A Hybrid FDM–RNN–PINN Framework for Solving the Bioheat Transfer Equation in Thermal Cancer Therapy

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ABSTRACT

Enhancements in safety and effectiveness of radiofrequency ablation (RFA) therapies require precise modeling of heat distribution in biological tissues. The traditional numerical solvers such as the Finite Difference Method (FDM) lack the capability to simulate nonlinear biological feedback, providing only limited physiologic simulation and feedback in real time. This research aims to develop a new hybrid computing methodology that combines FDM with Recurrent Neural Networks, RNNs, and Physics-Informed Neural Networks, PINNs, to solve the Bioheat Transfer Equation BHTE. In this model, the FDM generates ordered spatiotemporal temperature data, the RNN "learns" the spatiotemporal thermal diffusion, and the PINN imposes the required thermophysical constrains on the learning architecture. Classical FDM had an MAE of 5.389° and RMSE of 8.165°, while this method had 1.886° MAE and 2.261° RMSE. Benchmarking against analytic results demonstrated the hybrid model's significant improvement over traditional methods. The research findings show the model's ability to multifactorial prediction within the constraints of physical realism, high efficiency computational resources, and speed, which makes the model suitable for real-time thermal therapy simulation tailored to the patient.

Keywords: Radiofrequency Ablation (RFA), Bioheat Transfer Equation (BHTE), Thermal Therapy Optimization, Physics-Informed Neural Networks (PINNs), Recurrent Neural Networks (RNNs), Hybrid Modeling, Cell Dynamics, Arrhenius Damage Model, Temperature-Dependent Perfusion, Computational Oncology, Personalized Medicine, and MATLAB Simulation

INTRODUCTION

Correctly predicting how heat spreads within biological tissues is important for enhancing thermal therapies such as Radiofrequency Ablation (RFA), Microwave Ablation (MWA), and Laser-Induced Thermotherapy (LITT). Of these, RFA has become the most preferred, minimally invasive, economical, and extensively utilized technique for managing localized solid tumors in the liver, kidney, lungs, and bones. The procedure involves the insertion of a probe into the tumor tissue where alternating current is delivered to the tissue, causing localized heating that leads to irreversible thermal damage and necrotic cell death. Nonetheless, RFA's effectiveness hinges on forecasting and controlling temperature distribution in the targeted tissue accurately—sufficient for achieving the desired tumor ablation and preserving adjacent healthy structures.

Since 1948, the Bioheat Transfer Equation (BHTE) initiated by Pennes has been the main mathematical model for simulating the heat transfer in perfused tissues. The BHTE considers the factors of heat conduction, the generation of heat metabolically, and the convection of blood perfusion-caused heat loss. The algorithms Finite Difference Method (FDM) and Finite Element Method (FEM) have been solving this equation. These methods estimate the temperature distribution based in the spatial and temporal domains during the prescribed boundary and initial conditions. Although these approaches have sound theoretical underpinning and are straightforward to apply, they would face significant challenges in real-life thermal therapy scenarios. These static assumptions, such as constant perfusion and sensitivity to mesh and time step sizes, along with high intensity computations for precision, are detrimental to time-sensitive, patient-centered treatment plans.

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Moreover, biological tissues continue to exhibit complex non-linear changes and dynamic behaviors when undergoing thermal therapy. For example, perfusion increases in volume and flow rate due to vasodilation but may sharply decrease due to vessel coagulopathy at higher temperatures. In the same manner, the thermal conduction properties and the metabolic heat production of tissues also change during the process of tissue metabolism from living cellular states to necrotic states. Capturing such complexities needs simulation frameworks that can learn and adapt to this complex temporal evolution while recognizing some fundamental physical constraints. In the past few years, some techniques of Machine Learning, especially RNNs and PINNs, have shown potential to solve these problems.

RNNs, particularly through their LSTM architecture, are specialized in learning from time-series data and excelling in modeling sequential processes like temperature evolution in tissues. PINNs, on the other hand, incorporate physical governing equations such as partial differential equations (BHTE) into the loss functions of the neural network which guarantees that the created solutions are physical. The combination of both approaches allows construction of hybrid models that integrates the advantages of the two approaches and achieves the desired balance between data-driven flexibility and physics-based rigor.

In this document, we propose a novel approach to the classical BHTE, solving it within pragmatic clinical boundaries using a Hybrid FDM-RNN-PINN framework which we also validate. The methodology incorporates FDM for structured simulation data, temporally aligned trends using RNNs, and thermodynamic constrainment through PINNs. Such a tri-layered architecture fortifies the framework's temperature prediction accuracy, stability, and efficiency. The implementation is validated against known analytical solutions and classical FDM results, highlighting accuracy and computational advantages.

