Linear and non-linear modelling of Nigerian Inflation Rate

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***Abstract*: In order to model Nigeria's inflation rate, this analysis compared univariate linear models to univariate nonlinear models. The data for this analysis was gathered from the Central Bank of Nigeria statistical bulletin on a monthly basis from January 2006 to December 2019. The upward and downward movement in the series revealed by the time plot suggest that the series exhibit a regime-switching pattern: the cycle of expansion and contraction. At lag one, the Augmented Dickey-Fuller test was used to screen for stationary. For univariate linear ARIMA (p d q)) and univariate non-linear MS-AR, seven models were estimated for the linear model and two for the non-linear model. The best model was chosen based on the criterion of least information criterion, AIC (2.006612), SC (2.156581), and the maximum log-likelihood of(-150.5480) for the inflation rate were used to pick MS-AR (1) for the series. In analysing inflation rate data, the MS-AR model proposed by Hamilton outperforms the linear autoregressive models proposed by Box Jenkins. The model was used to predict the series' values over a one-year cycle (12 months).**

***Key word*: inflation rate, linear models, non-linear models and forecasting**

I. INTRODUCTION

Inflation has been an issue facing many countries around the world, especially undeveloped ones. It began as a result of the grouping of economic policies in the early 1960s as a measure to reduce the impact of inflation in societies. Most of these steps taken by developed countries to monitor the inflation issue are credit control instruments of the Central Bank. This is aimed at reducing and maintaining the amount of money in circulation to ensure low living costs. The problem of inflation is also faced by Nigeria as a developing nation. [13]. In Nigeria, inflation has been a concern for policymakers since the 1990s, and the rate of inflation has been strong since then. Inflation is neither new to the economy of Nigeria, nor to the world at large. Evidence has shown that inflation, with variations in severity or rates, exists in both advanced and unindustrialized countries. Inflation rates are higher in developing countries than those in developed countries. Inflation can be described as the process of the continuous increase in the price of goods and services as a result of a large amount of money in circulation used in exchange of few goods and services. This has resulted in high prices of imported goods resulting from an increase in foreign prices and the instability of international exchange rates, sub-charges due to congestion in the port, storage facilities, marketing arrangements, distribution networks, etc. Since the removal of subsidies, there has been an increase in the price of oil and this has led to an increase in the price of most items, such as an increase in transport prices is a living example at hand.

II. LITERATURE REVIEWS

[5] Analysed Nigeria's inflation rate using the 2003-2011 seasonal ARIMA mode. The time plot showed a secular trend in their discussion and the seasonal distinction showed a seasonality, but the course of the movement is not clear. A stationary series was created by the non-seasonal series differentiation. The ACF plot shows a spike indicating a seasonal MA portion at lag 12. At the beginning, the PACF plot showed no spike that suggested a non-seasonal MA portion. The SARIMA $(011)*(011)_{12}$ model follows the appropriate model for the inflation rate in Nigeria.[8] analysis of monthly inflation rate volatility using generalized autoregressive conditionally heteroscedastic (GARCH) mode in Nigeria from 2012 to 2013 examined the best-fitting model for a consumer price index (CRI) inflation rate in Nigeria from January 2012 to December 2013 as GARCH(1,0)+ARMA (1,0). For a diagnostic test, th researcher used R- Software and generalized autoregressive conditionally heteroscedastic, for the consumer price index (CRI) inflation rate, the series is non-stationary by the time plot. At first distinction, the log-transformation of the sequence shows non-constant variance. [10] Analyzed the relationship between exchange rate and inflation rate to assess the effect of the exchange rate on Romania's inflation rate using a vector autoregressive model. The result shows that a 1 percent change in the exchange rate produces a 0.36 percentage point shift to the index-based inflation rate of producer prices. [9] uses the regime-switching vector autoregressive method to study the inflation rate and the development of growth dynamics in South Africa. Their finding stated that the response of the production growth rate to the inflation shock depends not only on the regime but also on how the monetary authority responds to this shock. In the Federal Reserve Board staff forecast of the PCE price inflation rate, [13] Studies the inflation return using the Markov switching regime model to assess the duration of high levels of uncertainty and persistence. He concludes that the high variance rate likelihood of inflation is a highly persistent regime. [8] Studies the U.S stock returns through application of autoregressive conditional heteroscedasticity and regime shift. They've checked the volatility of the variable in different regime in the sequence. For the pre-test, they used the Gaussian and Student T distribution. The estimated model is

used in their conclusions to predict the values of the sequence with weak estimated values.

