Improving Problem Solving Skills in Calculus among the Grade 12 Learners' in Selected Public Secondary Schools in Lusaka District, Zambia

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Abstract: **The study sought to establish problem solving strategies that can improve problem solving skills in Calculus among the Grade 12 Learners' in Selected Secondary Schools in Zambia. The study employed a qualitative study approach, which followed a case study design. Hundred teachers (n=100) and two-hundred and fifty (n=250) Grade 12 learners from ten (n=10) public secondary schools making a total sample size of three-hundred and fifty (n=350) of Lusaka province, Zambia, participated. The sampling techniques employed were purposive sampling and simple random sampling. Data was collected using semi-structured interview schedule. Interviews are said to be the best way to collect data because it helps the researcher to have feelings, opinions, gestures, tone of voice, reactions, attitudes, views, and are useful in gathering in-depth data. Audio recordings were used to capture interviews, respectively, in their totality. The collected data was analyzed using qualitative techniques. Qualitative data was organized into themes and analyzed using narratives and direct quotations of the respondents' views, experiences and information. The study established that having pre-requisite knowledge in appropriate mathematics topics like Indices, Functions, Coordinate Geometry, Algebra (fractions, factorisation of quadratic expressions and equations), understanding the language of Calculus questions, understanding the development of Calculus formulas, and introducing Calculus symbols in early grades can improve Grade 12 learner's problem solving skills in Calculus. The study therefore, recommended that applications of basic concepts in earlier grades should be consolidated and revised on an on-going basis. Teachers should focus on the development of the formulas and introducing calculus symbols in early grades. The study further recommended that teachers should also be carrying out a diagnostic assessment to determine what learners know about pre-calculus topics such as Indices, Factorization, and Algebraic Fractions before learners are taught Calculus.**

Key words**: Calculus, Differentiation, Integration, Problem Solving Skills**

I. INTRODUCTION

On the advent of independence in the 1960s, many African nations revamped their school curricula with a view to satisfy the aspirations of their citizens (MOE, 1992, 1996). Zambia, like any other African countries, made changes to its National Curriculum and O-level Mathematics Syllabus that had been inherited from Britain (MSVTEE, 2013). However, recent changes to the revised O-level Mathematics Syllabus and the Revised Curriculum Framework were made in 2013 to align the two important documents to the current trends in the Zambian Education System (MESVTEE, 2013; ECZ, 2016). On the other hand, these changes did not only give birth to the 2013 revised O-level Mathematics Syllabus 2013 and the 2013 revised Curriculum Framework in Zambia but also gave birth to assessment, perspective, and stress on skills and values (MESVTEE, 2013; ECZ, 2016).

Consequently, one of the notable recent change in the 2013 revised O-level Mathematics Syllabus and the revised 2013 Curriculum Framework that has responded to the Zambian Educational Policy "Educating Our Future" document and the National Council of Teachers of Mathematics (NCTM) is the advocacy of making problem solving and problem solving strategy as a priority in everyday instruction (MESVTEE, 2013, p. x; MoE, 1996; NCTM, 1989, 2000, 2014). However, the article reported by the National Council of Teachers of Mathematics (NCTM) (2009) has recommended that problem-solving is the main expectation of Mathematics subject that improves students' performance in mathematics, improves a student's intellectual competence in logical reasoning, spatial visualization, develop numeracy, reasoning, thinking skills, problem solving strategies, and problem solving skills. In a supportive view, gurus on problem solving and strategy like Polya (1945) argued that problem solving and strategy helps students to apply the learnt knowledge in the classrooms to their real-life situations.

