the control group at two different Van Hiele's levels [$M_{diff} = 1.86, P = 0.000 < \alpha(0.001)$] for level 3; [$M_{diff} = 2.38, p = 0.000 < \alpha(0.001)$] for level 4. Meanwhile, no statistically substantial difference of achievement was revealed between the scores of the intervention group and the scores of the control group at the basic Van Hiele level 1 [$M_{diff} = 0.63, p = 0.074 > \alpha(0.001)$] and level 2 [$M_{diff} = 0.12, p = 0.737 > \alpha(0.001)$] and also at the highest Van Hiele level 5 [$M_{diff} = 0.5, p = 0.192 < \alpha(0.001)$]. There was a matched performance between the two groups at Van Hiele level 1, level 2 and level 5. Key to this finding of the study is that both control and experimental groups performed abysmally poor hence no significant difference between them at the highest Van Hiele's Level 5 suggesting that the students were not ready for formal geometric proofs in SHS. This has also been noted by other researchers (e.g. Armah et al., 2017; 2018; Chemuka, 2017).

These findings illustrated that students in the treatment group instructed with concept-based method performed better at higher Van Hiele levels than those in the control group who received traditional learning method, while there was a similar or matched performance between the two groups at the basic Van Hiele levels. The post-test results analysis indicates that teaching mathematical concepts can better enhance students' achievement at higher geometric thought than the teaching of procedures. Furthermore, the results evidenced that presenting mathematical concepts in an experiential way coupled with simple geometric proofs can benefit students and improve their achievement scores at advanced levels of Van Hiele's geometric reasoning.

An investigation was carried out by the researchers to examine students' scripts on individual items of the post achievement test. A detailed analysis of the individual questions of students' response to the mathematical task also shows a better improvement of the students who received concept-based instruction (experimental group) than students taught with traditional method especially on questions that were conceptually-oriented and demanded logical deductions. For example, students who were taught with the traditional method could not recognize the required theorem circle properties to answer questions that required logical deductions and informal arguments. Most students in the control group could not provide meaningful arguments of theorems in answering question (14) of the achievement test. This question recorded the second-lowest improvement in the post-test after the intervention activity. The question is as shown in Figure 2.

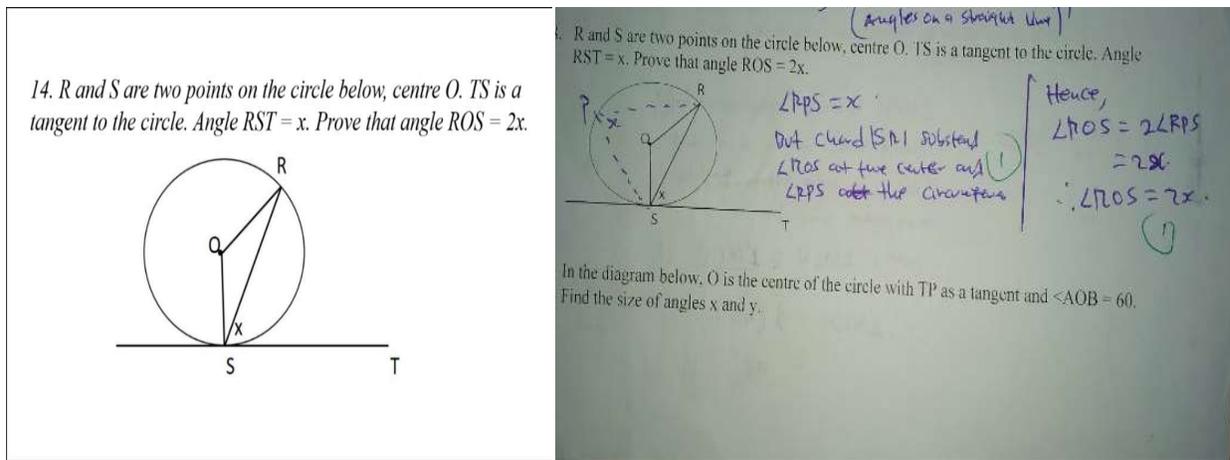


Figure 2-A Sample of Question (14) in the Post-Test (left) and a Snapshot Sample of Student Work for Question (14) in the Post-test (right)

The question assessed the students' level of geometric thinking that is Level 4 as proposed by Van Hiele. This question required students to apply the concept of either chord and alternative segment angle theorem or angle between a radius and tangent theorem. Most students, approximately (75%) in the control group could not attempt this question and the few students who attempted the question displayed poor knowledge of the concepts of circle theorem. The analysis of this question further provided evidence that only 4% of the students in the control group were able to answer this question correctly in the post-test. However, the majority of the students instructed with the concept-based method in the intervention group were able to apply the appropriate circle theorem properties and supplied meaningful arguments to prove that angle $ROS = 2x$. Only a few students, approximately (15%) gave an incorrect response in the experimental group. This is an indication that students have improved in their understanding of the concept of circle theorem in the experimental group. Figure 2 (right) shows a sample of student work on the question (14) that was correctly answered in the post-test of the treatment group but was initially incorrectly answered in the pre-test.

circle theorem involving the angle between a radius and tangent is 90 and some background knowledge of the sum of the interior angles of a triangle to answer the question. The question also required students to apply their previous knowledge on similar triangles by making some logical deductions to recognize that triangle MOB is similar to triangle BOT.