III. METHODOLOGY

3.1 Time Plot

When presented with any time series data the first step in the analysis is usually time plot of data, to examine a simple descriptive measure of the main properties of the series. The graph of the inflation rate was plotted against time to enable us to have an idea of the overall movement of the original data over the periods, whether the trend is constant or dies out with time. This plot also enables the researcher to know about the following, Trend (upward or downward) movement in the entire time, Seasonal fluctuation, Constant variances and the expansion and contraction period. [12]

3.2 Linear Models

3.2.1 Autoregressive (AR) Models of Order (p)

An autoregressive model is a time series model in which one uses the statistical properties of the past values of the series to predict future values. The general illustration of an autoregressive model of order p, AR(p) is [17]

$$Y_t = \sum_{i=o}^{p} \alpha_i Y_{t-i} + \varepsilon_t \qquad (3.1)$$

Generally, the Gaussian autoregressive of order p AR(p) process with mean μ is given by

$$Y_t - \mu = \sum_{i=o}^{p} \beta_i (Y_{t-i} - \mu) + \varepsilon_t \qquad (3.2)$$

Where the term ϵ_t is the error term and is called white noise, β_0 β_1 and β_p are unknown parameters, while y_t, y_{t-1} y_{t-2} and y_{t-p} are estimated from the sample. [3]

3.2.2 Stationarity Conditions for AR (P) Process

The characteristics polynomial of an AR (p) process, given as;

$$(1 - \beta_1 B - \cdots - \beta_2 B^p) y_t = \phi y_t = w_t \qquad (3.3)$$

And the process must satisfy certain conditions for the process to be stationary. For instance, the first-order autoregressive: [13]

$$(1 - \beta_1 B) y_t = w_t \qquad (3.4)$$

maybe written as;

$$y_t = (1 - \beta_1 B)^{-1} w_t = \sum_{j=0}^{\infty} \beta_1^j a_{t-1} \qquad (3.5)$$

Hence;

$$\psi(B) = (1 - \alpha_t B)^{-1} = \sum_{j=0}^{\infty} \beta_1^j B^j \qquad (3.6)$$

This implies that the parameter β_p, of an AR (p) process, must satisfy the condition $|\beta_p| < 1$ to ensure stationarity.

In general, for an $AR(p)$, the root of the characteristic equation $\varphi(B) = 0$ must lie inside the unit circle to ensure stationarity.[3]

3.2.3 Moving Average (MA) Model

In terms of deviation from the series, a series y_t is said to follow a moving average process of order MA (q). mathematical represented in the form

$$y_t = \varepsilon_t - \sum_{t=1}^{q} \theta_i \varepsilon_{t-i} \qquad (3.7)$$

Or

$$y_t = \theta(B) \varepsilon_t \qquad (3.8)$$

Where

$$\theta(B) = (1 - \theta_1 B - \theta_2 B^2, \dots, \theta_q B^q), \qquad (3.9)$$

ε_t is the white noise and $\theta_1, \theta_2, \dots, \theta_q$ are constants. This is known as finite moving average of order q, ma (q). It should be noted that defining the MA polynomial with a negative or positive sign does change the properties of the model but only changes the algebraic signs of the MA coefficients.