One other major amendment to the 2013 revised O-level Mathematics Syllabus is the introduction of a topic called Calculus. In Zambia, "Calculus has been introduced for the first time at Grade 12 in the Syllabus following the 2013 revised Curriculum Framework" (ECZ, 2016, p. 12). Calculus was newly introduced at Secondary level to learners of O-level Mathematics who were not allowed to study such advanced Mathematics before. Calculus concepts covered at this level include "differentiating functions from first principles, using the formula for differentiation, calculating equations of tangents and normals, explaining integration, finding Indefinite integrals, evaluating simple definite integrals, and finding the area under the curve" (MESVTEE, 2013, p. 13) (See Table 1 for details). The initiative was anchored on the usefulness of Calculus in all sectors of life. It was also the Ministry of General Education (MOGE) policy to introduce Calculus at this level to create a bridge for further

education, where it is demanded in many tertiary courses (MESVTEE, 2013).

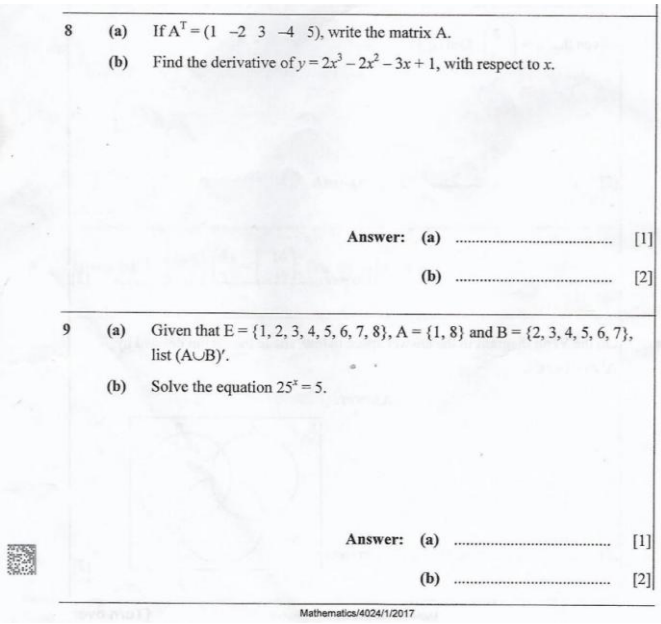
Table 2: Extract showing Calculus concepts covered at Grade 12 level in Secondary School

Topic	Subtopic	Specific outcome	Values
INTRODUCTION TO CALCULUS	12.7.1Differentiation 12.7.2 Integration	12.7.1.1Explain concept of differentiation 12.7.1.3Differentiate functions from first principles. 12.7.1.4 Use the formula for differentiation 12.7.1.8 Calculate equations of tangents and normals 12.7.2.1 Explain integration 12.7.2.3 Find Indefinite integrals 12.7.2.2 Evaluate simple definite integrals 12.7.2.3 Find the area under the curve	*Appreciation* of calculus. *Curiosity* in differentiating and integrating. *Critical thinking* in using rules for differentiation and integration.

Source: MESVTEE (2013) - "O" Level Mathematics Syllabus Grades 10 to 12.

Also, some topics like "Computer and Calculator, Arithmetic and Geometric progressions, Composite functions, Inverse functions, Graphs of cubic functions, Standard deviation in statistics, Trigonometric equations were included in the revised 2013 O-Level Mathematics Syllabus Grades 10-12 (ECZ, 2016, p.2). Similarly, the 2016, 2017, 2018, 2019, and 2020 examination at Grade 12 included questions from the new topics that were added to the revised 2013 O-level Mathematics Syllabus and Calculus questions were no exception (See Figure 1 and 2 for details).

Figure 1: Extract showing the 2017 paper 1 final examination Calculus question



8 (a) If $A^T = (1 \ -2 \ 3 \ -4 \ 5)$, write the matrix A.

(b) Find the derivative of $y = 2x^3 - 2x^2 - 3x + 1$, with respect to x.

Answer: (a) ..... [1]

(b) ..... [2]

9 (a) Given that E = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 8} and B = {2, 3, 4, 5, 6, 7}, list $(A \cup B)'$.

