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Stability And Disequilibrium Consideration in Structural Design of Multistorey Frame Systems

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ABSTRACT

Frame systems configurations are governed usually by geometrical and loading symmetry for the expected static equilibrium, and if this becomes infeasible there is tendency for unstable structural system associated with P-delta effect. The paper evaluates the significance of stability to safe structural load application by considering basic requirements for the static equilibrium state including effect of applied forces (F), reactive force ®, structural stiffness and suggested that if F < R the structure is stable and if F > R, the structure will be unstable which is non-equilibrium and instability state, and with respect to storey drift or rotation $\theta_a < \theta_p$ describes a stable system but if $\theta_a > \theta_p$ an unstable structural system (θ_a and θ_p are rotation due to applied load and permissible rotation respectively). The study invilved a structural design case study of a 12 storey multi-purpose building with Prota-structure software to determine the equilibrium factors associated with structural stability such as lateral displacement and rotation (or, storey drifts), the design presented a satisfactory results with respect to permissible limits included in the software programs algorithms. The paper also suggested that lateral displacement and torsion are phenomena associated with high-rise building basically due to asymmetric loadings and actions, which produces P-delta effect, and expressed that, if all anticipated forces are accurately evaluated and inputted correctly in software analysis, the design results will be very reliable with minimal lateral displacement and storey drifts

Key words: Stability, Equilibrium, Structural load, Building, displacements

INTRODUCTION

Frames are structural assemblage that comprise of beams and columns connected monolithically forming a system that can resist applied forces more effectively and with higher stability (Armenakas, 1998). Frame structures are classified as, (i) Rigid frame when the beams and columns are connected monolithically and act together to resist the applied structural loads, and (ii) Braced frame, when bracing are provided between the beams and columns, making the system more rigid, with increase resistance to lateral forces by preventing lateral movement and displacement..Frame system configurations (Corliss and Kearfott, 2007), is defined by their geometrical and loading symmetry, such that when either of this is lacking or inadequate, the system becomes asymmetric, which subject the system to instability or unstable equilibrium leading to sway displacement. Structures can only be subjected to tolerable or negligible deformation and therefore considered as rigid body system (Sung-chem et al, 2016). Rigid body dynamics is the study of movement of systems of interconnected bodies under the action of external forces, using the law of kinematics (ie, Force, motion and displacement), while constraint systems, are parameter that provides resistance to lateral movement of structural system (Eun H et al, 2004). In Hamiltonian mechanics, a primary constraint is a relationship between the coordinates and momenta (ie, m Δv), applicable without employing the equation of motion. Also the secondary constraint, is one that is not primary, and applicable when the equation of motion are satisfied (ie, F = ma). If a system of particles move parallel to a fixed plane, the system is said to be constraint to planar movement, because of the resistance to motion in the transverse direction. In this case, Newton's laws for a rigid system of N particles, ie, P_i, I = 1, 2, … N, is considered satisfied, because no movement (or, motion) is anticipated or expected in that direction.

Thus, the resultant force and torque at a reference point R can be defined, as,

$F = \Sigma\, m\, a = 0$, ------ (1) and,

$T = \Sigma\, (r-R)\, m.a = 0$, (2)

where, r_i, denotes the planar trajectory of each particle, m_i, = mass of particles and a_i = acceleration.

Structural stability is fundamental characteristics of dynamical systems which describe the quantitative behavior of trajectories unaffected by small perturbations or, load actions (Sabharwal R et al, 2023). Stability is significant property of load bearing system, it involves static equilibrium condition for load application on the structural system (eqns 1&2). In rigid body dynamics, structures are expected to be restraint from motion and remain at rest as expected for structural load application and performance. Newton's 3rd law state that, if two bodies exert forces on each other, these forces have the same magnitude but opposite directions.

Therefore, for structural stability following factors and parameters are important to ensure the rigid body system is in a state of rest (ie, static equilibrium) during load application

Applied force on structures F

Constraint Forces R

Structural systems including, elastic stiffness, configuration, rigidity etc Deformation, ie, stress and strain (σ, ϵ), fatigue, ultimate failure point

Hence if, $F \leq R$ the structure is in static equilibrium

And, if, $F \geq R$ the structure is not in static equilibrium

Stability theory addresses the solutions of differential equations (eg, $a = \frac{\partial^2 x}{\partial^2 t} \approx 0$), and of the trajectories (ie, a = f(x, t) of dynamical systems under small perturbations of initial conditions (Chen W F, 2000). Many parts of the qualitative theory of dynamical systems deal with asymptotic properties of solutions of associated differential equations and the trajectories with respect to time period. The simplest kind of behavior is exhibited by equilibrium points or fixed points and by periodic orbits. In the study of iterated functions, a periodic point, is a point which the system returns to after certain number of function iterations or certain amount of time.