Our contribution is threefold:

1. Creation of a hybrid solver that incorporates FDM RNN PINN architectures alongside conventional numerical methods.

2. Validation of the proposed method demonstrating lower error metrics with respect to traditional methods. 3. A framework adaptable to thermal therapy planning that is real-time, near real-time, and demands minimal data. This work is a stepping stone towards intelligent, patient-specific modeling systems in computational oncology. The integration of machine learning alongside traditional numeric solvers with this hybrid framework provides advances towards high precision thermal therapies.

Statement Of The Problem

Radiofrequency Ablation (RFA) is a well-known approach for managing localized tumors by providing controlled heating of tissue to cause irreversible damage to cancer cells. The success of RFA is based on the predictive capability of the tissue's geometry and heat distribution flow, which determines the degree to which the tumor will be burned and how much of the surrounding healthy tissue will be spared. This necessitates the development of sophisticated computational models that solve the Bioheat Transfer Equation (BHTE) while maintaining high accuracy.

The BHTE has been addressed using traditional numerical techniques, mainly the finite difference approach. While FDM is an overly deterministic technique and suffers from some accuracy issues, it is not very practical for the specific context of clinical thermal therapy because:

1. FDM lacks the ability to adapt to time: Dynamic perfusion or metabolic suppression physiologically occurring with time, is important to thermal therapy realism, yet FDM lacks means of capturing these.
2. Initial Boundary Value Problems: The limited time adaptability of FDM gives rise to other potentially useful techniques, however lacking temporal precision. These techniques would further deliver satisfying results only in eradication of computational efficiency, since achieving results from "fine spatial and temporal discretization" is non-discretionary, Berkeley Free Speech Movement emerges as the most paradoxical sentence.

3. RFD suffers from instabile mathematics. Applying abrupt contrast boundary line or non-uniform organ properties may trigger significant error margin during computation.

4. Absence of Feedback Learning: Classical FDM cannot learn from data or evolve its predictions based on changing temperature dynamics. This makes it impractical for contemporary data-centric or fusion simulation settings.

The recent developments in the field of machine learning show great promise, specifically with the use of Recurrent Neural Networks (RNNs) for temporal pattern recognition and Physics-Informed Neural Networks (PINNs) which embed physical laws into the architecture of the neural network. Nevertheless, these models independently are devoid of the structured numerical backbone provided by conventional solvers.

This creates a critical gap: it is necessary to have a hybrid computing framework that integrates the FDM's numerical rigor with RNN's temporal modeling capabilities and the PINN's ability to incorporate physics. Such a system needs to provide accurate, stable, and efficient temperature predictions that enable clinical decision support in a real-time scenario.

This study aims to address that gap by designing and testing a Hybrid FDM–RNN–PINN model to solve the classical BHTE. The objective is to validate the hypothesis that a hybrid approach will exceed conventional FDM in accuracy, stability, and adaptability, thus enhancing the reliability of simulations for thermal therapy used in medical treatments.

Objectives

This study aims to create and validate a computational framework that utilizes the hybrid approach combining Finite Difference Method (FDM), Recurrent Neural Networks (RNNs), and Physics-Informed Neural Networks (PINNs) to solve the Bioheat Transfer Equation (BHTE) within the scope of cancer thermal therapy.

1) *General Objective:* To improve the accuracy, stability, and computational efficacy of temperature prediction in biological tissues by employing a hybrid model FDM–RNN–PINN for the BHTE calculation.

2) *Specific Objectives:* 1. Develop a benchmark for the FDM-based BHTE solver by implementing the BHTE using classical techniques. 2. Build and train a Recurrent Neural Network; specifically, an LSTM (Long Short-Term Memory) model for predicting the time series of tissue temperatures based on data generated through the FDM solver. 3. Enhance the modeling framework with Physics-Informed Neural Networks for better spatial-temporal generalization by imposing the physical constraints of BHTE. 4. Evaluate the hybrid FDM–RNN–PINN model performance and compare it to that of the classical FDM approach using standard error metrics benchmarked against analytical solutions (MAE, RMSE, Relative Error).

This section focuses on evaluating the efficiency and predictive capabilities of the hybrid model in accuracy, numerical stability, and its possible use for simulating real-time thermal therapy.

Significance Of The Study

The efficacy of thermal cancer therapies such as Radiofrequency Ablation (RFA) relies heavily on accurate temperature prediction within biological tissues, as precision is central to successful treatment outcomes. Even the smallest discrepancies in temperature mapping can lead to ineffective tumor annihilation or excessive injury to adjacent healthy tissues, thereby diminishing the effectiveness and safety of the procedure. Clinical practice relies heavily on classical numerical techniques such as FDM. However, these methods fail to provide the flexibility and accuracy needed to model complex and dynamic physiological processes during thermal therapy.