(b) Solve the equation $25^x = 5$.

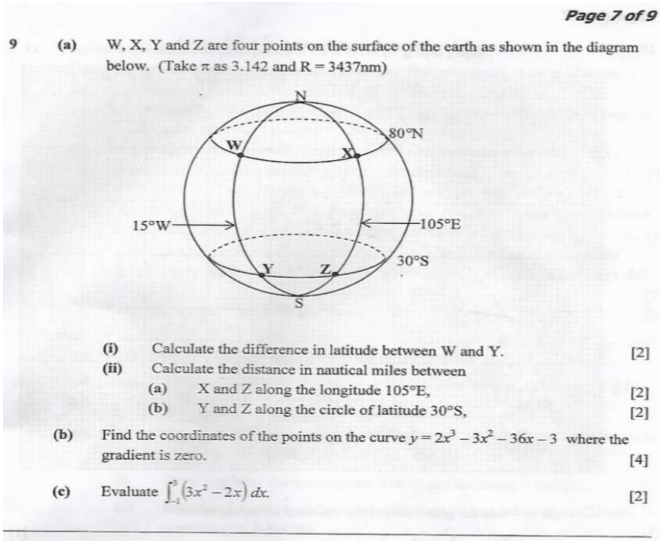
Answer: (a) ..... [1]

(b) ..... [2]

Mathematics/4024/1/2017

Source: 2016 ECZ Examination Past Paper

Figure 2: Extract showing the 2017 paper 2 final examination Calculus question



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9 (a) W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. (Take π as 3.142 and R = 3437nm)

(i) Calculate the difference in latitude between W and Y. [2]

(ii) Calculate the distance in nautical miles between

(a) X and Z along the longitude 105°E, [2]

(b) Y and Z along the circle of latitude 30°S, [2]

(b) Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero. [4]

(c) Evaluate $\int_{-1}^{3}(3x^2 - 2x)\,dx$. [2]

Source: 2016 ECZ Examination Past Paper

At the time of the study, the topic ran from Grade 12 up to tertiary level. Calculus was then a new term in the Senior Secondary School sector, although some of the concepts implicitly existed at Secondary School level under 'Coordinate Geometry" in Grade 11 which covered "Coordinate and Mid-point, Length of a straight line between points, Gradient, Equation of straight lines, and Parallel and Perpendicular lines" (MESVTEE, 2013, p.18) (See Table 2 for details).

Table 1: Extract showing Coordinate Geometry content in Secondary School Syllabus

Topic	Subtopic	Specific outcome	Values
COORDINATE GEOMETRY	11.6.1 Coordinate and the mid point 11.6.2 Length of a straight line between two points 11.6.3 Gradient 11.6.4 Equation of a straight line Parallel and perpendicular lines	11.6.1.1 Calculate the mid-point of two points 11.6.1.2 11.6.2.1 Calculate the length of a straight line 11.6.3.1 Calculate the gradient of a line segment 11.6.4.1 Find the equation of a straight line 11.6.4.1 Find the gradients of parallel and perpendicular lines Use gradients of parallel and perpendicular lines to find equations	• Problem solving • Application • Appreciation • Reasoning • Recognition • Interpretation • Relation

Source: MESVTEE (2013) - "O" Level Mathematics Syllabus Grades 10 to 12.

Nevertheless, since the introduction of Calculus at Grade 12 in 2013, the 2013 revised O-level Mathematics