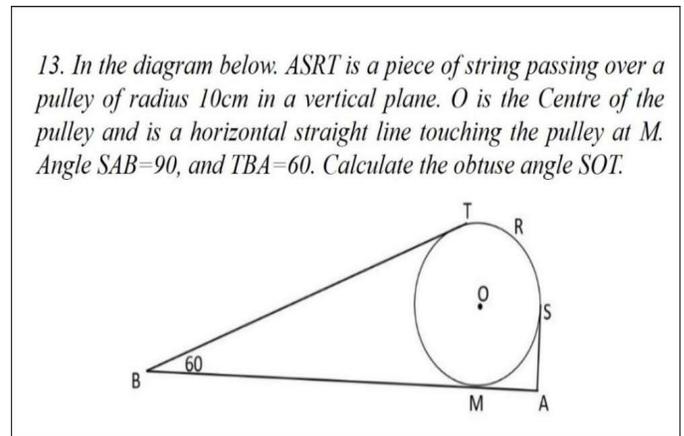


Figure 3-A Sample of Question (14) in the Post-Test

Conceptually and visually, the student drew two lines to form an angle at the circumference of the circle and applied the alternative segment angle theorem to solve the problem. The students' ability to recognize the appropriate circle theorem property and construct meaningful arguments to solve the question correctly could be associated to the teaching environment in which students explored and discovered the various properties of circle theorem (concept-based method).

Only a few students representing 9.5% in the control group attempted this question and were unable to recognize that the two triangles are similar and therefore have equal angles. The reason might be that these students do not have the conceptual understanding of the theorems involved. However, it is observed that approximately 65.9% of the students in the treatment group responded to this question correctly. Some of them re-sketched the diagram to produce two similar triangles. However, only eight (8) students in the intervention group responded incorrectly. These students had some difficulties in prerequisite concepts for learning circle theorem (e.g. the sum of interior angles of a triangle and similar triangles).

IV. DISCUSSION AND CONCLUSION

Based on the research hypothesis tested in this study, it appears that the use of concept-based instruction in teaching

circle theorem had a significant impact on students' achievement in geometry as compared to the traditional method. This was evidenced by the difference in performance between control and experimental groups at higher Van Hiele levels. Concept-based instruction helps students understand the mathematical concepts and the reasons behind using each procedure or formula and how they relate to each other. The traditional teaching method, similar to procedural teaching in which students are taught the rules of procedures, without necessarily knowing the reasons behind each step of the procedure, had a less significant impact on mathematics performance of learners as compared to conceptual instruction.

Moreover, the findings of the study had statistically demonstrated that a conceptual learning environment will not only improve achievement on the basic Van Hiele's levels but, will also help students learn better on the advanced levels such as the abstraction and deduction of Van Hiele's geometric thought. The study reveals that students achieve higher results and higher geometric thinking when taught with concept-based instruction involving the concepts of discussion, hand-on activity and guided discovery. The hand-on activities investigations enable the attainment of the characteristics level 1 and 2 of Van Hiele geometric thinking. Similarly, constantly verifying and justifying elements of procedures and each concept of circle theorem property in an experiential way using various geometric and algebraic methods ensure that students attain the characteristics of informal deduction level 3 and formal deduction level 4 of Van Hiele geometric thinking.

Contrary to several studies that examined the impact of instructional methods on students' performance using Van Hiele levels that yielded significant impact on only the basic Van Hiele levels (1 and 2), this study has shown remarkable increase even at higher levels of Van Hiele. For example, Chemuka (2017) in his study using Geogebra to improve students' performance in circle theorem found that using Geogebra only improves significantly specific levels of Van Hiele that is, the basic level 1 and 2. In his study, students could not have attained higher levels (abstraction, deduction and rigor) of Van Hiele geometry thought because Geogebra was ineffective to help students to be able to "abstract, deduct and carry out rigorous circle theorem postulates". Conversely, this study has shown significant improvements even at the higher levels of Van Hiele.

The study reveals that students achieved higher geometric thinking when taught with concept-based instruction involving the concepts of discussion, hand-on activity and guided discovery. The study calls for mathematics teachers to focus on concept development rather than emphasising the teaching of skills, procedures and algorithms. That is, "when a concept is taught in class, various graphical, algebraic and real life examples should be given" (Mahir, 2009). The study also adds to the argument that mathematics teachers should design tasks using Van Hiele's levels of geometric reasoning as well

as adopting these levels in assessing students. This will help them appreciate the need to design appropriate interventions for the attainment of each Van Hiele level.

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