3.2.4 Invertibility Condition for (Ma) Model

For an $MA(q)$, the invertibility condition is that the roots of the characteristic polynomial lie inside the unit root circle.Box and Jenkin (1975)

$$|1 - \theta_1 B - \theta_2 B^2 - \cdots \dots \dots \dots \dots . - \theta_p B^p| < 1 \qquad (3.10)$$

3.2.5 Autoregressive-Moving-Average Models (ARMA)

We have seen from above that the AR model includes lagged terms on the series itself and that the MA model includes lagged terms on the error term. By including both types of lagged terms, we arrive at the ARMA model. Therefore ARMA (p,q), where p is the order of autoregressive term and q the order of the moving-average term, these can generally be represented as

$$Y_t = \sum_{i=o}^{p} \alpha_i Y_{t-i} + \varepsilon_t - \sum_{t=1}^{q} \theta_i \varepsilon_{t-i} \qquad (3.11)$$

A series $\{y_t\}$ is said to follow an autoregressive moving average model *of orders p and q*, designated ARMA (p, q), where α_i and θ_i are constants such that the model is stationary as well as invertible and ε_t is a white noise process. Let model (3.11) be written specifically as;

$$A(B) y_t = B(L) \varepsilon_t \qquad (3.12)$$

$$\text{Where } A(B) = 1 - \beta_1 B_1 - \beta_2 B_2 - \dots - \beta_p B_p \qquad (3.13)$$

$$B(L) = 1 + \theta_1 B_1 + \theta_2 B_2 + \dots + \theta_q B_q \qquad (3.14)$$

B is the backshift operator defined by

$$B_t^k y_t = y_{t-k} \qquad (3.15)$$

3.2.6 Arima Model with Differencing

Many series are non-stationary. For a non-stationary series $\{Y_t\}$ Box – Jenkins proposed that differencing up to an appropriate order make it stationary. Suppose d is the minimum order of differencing necessary for stationary to be attained. The d[th] difference of $\{y_t\}$ is denoted by $\{\Delta^d y_t\}$ where Δ^d is the difference operator defined by Δ^d=1 – B.[3] If the series $\{\Delta^d y_t\}$ follows the model (3.12), then $\{y_t\}$ is said to follow an autoregressive integrated moving average model of order p, d and q designated as ARIMA (p, d, q).

In a general will, can write the model as

$$\varphi(B)(1-B)^d y_t = \theta(B)\varepsilon_t \qquad (3.16)$$

3.3 Non-Linear Modelling

3.3.1 Markov Switching Model

In certain situations, the regime in operation at any point in time is directly observable. More generally if the regime is unobserved, the researcher must conduct inference about which models are allowed to switch from one state to another in each fixed number of regimes. A stochastic process assumed to have generated the regime shifts as part of the model, which allows variables to switch between regimes according to an unobserved Markov chain the process is represented by a past and present time. Regime switching models can be divided into two categories, Threshold models and Markov-switching models. The primary difference between these approaches is how the state process is modelled. Threshold models, introduced by [14], assume that regime shifts are triggered by the level of observed variables about an unobserved threshold. Markov-switching models, introduced by [7], assume that the regime shifts according to a Markov chain.

Markov-switching models also assume that S_t is the unobserved variable and y_t an observed variable. Contrast to threshold models, Markov-switching models assume that, S_t follow a particular stochastic process, namely an *N* state Markov chain. The development of Markov chains is described by their transition probabilities, given by:

Pr($S_t = i/S_{t-1} = j, S_{t-2} = q \dots\dots$) =Pr$S_t = i/S_{t-1} = j$) =Pij (3.17)

Where conditional on a value of *j*, we assume$\sum_{i=1}^{n} Pr_{ij} = 1$. That is, the process specifies a complete probability distribution for S_t. In general, the Markov process allows regimes to switch from one state to another. More than once restrictions can be placed on Pr_{ij} to restrict the order of regime shifts. [7]

3.3.2 Markov Switching Autoregressive Model (MS-AR)

The technique of using switching probability in non-linear models was first discussed by and a similar idea of modelling a non-linear series was also develop my [8] which emphasizes on the aperiodic transition between the various state of the economic variable. The transition is driven by a hidden Markov state. A time series y_t follow an MS-AR model if it satisfies the following models.