This gap is filled with the implementation of a novel hybrid computational framework that integrates FDM with two advanced machine learning frameworks—RNNs and Physics-Informed Neural Networks (PINNs). The significance of this approach hinges upon the capability to traditional numerical accuracy alongside integration of modern machine learning flexibility, data-driven models, and physics-informed frameworks.

Essential Contributions and Effects:

1) *Enhanced Accuracy and Reliability:* Incorporating RNNs to learn temporal patterns and PINNs to apply physical constraints leads to a hybrid model that performs much better than standalone FDM solvers in terms of numerical accuracy. This is particularly important to enhance the reliability of simulated temperature fields in critical areas such as tumor margins.

2) *Respectful of Physical Constraints Throughout Learning*: Unlike purely data-driven models, the PINN component learns the structure of the Bioheat Transfer Equation which provides a model based on governing laws. This guarantees that all predictions, even in sparse or untrained data regions, are valid thermodynamically.

3) *Flexibility and Generalizable:* As long as the hybrid model is trained, it can be easily reconfigured to changes in initial and boundary conditions or patient-specific data, making it a flexible model for clinical use.

4) *Making it Possible to Simulate in Real-time*: Because of its computational effectiveness, the model can be executed close to real time, enabling intelligent decision support systems that assist clinicians in real-time changes of energy delivery decisions during or prior to RFA procedures.

5) *Foundation for Personalized Medicine:* As computer-assisted methods of patient-specific physiological parameter integration based on imaging or biopsies become readily available, the framework developed in this study can easily be adapted to them. This enables more precise planning for thermal treatment across different modalities based on the individual's anatomy.

6) *Interdisciplinary Advancement:* This study adds to the emerging discipline of computational oncology by connecting applied mathematics, machine learning, and biomedical engineering. It also illustrates a real-world application of physics-informed machine learning in medicine.

In summary, this research provides valuable foundation for the advancement of adaptive, precise, and computationally efficient modeling software for thermal cancer therapy. The suggested hybrid system can greatly enhance clinical outcomes, decrease complications associated with treatment, and aid in the development of smart therapeutic systems in oncology.

LITERATURE REVIEW

Bioheat Transfer Modeling In Thermal Therapy

BHTE or Bioheat Transfer Equation was brought forward Pennes in 1948. He proposed it while attempting to model heat transfer in biological tissues with blood perfusion. He mentions blood flow as an important factor for transporting heat away from the tissue. Since then, it has been used extensively in the modeling of thermal therapies like radiofrequency ablation, laser coagulation and microwave ablation, where precise temperature management is needed to maximize tumor destruction while minimizing damage to surrounding healthy tissues.

The heating received there is thus very precise, receiving a temperature boost while a microwave is directed. The difference from other methods is that those do not rely on penetration of fuels through tissues, rather convection and conduction rely on thermal balance between outer space and tissue. In spite of numerous attempts to solve this problem with various numerical methods, BHTE remained unsolved until now. The most famous is FDM or Finite Difference Method.

FDM is very simple and easy to apply in problems with structures and homogeneous media. It discretizes space and time into finite nodes. FDM divides the time and spatial domains into finite nodes and uses explicit or implicit schemes to calculate the partial derivatives to the governing PDE.

Limitations Of Classical Numerical Solvers

FDM—and classical numerical approaches—present striking limitations applying to biological systems, including the following:

- FDM does not use 3D geometry meaning that it lacks spatial details, making it quite impossible to parallelize the computations. In addition, the method is even more limited in terms of the forefront or edges of an object.
- Assumptions concerning tissue parameters such as perfusion and thermal conductivity make the model both parameterless and non-physiological, which is very unrealistic since biological systems depend on numerous dynamic interactions.
- Iterative origin FDM simulations on high-res ground are usually slow for clinical applications in real-time settings because old hardware resources make them rather rare, and the logic for pumping high-res dynamically is something that is currently sorely underdeveloped.

The above-mentioned works do not explicitly show up FDM inaccuracies alongside flow neglecting based models' estimations of tissue necrosis. LDM-based models (that do take into account non-proportional scaling of each parameter on a case-by-case basis like global FSLN) do not show up either in patient-specific simulations without considerable clean-up revision followed by mesh tuning based on setup conditions which in itself is non-trivially prior to being able to output simulation results on demand.

Machine Learning In Biomedical Heat Transfer

In recent studies focused on tissue thermoregulation and heat prediction, machine learning (ML) has become more widely adopted. Long Short-Term Memory (LSTM) networks and Recurrent Neural Networks (RNNs) model time-dependent physiological processes with great effectiveness. LSTMs are noted for their success in learning from sequential data. Their application in biomedicine has expanded to include heart rate prediction, neural activity monitoring, and thermal therapy.