Syllabus, the 2013 revised Curriculum Framework and examinations have undergone five cycles in secondary schools in 2016, 2017, 2018, 2019, and 2020 (ECZ, 2016; ECZ, 2017; ECZ, 2018; ECZ, 2019; ECZ, 2020). This means that the 2016, 2017, 2018, 2019, and 2020 examination at Grade 12 included questions from the Calculus (ECZ, 2016, p. 12; ECZ, 2017, p. 25; ECZ, 2018, p. 22; ECZ, 2019, p.118, & ECZ, 2020, p. 121). In the final Mathematics examination for 2016, Calculus was awarded 2 marks out of 80 in paper 1 which is 2.5% of the whole examination and 5 marks out of 100 in paper 2 which is 5% for the whole examination. In the final Mathematics examination for 2017, Calculus was awarded 2 marks out of 80 in paper 1 which is 2.5% of the whole examination and 3 marks out of 100 in paper 2 which is 3% of the whole examination. In the final Mathematics examination for 2018, Calculus was awarded 4 marks out of 80 in paper 1 which is 5% of the whole examination and 3 marks out of 100 in paper 2 which is 3% of the whole examination. In the final Mathematics examination for 2019, Calculus was awarded 5 marks out of 80 in paper 1 which is 6.3% of the whole examination and 4 marks out of 100 in paper 2 which is 5% of the whole examination, and in the final Mathematics examination for 2020, Calculus was awarded 3 marks out of 80 in paper 1 which is 3% of the whole examination and 3 marks out of 100 in paper 2 which is 3% of the whole examination. Hence, the Calculus theme is quite significant in the Zambian Mathematics curriculum as it is international.

Despite the integration of Calculus at Grade 12 level, learners' "performance in Calculus has not improved significantly and "some candidates scored zero" (ECZ, 2016, p. 12 &116; ECZ, 2017, p. 11 & 121; ECZ, 2018, p.14; ECZ, 2019, p.12). The report by the 2016-2020 examinations indicates that poor performance in Calculus was as the result of lack of problem solving skills in Calculus (ECZ, 2016, p.28; ECZ, 2017, p.25). We studied the Mathematics examiners' reports of the Examination Council of Zambia for Senior Secondary Certificate from 2016 to 2020. The objective was to find areas of poor learner performance with the ultimate goal of identifying counter measures which would enhance problem solving skills in Calculus. The purpose of the examiners' reports is to provide feedback to teachers, learners, policy makers and other stakeholders on learner performance in the examination with recommendations on how any issues identified may be addressed (ECZ, 2014; ECZ, 2015). A repeating theme in the examiners' reports was lack of problem solving skills among the Grade 12 learners' when solving Calculus problems. The following are some of the notable extracts from the reports (See Table 3, 4, 5, 6, and 7 for details):

Table 3: Extract showing an analysis performance in Calculus

On	Topic	General performance	Comments
6	(a) Differentiation	Poor	- Most candidates lacked knowledge of the use of differentiation to find the equation of the normal. Poor problem solving skills among the candidates.
	(b) Application of differentiation	Poor	-Some just equated the original equation to zero and solved for x leading to wrong solutions. -Poor problem solving skills was a challenge among the candidates.

Source: ECZ (2016)-Examination Council of Zambia, 2016 Performance Review Report

Table 4: Extract showing an analysis performance in Calculus

On	Topic	General performance	Comments
6	(a) Differentiation	Poor	-Most candidates lacked knowledge of working from first principles -Most candidates lacked problem solving skills
	(b) Integration	Poor	- Poor problem solving skills was a challenge among the candidates.

Source: ECZ (2017)-Examination Council of Zambia, 2017 Performance Review Report

Table 5: Extract showing an analysis performance in Calculus

On	Topic	General performance	Comments
6	(a) Differentiation	Poor	-Most candidates lacked problem solving skills.
	(b) Integration	Poor	-Most candidates lacked problem solving skills. -Most candidates lacked knowledge of finding the upper and lower limits.
	(c) Application of integration	Poor	-Candidates lacked knowledge of finding the area and problem solving skills.