$$y_t = \begin{cases} c_1 + \sum_{i=1}^{p} \theta_{1,i}\, y_{t-i} + \varepsilon_{1,t} & if\ s_t = 1 \\ \vdots & \\ c_2 + \sum_{i=1}^{p} \theta_{2,i}\, y_{t-i} + \varepsilon_{2,t} & if\ s_t = 2 \end{cases} \qquad (3.18)$$

In general, Markov switching autoregressive model of order p is represented by [2]

$$y_t = c_{s_t} + \sum_{i=1}^{p} \theta_{s_t,i}\, y_{t-i} + \varepsilon_{s_t,t} \qquad if\ s_t = m \qquad (3.19)$$

s_t follow a first-order Markov chain with transitional probability

Pr($s_t = j/s_{t-1} = i$) $= w_1$,Pr($s_t = i/s_{t-1} = j$) $= w_2$ (3.20)

$\varepsilon_{s_t,t}$ is the error terms iid random variable with mean zero and infinite variances and independent of each other.[1] If p_{ij} is small, it's mean that the model tends to stay longer in a state i then in state j. The expected duration of the process is given by $\frac{1}{w_i}$ (the period the switching is to stay in state i) the Markov switching autoregressive models uses a hidden Markov chain to govern the transition probability from one conditional mean function to another.[9]

IV. RESULT

The data used for this work is the average monthly inflation rate data (Y_t) in Nigeria between 2006-2019 (168 observations) The raw data of inflation rate was obtained from website httpt.www.centralbank.com. the analysis is carried out using eviews 11software.

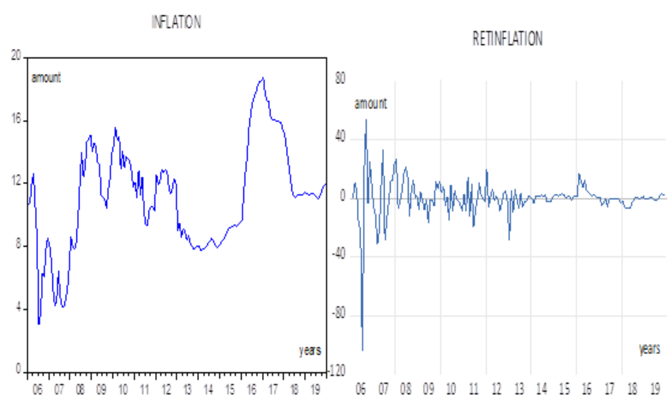

Figure 4.1: Time Plot of the Inflation Rate at Level and first difference

4.1 Time Plot of Inflation Rate

The time plot of the series is shown in figure 4.1. A critical look at the time plot revealed upward and downward movement in series, this means that the series exhibit a regime-switching pattern (a period of expansion and contraction in their movement). This shows a period of 2 regimes in the variable of our studies. Therefore, a linear trend is present in the data. The presence of a trend in a series will make it not to be stationary (a series is said to be stationary if it has constant mean and variance). The time plot of first difference is shown in figure 4.1 showing a stationary process, (meaning that the series has constant mean and variances)

Table 4.1: Augmented Dickey-Fuller (ADF) Unit Roots Test

Variable	Levels		1st Difference	
	Intercept	A Intercept, Linear Trend	Intercept	Intercept Linear Trend
Inflation rate (INF_t)	-2.10573 (0.2428)	-2.296291 (0.4332)	-5.50677 (0.00)	-5.48844 (0.00)
Test critical values: 1% level 5%level 10% level	-3.4731 -2.8802 -2.5768			