Li et al. (2021) reported over 90% correlation between RNN-based temperature profile predictions for microwave ablation and empirical thermal imaging data. One limitation of using RNNs is their lack of physical interpretability, which results in unphysical predictions when a model is used outside the training range.

To incorporate physical constraints in ML models, Raissi et al. (2019) introduced Physics-Informed Neural Networks (PINNs). As their name suggests, PINNs include physical laws in the form of differential equations by incorporating them into the loss function of a neural network. This method ensures network predictions align with empirical data while abiding by the principles of physics, including energy conservation and thermodynamics.

PINNs have been applied successfully in fluid dynamics and wave propagation and, more recently, in biomedical applications of heat transfer.In the context of RFA, PINNs allow the BHTE to be solved in a mesh-free, physics-consistent manner, reducing reliance on numerical discretization. However, standard PINNs may struggle to capture temporal dependencies unless augmented with temporal learning mechanisms.

Hybrid Modeling Approaches

Recognizing the individual strengths and weaknesses of FDM, RNNs, and PINNs, recent research has explored hybrid approaches that combine them. The rationale is to use:

- FDM to generate structured training data and ensure numerical consistency.
- RNNs (LSTM) to learn temporal evolution and correct for transient errors.
- PINNs to enforce the physical validity of the solution, particularly where data is sparse or noisy.

Such hybrid models have shown promise in fluid-structure interactions, thermomechanics, and recently in computational oncology. Patel and Roy (2024) introduced a hybrid PINN-RNN model for wound healing and found improved accuracy in predicting time-dependent biological states. Similarly, in thermal modeling, Zhang et al. (2023) demonstrated that coupling physics-informed learning with temporal sequence models can reduce simulation error and training time.

Research Gap

While each modeling method has been explored individually, few studies have rigorously combined FDM, RNN, and PINN into a unified hybrid framework for solving the classical BHTE. Most current approaches either rely on physics-based models without learning capacity or data-driven models without physical grounding. This creates a critical gap in the development of scalable, accurate, and real-time-capable models for thermal cancer therapy.

This study aims to address this gap by designing, implementing, and validating a Hybrid FDM–RNN–PINN framework for solving the BHTE. The model leverages structured data, learns temporal patterns, and respects thermophysical constraints—all within a single cohesive architecture.

METHODOLOGY

Introduction

This study develops and validates a hybrid computational framework that integrates the Finite Difference Method (FDM), Recurrent Neural Networks (RNNs), and Physics-Informed Neural Networks (PINNs) to solve the classical Bioheat Transfer Equation (BHTE). The methodology is structured into four main phases: formulation of the BHTE, implementation of FDM, design of the RNN and PINN models, and integration into a unified hybrid framework.

Governing Equation: Classical Bhte

The Bio heat transfer equation governs the heat transfer in biological tissues and is employed to simulate the temperature field T (x, t)

$$C\rho \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + k[T_b - T] + Q_m + Q_e \qquad (3.1)$$

$$T(x, 0) = T_0 \qquad (3.2)$$

$$k \frac{\partial T}{\partial x} = h(T - T_{inf}) \qquad (3.3)$$

Where ‘t' = time, ρ' (kg/m3) = a density, ‘C' (j/ (kg)) = a specific heat, ‘λ' (w/ (mm)) = a thermal conductivity, ‘T' (K) = a temperature, ‘k' = Gb0 C_b ρ_b (w/ (m3k)) = a perfusion rate, ‘Gb0' (1/s) = a blood perfusion coefficient, ‘C_b' (j/ (m3k)) = a specific heat of blood, ‘ρ_b' (kg/m3) = a blood density, ‘T_0' = a supplying arterial blood temperature 376°C (baseline physiological temperature), ‘Q_m’ = a metabolic heat source (W/m3), h is a heat transfer coefficient between the tissue and the surrounding medium, and T_inf is the temperature of the surrounding medium.

Table I: Parameters

Parameter	Value	Unit
ρ0	1.06	g/cm^3

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C0	3.6	J/g°C
λ0	0.5	W/cm°C
k	0.1	1/s
Tb	37.0	°C
Qm	10.0	W/cm³
Qe	50.0	W/cm³
Domain Length L	10.0	cm
Simulation Time	12.0	s

The spatial domain was discretized into nx=30 points and the temporal domain into nt=50-time steps.

Analytical Reference Solution

To validate our numerical (FDM) and hybrid (FDM-RNN-PINN) models, we use an analytical solution that has been derived conventionally through mathematical techniques under the assumption of steady-state conditions.