Source: ECZ (2018)-Examination Council of Zambia, 2018 Performance Review Report

Table 6: Extract showing an analysis performance in Calculus

On	Topic	General performance	Comments
6	(a) Integration	Poor	-Most candidates lacked knowledge of finding the upper and lower limits Most candidates lack problem solving skills
	(b) Integration	Poor	-Most candidates lacked problem solving skills
	(c) Application of integration	Poor	-Candidates lacked knowledge of finding the area and problem solving skills

Source: ECZ (2019)-Examination Council of Zambia, 2019 Performance Review Report

Table 5: Extract showing an analysis performance in Calculus

On	Topic	General performance	Comments
6	(a) Differentiation	Poor	- Most candidates lacked knowledge of the use of differentiation to find the equation of the normal.
	(b) Integration	Poor	-Most candidates lacked problem solving skills

Source: ECZ (2020)-Examination Council of Zambia, 2020 Performance Review Report

In all the reports, lack of problem solving skills was echoed as a major deficiency in Grade 12 learners' work, in particular, when solving Calculus problems. The fact that lack of problem solving skills has continued to cause learners to under-perform in Calculus since 2016, calls for serious investigation. Thus, lack of problem solving skills in Calculus necessitated the authors to explore problem solving strategies that can improve problem solving skills in Calculus at Grade 12 level.

II. RESEARCH METHODOLOGY

The study employed a qualitative study approach, which followed a case study design. One Hundred teachers (n=100) and two-hundred and fifty (n=250) Grade 12 learners from ten (n=10) secondary schools making a total sample size of three-hundred and fifty (n=350) in Lusaka Province. The sampling techniques employed were purposive sampling and simple random sampling. Merriam (1998) says purposeful sampling allows the researcher to select those participants who will provide the richest information, those who are the most interesting, and those who manifest the characteristics of most interest to the researcher. Data was collected using semi-structured interview schedule. Semi-structured interviews are said to be the best way to collect data because it helps the researcher to have feelings, opinions, gestures, tone of voice, reactions, attitudes, views, and are useful in gathering in-depth data (Zulu, 2019; Zulu, 2021;Patton, 1990). Audio recordings were used to capture interviews, respectively, in their totality. The collected data was analyzed using qualitative techniques. Qualitative data was organized into themes and analyzed using narratives and direct quotations of the respondents' views, experiences and information.

III. FINDINGS AND DISCUSSION

In this section, we present results on problem solving strategies that can improve problem solving skills in Calculus among the Grade 12 Learners' in Selected Secondary Schools in Lusaka Province, Zambia. Learners have been coded as A_1 to A_{25} (Learners from school A), B_1 to B_{25} (Learners from school B), C_1 to C_{25} (Learners from school C) and D_1 to D_{25} (Learners from school D), E_1 to A_{E5} (Learners from school E), F_1 to F_{25} (Learners from school F), G_1 to G_{25} (Learners from school G), H_1 to H_{25} (Learners from school H), I_1 to I_{25} (Learners from school I) and teachers have been coded as A (Teacher from school A), B (teacher from school B), C (Teacher from school C), D (teacher from school D), E (Teacher from school E), F (teacher from school F), G (Teacher from school G), H (teacher from school H), and H (teacher from school H). Four (4) problem solving strategies were revealed:

Pre-requisite knowledge in appropriate mathematics topics

Majority (197 out of 250) of the learners representing 79% recommended that having a strong background knowledge in topics like Indices, Functions, Coordinate Geometry, Algebra (fractions, factorisation of quadratic expressions and equations) can improve their problem-solving skills in solving Calculus problems. Generally, learners spoke zealously about having pre-requisite knowledge in order to be good in Calculus. The following quotations by learners (C_3, F_5, and $H_{7)}$ provided below act as typical examples:

> *"…I think Sir one needs to understand topics like Factorisation and Indices"* (A_3). *…By studying topics like Coordinate Geometry, Factorisations, functions and relations, and Algebra and more especially on factorising the quadratic expressions and equations our problem solving skills can be improved. For example, in class we were given to find the equations of the tangent and normal to a curve, the concepts we learnt from our previous Grades were needed to be applied. So, Sir we have to be good in some of these topics* (F_5). *By studying it different composition such as Coordinate Geometry, Indices, Factorisations and others* (H_7).