The variables involved in this study were tested for stationarity, since the variables of the study cannot be applied for analysis unless it is established that the variables are stationary. Data on each series were tested for stationarity to avoid the problem of spurious regression. The Augmented Dickey-Fuller (ADF) test was used to test for unit root on each of the variable, at level and first differences, constant, linear trend and probability values in brackets, the probability values (p-values) at level is greater than 0.05 (p-values >0.05), the result showed the presence of unit root.[6]

Table 4.2: Comparing Linear and Non-Linear Models of Inflation Rate in Nigeria Using ((AIC, (SC) and Log-Likelihood)

S/N	ARIMA(p.d.q)	AIC	SC	LOG-LIKELIHOOD
	Autoregressive Integrated Moving Model Of Inflation Rate			
1	ARIMA(111)	2.833298	2.87980	-232.5804
2	ARIMA(110)	2.82112	2.881124	-232.8968
3	ARIMA(210)	2.826102	2.900784	-231.9795
4	ARIMA(011)	2.829069	2.885681	-232.2273
5	ARIMA(112)	2.824053	2.917400	-230.8084
6	ARIMA(012)	2.814596	2.889579	-231.0438
7	ARIMA(212)	2.790670	2.902694	-227.0209
	Markov Switching Autoregressive Of Inflation Rate			
8	MS-AR(1)	2.006612	2.156588	-158.5488
9	MS-AR(2)	2.00676	2.194316	-155.5313

Seven models were estimated for univariate linear (ARIMA (p d q)) models and Two model univariate non-linear (MS-AR(p)), All variables were stationary at lag 1, the best model was selected base on Minimise Information Criterion. MS-AR(1) was chosen for the series with the following information criterion AIC (2.006612), SC (2.156581) and the largest log-likelihood of (-150.5480) for data. The MS-AR model of [7] performs better than the linear autoregressive integrated moving average models proposed by [3] in examining the data of inflation rate in Nigeria.

4.2 Markov Switching Autoregressive Model of Inflation Rate

The MS-AR (1) model of inflation rate in both regime 1 & 2 can be represented mathematically as follow:

$$inf_t = -0.0256690 + 0.121892\, inf_{t-1} \text{ if } s_t = 1(regime\ 1) \qquad (5.1)$$

$$inf_t = 0.077932 + 0.462990\, inf_{t-1} \text{ if } s_t = 1(regime\ 2) \qquad (5.2)$$

Where inf_t represent inflation rate at current values and $s_t = 1$ represent expansion and $s_t = 2$ denote contraction in differences regime.

4.3 The transition Matrix of Inflation Rate

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.95954 & 0.040457 \\ 0.065162 & 0.934338 \end{bmatrix} \qquad (5.3)$$

Where $P_{11} + P_{12} = 1$, $P_{21} + P_{22} = 1$

P_{11} Denote the probability of transitioned to expansion in the next period given that the current state is in expansion. P_{12} denote the probability of transitioning to a contraction in the next period given that the current state is in expansion. P_{21} denote the probability of transitioning to expansion in the next period given that the current state is in contraction.

P_{22} Denote the probability of transitioning to a contraction in the next period given that the current state is in contraction.

4.4 Expected Duration Inflation Rate

The expected time spent in each state is called the expected duration. If D_1 is the expected duration spend in state 1, is denoted as

$$E(D_1) = \frac{1}{1 - p_{11}}$$

The closer P_{11} is to 1 the higher is the expected duration of state 1.

Table 4.3: The Transition Probability from Regime 1 to Regime 2 is Presented below

$$P_r(1, 2) = p_r(s(t)) = 2 / s(t-1=1)[2]$$

Variables	Transition Probability				Expected Duration			
	P_{11}	P_{12}	P_{21}	P_{22}	E_{11}	E_{12}	E_{21}	E_{22}
Inflation Rate	0.95954	0.040457	0.065162	0.934338	24.7176	1.04228	1.07027	15.3468