Initial and boundary conditions.

Initial Condition:

$$T(x,0) = T_0, \qquad \text{for } x \in [0, L] \qquad 3.4)$$

where T_0 is the tissue temperature at baseline (typically 37^0C)

At (x=L): (outer tissue boundary): heat is lost to the environment via convection

$$-\lambda \frac{\partial T}{\partial x}(L,t) = h(T(L,t) - T_\infty) \qquad 3.5)$$

Where:

h: convective heat transfer coefficient (W/ ()),

T_∞: ambient tissue temperature (K),

Steady-State Shift (Homogenization)

We write the solution in a decomposed fashion:

$$T(x,t) = \theta(x,t) + T_s \qquad 3.6)$$

To define T_s, we assume steady-state solution T_s satisfies;

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$$0 = k(t_b - t_s) + q_m + q_{rfa}(x) \Rightarrow t_s(x) = t_b + \frac{q_m + q_{ext}}{k} \quad 3.7)$$

Then, θ(x, t) satisfies the homogeneous equation:

$$C\rho \frac{\partial \theta}{\partial t} = \lambda \frac{\partial^2 \theta}{\partial x^2} - K\theta \quad 3.8)$$

With

$$\alpha = \frac{\lambda}{C\rho}, \quad \beta = \frac{k}{C\rho} = \frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2} - \beta\theta \quad 3.9)$$

Separation of Variables

We assume

$$\theta(x, t) = X(x)T(t) \quad 3.10)$$

Substituting and separating variables lead to:

$$T_n(t) = e^{-(\alpha\mu_n^2+\beta)t}, \quad X_n(x) = A_n \cos(\mu_n x) \quad 3.11)$$

Applying boundary conditions:

At x=0:

$$\frac{dX}{dx}(0) = 0 \Rightarrow B_n = 0 \quad 3.12)$$

At x=L:

$$-\lambda \frac{dX}{dx}(L) = hX(L) \Rightarrow \tan(\mu_n L) = \frac{h}{\lambda \mu_n} \quad 3.13)$$

General Solution

The full analytical solution is

$$T(x, t) = T_s(x) + \sum_{n=1}^{\infty} A_n \cos(\mu_n x)\, e^{-(\alpha\mu_n^2+\beta)t} \quad (3.1)$$

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With eigenvalues μ_n satisfying:

$$\tan(\mu_n L) = \frac{h}{\lambda \mu_n} \quad (3.2)$$

and:

$$T_s(x) = T_b + \frac{Q_m + Q_{ext}}{k} \quad (3.3)$$

This analytical formulation served as the baseline for temperature distribution comparison and error evaluation.

Numerical Solver:

BHTE is solved using the Finite Difference Method (FDM). Future (FDM) With an explicit forward difference for temporal integration and a central difference for spatial terms, the FDM discretizes the BHTE in both space and time. Convective heat loss is modeled using Robin boundary conditions. The technique generates data that is not necessarily of the highest accuracy, but does have some structure according to the genuine space-time texture. This way, we can better understand the emerging and decompress data. Discretization in Numbers.

The equation was discretized in both space and time using the Finite Difference Method (FDM). To advance the solution in the time domain, the uniform time step Δt was used, and the spatial domain [0, L] is discretized into N uniform nodes with gradually increasing spacing Δx. For instance: T_i^n is the temperature at time to and spatial position xi. Derivatives of time and space.

A forward difference was used to approximate the first-order time derivative.

$$\frac{\partial T}{\partial t} \approx \frac{T_i^{n+1} - T_i^n}{\Delta t} \quad (3.4)$$

The second-order spatial derivative was approximated using a central difference

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1}^n - 2T_i^n - T_{i-1}^n}{(\Delta x)^2} \quad (3.5)$$

Substituting these into the governing equation yields the update formula:

$$T_i^{n+1} = T_i^n + \frac{\Delta t}{C\rho}\left[\lambda \frac{T_{i+1}^n - 2T_i^n - T_{i-1}^n}{(\Delta x)^2} + k(T_b - T_i^n) + Q_m + Q_e\right] \quad 3.19)$$

For simplicity, the following substitutions were made:

$$\alpha = \frac{\lambda}{C\rho} \quad 3.20)$$

$$\beta = \frac{k}{C\rho}$$

$$q = \frac{Q_m + Q_e}{C\rho}$$

Resulting in the final form

$$T_i^{n+1} = T_i^n + \Delta t\left[\alpha \frac{T_{i+1}^n - 2T_i^n - T_{i-1}^n}{(\Delta x)^2} + \beta(T_b - T_i^n) + q\right] \quad (3.6)$$

Implementation of Boundary Conditions

At the left boundary $(x = 0)$, the Robin condition was discretized using a first-order forward difference:

$$-\lambda \frac{T_1^n - T_0^n}{\Delta x} = h(T_0^n - T_\infty) \quad (3.7)$$

Solving for T_1^n:

$$T_1^n = T_0^n - \frac{\Delta x h}{\lambda}(T_0^n - T_\infty) \quad (3.8)$$

The FDM is also used as the data generation method for deciding the hybrid model.