The same understanding was held by both teacher G and teacher J who expressed great concern on pre-requisite knowledge. The following excerpts are provided as typical examples:

> *I think from my vast experience, one of the challenge one would be lack of pre-requisites because a topic like Calculus I feel there are prerequisites which should be looked briefly before teaching the topic. I know these kids at Grade 10, even at Grade 9 they learn Indices and by the time they reach Grade 12 a lot of them may have forgotten the concepts. So, it is very important before introducing or teaching Calculus that such prerequisites topics such Indices should be tackled first briefly off course for maybe a period or even the full period then the next period you begin teaching Calculus. "In addition, another prerequisite of knowledge to improve learners' problem solving skills in Calculus according to my experience is a topic at Grade 9 which is evaluating algebraic expressions where you have letters representing numbers so if you have an algebraic expression and you want to evaluate you substitute those letters with figures, integers given. So, it is important to briefly go back to that, because for example evaluating definite integrals, you replace variables like x with integers that you may be given, so it is important as a teacher you go back to that"* (Teacher G). *One good strategy is that learners should have a strong background in algebra. Teachers should be emphasizing*

on the importance of algebraic skills and how important are. I have noticed that the majority of the pupils have serious problems with applying the rules of indices and the concepts of factorisations and in order to improve their skills in Calculus, I think topics like Indices and functions also are key because algebraic skills is what drives mathematics (Teacher J).

Understanding the language of Calculus questions

Learners (223 out of 250) representing 89% suggested that understanding the language of Calculus questions before solving can improve their problem-solving skills in Calculus. For instance, the following verbatim by learners (A_2, D_4 and G_9) act as typical examples:

... Since the first step when solving in any mathematical question, you have to read and understand the question, then you develop a plan, so as for me I think reading and understanding the question and also coming up with a plan in that way you will have a clear picture of the kind of the question you are solving" (A_2). *...One way could be understanding the language of the problem first"* (D_4). *I think by understand Calculus questions before starting to solve Calculus problems can improve our solving skills* (G_9).

Similarly, teacher K expressed that:

...I feel one of a strategy that can improve our children's problem-solving skills in Calculus can be through understanding the Calculus questions. It's like most of us our children just start solving Calculus problems without understanding what the problem is saying this becomes very hard for them to solve most the questions.

Understanding the development of Calculus formulas

A large portion (209 out of 250) of the learners representing 84% suggested that in order to improve their problem-solving skills in Calculus, Calculus formulas should be understood and applied appropriately. The following excerpts by learner (J_6 and K_7) acts as typical examples:

".....Understand how to use Calculus formulas Sir" (J_6). *"……. Know different Calculus formulas and how to apply them"* (K_7).

Introducing Calculus symbols in early grades

About 202 out of 250 of the learners representing 81% during the interviews cited that Calculus symbols such as $\frac{dy}{dx}, f'(x) or\ y', \lim h \to 0$ should be introduced in early Grades that is 10 or 11 in order to improve their problem solving skills as evidenced by the following excepts by learner (K2 and L5) acts as typical examples:

"I think symbols like $\frac{dy}{dx}$ *should be introduced in previous grade. In that way we will be good in Calculus* and our problem solving skills will be improved (K_2). *"…….I think Calculus symbols should be introduced in early grades system so that lets say for example you are in Grade 8 or 10, you begin from there knowing these symbols by the time you get there in Grade 12, you have knowledge. This way our problem solving skills will be improved"(*L_5).