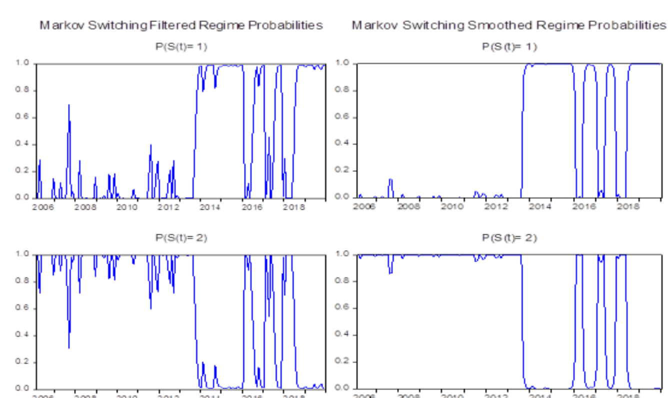

Figure 4.2: Filtered and Smoothed Probabilities of Inflation Rate in Both Regime 1 & 2

The smoothed probabilities provide inference on s_t conditional on all available sample information, example exogenous and endogenous switching process. In regime one $pr(s_t = 1)$, the expansion state, we find out that smoothing process is low from the being and high at the end, in regime two $pr(s_t = 2)$, contraction smoothing is in opposite direction from the being and low to high and low at the end. However, during the financial crisis smoothing is away in a high regime of variance which corresponds to a high level of market volatility. The Filtered probabilities showed that the unobserved state for switching is in a particular regime at time t condition on observing sample information up to time T. The filtered probability have a similar pattern with the smoothed probability15]

4.5 Forecast Equation of Inflation Rate for January to December 2020 For Regime 1 Using MS-AR(1)

inf_t=$-0.0256690 + 0.121892\, inf_{t-1}$, January 2020 (5.4)

$inf_{t+1} = -0.0256690 + 0.121892\, inf_t$, February 2020 (5.5)

inf_{t+11} =$-0.0256690 + 0.121892\, inf_{t+10}$, December 2020 (5.6)

4.6 Forecast Equation of inflation Rate for January to December 2020 for Regime 2 Using MS-AR(1)

inf_t=$0.077932 + 0.462990\, inf_{t-1}$, January 2020 (5.7)

inf_{t+1}=$0.077932 + 0.462990\, inf_t$, February 2020 (5.8)

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inf_{t+11}=$0.077932 + 0.462990\, inf_{t+10}$ December 2020 (5.9)

Table 4.4: Forecast Value of Inflation from January 2020 to December 2020

Month	Forecast Value for 2020 January -2020 December of the Inflation Rate		
	Regime1	Regime2	Average
January	11.84	11.98812	11.91406
February	11.8133	12.13	11.97165
March	11.784367	12.27362	12.0289935
April	11.75517	12.41802	12.086595
May	11.72595	12.56282	12.144385
June	11.69672	12.70779	12.20..
July	11.66749	12.85279	12.26
August	11.63826	12.99788	12.31807
September	11.60903	13.14298	12.376
October	11.5798	13.28809	12.433945
November	11.55057	13.4332	12.491885
December	11.52134	13.57832	12.5433

V. CONCLUSION

The researcher used [3] autoregressive integrated moving average (ARIMA) and [7] Markov switching autoregressive (MS-AR) models to model Nigeria's inflation rate from 2006 to December 2019. The details came from the CBN statistical Bulletin. The plot of the original series shows that the data is traveling in an unusual pattern. The existence of irregularly moving objects can lead to erroneous regression. Autoregressive integrated moving average (ARIMA) and markov switching autoregressive models were used to model the sequence (MS-AR). The Akaike knowledge criterion (AIC) was used to pick the best models. This analysis compared two types of univariate models to models of Nigerian inflation rate. The series' time plot reveals two "distinct" regimes (expansion and contraction). Diagnostic tests were conducted using Augmented Dickey Fuller. For univariate linear univariate models, seven models were calculated, and two for non-linear models. The best model was chosen based on the criterion of least knowledge. MS-AR(1) was chosen to model the inflation rate in Nigeria. From January to December 2020, the best model was used to forecast the value of the collection.

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