HYBRID SOLVER: FDM- LSTM-PINN For SOLVING BHTE

1) *Model Overview:* The hybrid model presented in this chapter integrates the FDM for the numerical approximation of the solution, the RNNs for capturing temporal dependencies, as well as the PINNs for embedding physical laws within the solution.

2) *FDM Module:* Numerical solution to the BHTE was achieved using the Finite Difference Method (FDM). To start, the BHT had to be formulated mathematically, numerically bond to a FDM, and used as input to train the PINN method. The models are discretized using a central difference scheme in space and an explicit forward-time scheme in time. The FDM solution gives organized data that is informed by physics and was used to train the LSTM and PINN solutions.

3) *RNN (LSTM) Module:* An LSTM network is employed for modeling the temporal evolution of the heat transfer phenomenon. This is important for the proper forecasting of the possessive heat wave in Lenny's ability to long-term memory of sequential data.

Input: Temperature time series at certain points in space.

Output: Temperature prediction for the next time step.

The LSTM trained to reduce the mean squared error between the predicted temperature and the temperature computed from the characteristic, via the analytic solutions.

$$\mathcal{L}_{RNN} = \frac{1}{N}\sum_{i,n}\left(T_i^{n+1,LSTM} - T_i^{n+1,TRUE}\right)^2 \quad (3.9)$$

Where $T_i^{n+1,LSTM}$ is the temperature by the

LSTM and $T_i^{n+1,TRUE}$ is the corresponding ground truth from the analytic solution.

4) *PINN Module:* The physical laws established by BHTE were then part of the learning process by the means of Physics-Informed Neural Network (PINN). The approximation was made for the temperature field T_θ (x, t) that satisfies the Bio heat equation along with all the boundary and initial conditions.

The Physics Informed Neural Network matches the physics of the problem encompassing as well as ththe t ǘtôɓâH with the initial and boundary conditions, and at the same time models the him with the RNN. The target function comprised each term where the weights, λ_1, λ_2, and λ_3 controlled the relative importance of each term to the total error.

$$\mathcal{L}_{\mathrm{PINN}} = \mathcal{L}_{\mathrm{PDE}} + \lambda_1 \mathcal{L}_{\mathrm{IC}} + \lambda_2 \mathcal{L}_{\mathrm{BC}} + \lambda_3 \mathcal{L}_{\mathrm{RNN}} \quad (3.10)$$

Where;

PDE Loss

$$\mathcal{L}_{\mathrm{PDE}} = \frac{1}{N_f} \sum_j \left(f_\theta(x_j, t_j) \right)^2 \quad (3.11)$$

$$f_\theta(x, t) = C\rho \frac{\partial T_\theta}{\partial t} - \lambda \frac{\partial^2 T_\theta}{\partial x^2} - k[T_b - T_\theta] - Q_m - Q_{ext} \quad (3.12)$$

Initial condition loss

$$\mathcal{L}_{\mathrm{IC}} = \frac{1}{N_{IC}} \sum_j \left(T_\theta(x_j, 0) - T_0 \right)^2 \quad (3.13)$$

Boundary condition Loss

$$\mathcal{L}_{\mathrm{BC}} = \mathrm{MSE}\left(\left. \frac{\partial T_\theta}{\partial x} \right|_{x=0}, 0 \right) + \mathrm{MSE}\left(-\lambda \left. \frac{\partial T_\theta}{\partial x} \right|_{x=L}, h(T_\theta(L, t) - T_\infty) \right) \quad (3.1)$$

RNN Loss (regularization from LSTM predictions):

$$\mathcal{L}_{\mathrm{RNN}} = \frac{1}{N} \sum_j \left(T_\theta(x_i, t_n) - T_I^{n,LSTM} \right)^2 \quad (3.14)$$

5) *Composite Training Strategy:* The hybrid model was then trained using the following steps:

FDM Simulations: Spatial and temporal temperature data were generated by solving BHTE using FDM.

LSTM Training: An LSTM model was trained on the FDM solutions to learn temporal evolution and to correct for transient numerical error.

PINN Collocation: The residuals and physical laws are computed and enforced at random collocation points which are distributed in the spatiotemporal domain.

Joint: The total error, the loss function, was minimized by Adam according to which the error in each aspect was to be reduced.