Problem solving strategies to improve learners' problem solving skills in Calculus

Pre-requisite knowledge in pre-calculus topics

The current study established a need for Grade 12 learners to have pre-requisite knowledge in appropriate Mathematics topics can improve their problem solving skills in Calculus. Among the topics learners suggested during interviews that can improve their problem-solving skills in Calculus include: Functions and Relations, Coordinate Geometry, Algebra, Indices, Quadratic expressions, Quadratic equations, Fractions, and Factorisation. However, the concerns expressed by the Grade 12 learners are in harmony with reviewed literature by different scholars in Mathematics (Herbert, 2009; Rickard, 2005) who argued that Calculus is just a combination of different topics in Mathematics such as Algebra, Indices, and Functions and Relations.

Moreover, learners concerns also comprehended with teachers responses concerning the need to understand pre-calculus topics such as Functions and Relations, Coordinate Geometry, Algebra, Indices, Quadratic expressions, Quadratic equations, Fractions, and Factorisation in order to improve problem solving skills in Calculus. This calls for learners to know and understand that in order to have good problem solving skills in Calculus, appropriate pre-calculus topics such as Functions and Relations, Coordinate Geometry, Algebra, Indices, Quadratic expressions, Quadratic equations and Factorisation are cardinal, and teachers of Mathematics of should always ensure that these pre-calculus topics are emphasised and revised on an ongoing basis. Apparently, having pre-requisite knowledge in these pre-calculus topics can help learners understand how these topics and a topic like Calculus interlink because prior knowledge is another aspect that plays a significant role in Calculus solving. In fact, the importance of prior knowledge is highly emphasized by Gestalt theory of problem solving (Wertheimer, 1959).

Understanding the Language of Calculus questions

Results from the study established that understanding the language of Calculus questions can improve learners' problem solving skills in Calculus. Majority of the learners during semi-structured interviews expressed concern that understanding the language of Calculus can be one of the good strategy that can improve their problem-solving skills in Calculus. Learners' responses are in concordance with studies done by numerous researchers who have argued that familiarity with mathematical terminology improves problem-solving ability contexts (Verschaffel et al., 1999). Besides, literature across the globe has revealed that students with a sufficient understanding of conventional Mathematics terminology are apt to solve problems because they understand the meaning of the words they read (Ball & Bass,

2003; Pape, 2004). Thus, researchers argue that Mathematics is a language that relies on symbols but it also includes graphs, charts, and texts to decode. "Reading completely depends on being able to understand the structures of texts and nuances of language; to interpret authors' ideas; and to visualize, evaluate, and infer meanings" (Ball & Bass, 2003; p. 29).

Understanding the development of Calculus formulas

Learners and teachers recommended that understanding the development of Calculus formulas such as the rule of differentiation, first principles or rule of integration can improve learners' problem solving skills in Calculus. Participants responses correlates with reviewed literature (Schoenfeld, 2013; Kilpatrick et al., 2001; Lam, 2009) which has shown that formulas in Mathematics are key to finding answers and improving problem solving skills. Arguably, it should be emphasized that in order to ensure that learners cite correct first principles formula or work from first principles formula, teachers of Mathematics should focus on the development of formulas when teaching Calculus and solving Calculus problems. In order to illuminate memorizing Calculus formulas that is first principles when solving Calculus problems by the learners, the development of first principles can be developed as follows:

The word tangent is derived from the Latin word which means touching. Thus, the tangent to a curve is a line that touches the curve. The tangent has to have the same direction as the curve at the point of contact. The essence of Calculus is the limit and derivative (Makonye, 2011). Figure 1 depicts how the derivative is the instantaneous rate of change of a function with respect to one of its variables. This is equivalent to finding the slope of the tangent line to the function at a point. However, as the derivative can be interpreted in three ways;

As the slope of a tangent:

$$f'(x) = \lim_{h\to 0} \frac{f(a+h) - f(x)}{h}$$

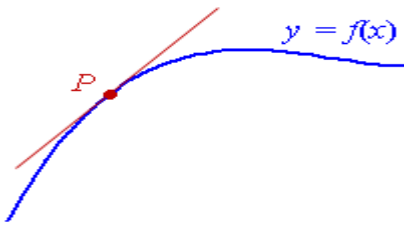

Figure 1: Graph showing the slope of a tangent

Secondly, the derivative at point a can be written as:

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{h}$$

Thirdly, the derivative can be visualized in terms of infinitesimals, small increments in x resulting in a small increment in y (Figure 2):

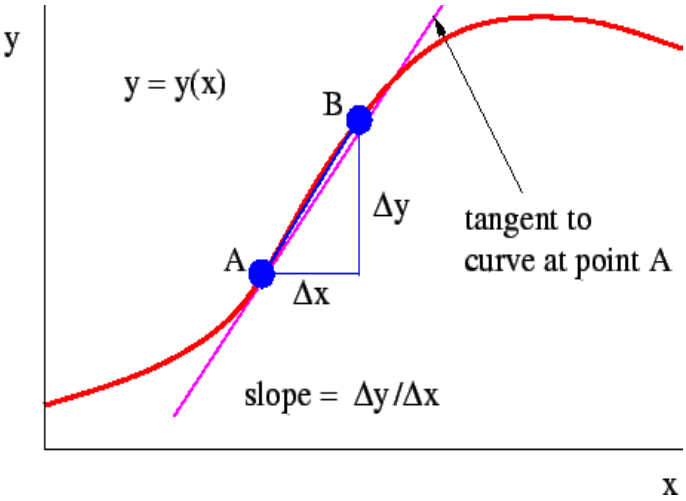

Figure 2: Graph illustrating the basis of derivative

This is usually written as;

$$\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{x\to a} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x}$$

Therefore, researchers suggest that if the concepts of developing first principles can be illustrated as shown above, learners will be able to understand first principles and know how to apply it. This will enhance their problem solving skills in Calculus and illuminate memorization.

Introducing Calculus symbols in early Grades

Learners suggested that Calculus symbols such as $f'(x), f(x), y, y', \frac{dy}{dx}, \lim_{h\to 0}$ *and* $\int f(x)\,dx$ should be introduced in early Grades that is Grade 10 or Grade 11. Learners also expressed that introducing Calculus symbols in early grades will help them know the symbols before they get to Grade 12. Learners further expressed that understanding of these Calculus symbols in early Grade will improve their problem-solving skills in Calculus as they will be more familiar with the symbols. Learners suggestions are in harmony with the Department of Basic Education in South Africa that reported that understanding mathematical symbols can help learners appreciates Mathematics (DoBE, 2015). Learners' responses are in agreement with reviewed literature by Makonye (2011) who said understanding of Calculus symbols such as $\frac{dy}{dx}$ *and* $\lim_{h\to 0}$ can improve performance in Calculus.

IV. CONCLUSION AND RECOMMENDATIONS

The study sought to establish problem solving strategies that can improve problem solving skills in Calculus among the Grade 12 Learners' in Selected Secondary Schools in Zambia. The study established that having pre-requisite knowledge in appropriate mathematics topics, understanding the language of Calculus questions, understanding the language of Calculus, understanding the development of Calculus formulas, and introducing Calculus symbols in early grades can improve Grade 12 learners' problem solving skills in Calculus. Authors therefore, recommended that applications of basic concepts in earlier grades should also be consolidated and revised on an on-going basis. Teachers should focus on the development of the formulas and introducing Calculus symbols in early grades. The ministry of Education through Curriculum Development Centre need to consider introducing Calculus symbols such as $\frac{dy}{dx}, y' or\ f'(x)\ and\ \lim_{h\to 0}$ in early Grades that is Grade 10 or 11 so that learners can be more familiar with Calculus symbols before coming across them in Grade 12. Teachers should also be carrying out a diagnostic assessment to determine what learners know about pre-Calculus topics such as Indices, Factorization, and Algebraic Fractions before learners are taught Calculus.

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