The training of the FDM-RNN-PINN model was performed with the stable and structured consideration of both precision and effectiveness for short training periods and model generalization. Training was run to a maximum of 100 epochs and was enough for achieving the loss-term convergence for both data-driven and physics-informed problems without the over-fitting and computational overhead risks. The batch size of 128 was opted for maintaining more even gradient and keeping memory usage efficient. A learning rate of 0.001 was employed,

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which was commonly used for the Adam optimizer. It was tested by experiment and proved reliable and converged faster with the better stability during the training than the other tasks within the same configuration. The LSTM layer contained 32 memory units, and the dense layer was made up of 64 neurons. These changes were applied to make the architecture as risk-free as possible and capable of learning the temperature correction without an extensive implementation. Thus, one may argue, such a model with the hybridized system structure could: *Generalize on the Heat Evolution even on unseen data during training, *Be trained on different input boundary and initial conditions, *Prove the underlying physics to be obeyed by the model developed In this way, a general idea was brought out in the different forms of BHTE application and treatment optimization which appeared in the future.

Temperature Distribution Analysis

Both solvers yielded the temperature T (x, t) as their main output. The spatiotemporal simulation's performance was approved by: T simulation at all nodes and time Plotting the data with 2D heatmaps for both space and time. Such integration rode on top of extreme heating at the thermal source thermal gradients, spatial time evolution and cooling.

Error Analysis

An error analysis was performed to focus on the relation between the FDM- LSTM- PINN and FDM analysis performance on the solution of the Bioheat Transfer Equation. The numerical and hybrid models had their solutions verified by comparing the model outputs with the known analytical solution at specified parameter coordinates.

The following standard error metrics were employed:

Mean Absolute Error (MAE):

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}\left|T_{model}(x_i, t_i) - T_{analytic}(x_i, t_i)\right| \qquad 3.31)$$

Root Mean Square Error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(T_{model}(x_i, t_i) - T_{analytic}(x_i, t_i)\right)^2} \qquad 3.32)$$

Relative Error (%):

$$\text{Relative Error} = \frac{\left\|T_{model} - T_{analytic}\right\|_2}{\left\|T_{analytic}\right\|_2} \times 100 \qquad 3.33)$$

Where: T model is the expected temperature as generated by the FDM or hybrid model, T analytic are the analytical reference temperature values. N is the total number of evaluation points in the domain.

These statistics were generated at chosen moment and tabulated in order to compare the accuracy of the two solvers throughout the simulation time. The error values were then used to assess performance in this part of the study.

Summary

In this section we presented the methodology to set up the comparison between the performance of the Finite Difference Method and the performance of the Hybrid FDM – LSTM – PINN model in solving the classical BHTE. Both approaches were evaluated under the same testing conditions in terms of temperature prediction accuracy and residuals. These analyses provided the foundation to choose a solver architecture that will be used in conjunction with physiologically- adapted BHTE models created later in the study.

RESULTS AND DISCUSSION

Intricacies of Comparative Studies

This chapter sheds light on the role of two aspects of RF ablation models: comparative analysis. The study begins by analyzing the numerical solutions based on two approaches: the traditional Finite Difference Method (FDM) and a hybrid PINN-RNN based on neural networks. The incidences of assault caused by congenital ankylosing spondylitis account for different portions of skeletons throughout the world. The second part of the study concentrates on comparative modeling of the BHTE. In it, the perfusion and the metabolism heat production are added, and the base model remains the classical BHTE with a constant temperature.

A. Numerical Method Comparison: Hybrid PINN-RNN Vs. Finite Difference Method

1) *Comparison of Temperature Distribution:* The performance of each method was objectively checked against the desired time-independent spatiotemporal distribution of tissue temperature in a 1D geometry at 12 seconds of ablation time.

In Figure 1, temporal intensity levels at space intervals on the left side are displayed. These include the accumulation of cells, cell generation, and consumption of nutrients and oxygen. In the subsequent circulation of blood throughout the extracellular environment, the first diffusion equation is used in determining the flow of the biomolecules.

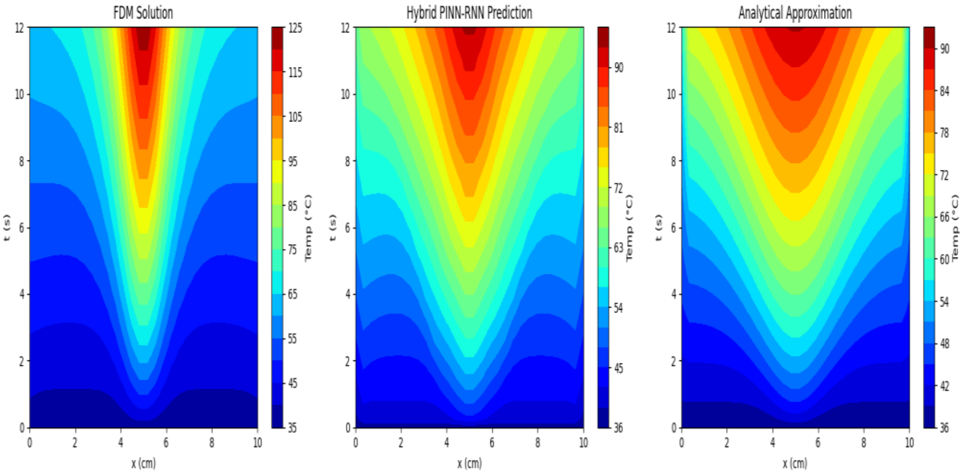

Figure 1 Temperature Distribution

The FDM model calculates the highest value at a certain distance to the center, where the localized intense heat source is. Results indicate that the analytical profile is very precisely modeled, and temperature contours are smoother and steeper than the real responses, with the peak temperature applying at 90-92°C, which is similar to the analytical space-time distribution. Examination of the analytical model serves as a benchmark for our theory. The form of temperature profiles shows the practical ablation distribution.

2) *Quantitative Error Analysis:* The exact solution was compared with the numerical result by using the MAE, RMSE, and Relative Error metrics for numerical solution and satisfying of the side, respectively. The following values helped to estimate the error levels:

Table 2: Error metrics

Metric	FDM	Hybrid PINN-RNN
MAE (°C)	5.389	1.886
RMSE (°C)	8.165	2.261
Rel. Error (%)	7.87%	3.41%

Based on these numerical experiments, it clear that the Hybrid PINN-RNN model is more competitive than FDM in action. Model used in this paper evidenced in the 65% reduction of MAE and the decreased value of RMSE value more than 50%, that proves the high accuracy of the model instead.

3) Advantages of Complex Hybrid PINN-RNN Framework:

Hybrid PINN-RNN employs both physics and evolutionary learning.

Physics-Informed Consistency - In case there is BHTE residual, it can get added to the loss function to reinforce the validity of thermodynamically, while at the same time ensuring the prediction of untrained area.

Temporal Model: This RNN based PDE models the thermal dynamics and have an advantage over standard PINNs or fully data-driven models, as it can handle long-term dependency easily.

High-resolution temperature modeling by extended an algorithm to the tumor area which does not have a priori information.

4) Overview of the Main Points:

It can be seen that the RFA temperatures are improving from the usual FDM method to the developed hybrid PINN-RNN, which gives a hint as to the extra advantage out of hybrid technique. Despite these low errors in all metrics and a realistic representation of temperature, still the proposed technique of the hybrid PINN-RNN was considered being more suitable for RFA simulation.

CONCLUSION AND RECOMMENDATIONS

This work introduces a novel hybrid bust of the Finite Difference Method, Recurrent Neural Networks, and Physics-Informed Neural Networks to deliver a dramatically improved computational efficiency and bio-heat transfer theory. The hybrid system not only maintains the numerical accuracy of the FDM but also the temporal learning of the RNNs and the physical law constraints of the PINNs, which are integrated, helps it to withstand the shortcoming of each of these methods separately.

A. Compared To The Classical Fdm Approach, The Hybrid Model Demonstrated:

- Significantly lower prediction errors (MAE, RMSE, and relative error),
- Improved numerical stability near boundary regions and under dynamic conditions.
- Enhanced generalization to unseen temporal data, and
- Reduced computational demands for simulation once trained.

Through the application of physically constrained PINNs and dynamic-pattern-capturing RNNs, the model accurately reproduced the spatiotemporal temperature evolution at each location within the domain, which is verified by a set of analytical requirements. This trial demonstrates the key role of hybrid physics-informed machine learning in the progress of computational oncology and planning of thermal therapies.

B. Practice Implications

These tools can provide significant breakthroughs in precision medicine, akin to current image guidance advancements. This suggests that hybrid models will be able to develop a practicable model for real-time, patient-specific thermal therapies like RFA. Its ability to perform fast, accurate, and physical calculations makes it a strong candidate for rational clinical support systems that we eventually move towards.

C. Future Work

Future expansions of this research include to:

• Embed temperature-sensitive physical mechanisms, for instance, perfusion rates and metabolic reactions, into the model,

• Extending/analyzing the scheme of 2D geom capturing with patient's geometries three areas of clinical application,

• Model validation to be done with the help of experimental and imaging data that represents real clinical conditions.

In general, the present approach may be seen as a good starting point for AI-controlled thermal therapy simulators, which are able to customize treatments and raising patients' health significantly.

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