Given a mapping f from a set of X into itself

$f: X \rightarrow X$,

a point x in X is called period point, if there exists an $n > 0$ so that $f_n(x) = x$

Where f_n is the nth iterate of f.

The smallest positive integer n satisfying the above is known as the prime period or least period of point x

Stability theory addresses the following 2 questions

Will a nearby orbit indefinitely stay close to a given orbit?

Will it converge to the given orbit?

In the former case, the orbit is called stable, while in the latter case it is called asymptotically stable and the given orbit is said to be attracting

An equilibrium solution to the autonomo9us system is known to be Stable (Pukdeboon C, 2011), if for every (small) $\epsilon > 0$, there exists a $\delta > 0$ such that every solution f(t) having initial conditions within distance δ, ie, $|f(t) - f_e| < \delta$, if the equilibrium remains within distance $\epsilon|\theta$, $|f(t) - f_e| < \epsilon$ for all $t \geq t_0$

Asymptotically stable if it is stable and in addition there exists $\delta_o > 0$

such that whenever $|f(t_o) - f_e| < \delta_o$ then $f(t) \rightarrow f_e$ as $t \rightarrow \alpha$

LITERATURE REVIEW

2.1: Instability and disequilibrium are unsatisfactory conditions associated with structural dynamics, which is the study of structural response to dynamic loading such as wind, seismic and vibrations etc. These loads vary with time and produces instability involving structural oscillations with tendency to change the equilibrium state if not properly controlled. Instability is a condition associated with lower integrity and strain energy (U) of the system which eventually may lead to structural failure. The structural resistance (ie, strain energy) reduces over period of load application

Ie, $U_r = U - \Delta W$

ΔW is a function of applied load over period of time or workdone by structural components in sustaining the load applied and U_r the strain energy available after period of load application

Principle of least displacement (ie, $\Delta s \approx 0$), or more precisely the principle of minimal displacement action, indicates that, the displacement of a rigid body must be relatively minimal and negligible, to be assumed stationery or at rest in static equilibrium state (Eugster, S E et al, 2020). Likewise in relativity theory, a different action must be maximized or minimized, as functional to attain a stable equilibrium. Functionals, are often expressed as definite integrals involving functions and their derivatives. Therefore in minimizing lateral displacement, the action by the applied forces on the structure can be optimized, in order to determine, the extrema functions, that will make the functional attain critical state. Motion S of a physical system (Eugster et al, 2020), is defined as the integral of the Langrangian L between two instants of time t_1 and t_2 which are functional of N generalized coordinates $q = (q_1\ q_2\ \ldots..q_N)$, the configuration of the system,

$$S\lceil q, t_1\ t_2 \rfloor = \int_t^t L\,(q(t)q^{\shortmid}(t)\ t)\delta t$$

Hence, for least action, $\delta S \approx 0$ (ie the force action can produce only negligible displacement because of static equilibrium requirement for structural performance)

2.2: Sabharwal R et al (2023), evaluated structural stability analysis of multistory building structure under wind load conditions, using techniques of computational fluid dynamics. CFD analysis on building is conducted using ANSYS simulation package. The fluid structure interaction (FSI) simulations considered the complex interactions between structural components and the surrounding fluid flow, providing valuable insights into the structural response and its vulnerability to external loading conditions. Through computational tools and numerical methods, the study was able to accurately predict the structural response including deformations, displacements, stresses and critical failure points. Zavila D et al (2022), investigated by analysis, the influence of P-delta effect and stiffness irregularity on structural behavior of RC buildings. They started by performing the linear dynamics analysis to determine the structural response in terms of drifts, shear force and moment per floor. Them proceeded with nonlinear static analysis procedure to obtain the capacity curve (structural resistance) of the structure, and post elastic stiffness and overall ductility of the structure was determined from the capacity curve. Lastly, a comparative analysis of responses from the linear and nonlinear analysis was performed to determine the percentage variation of the results. They observed that when p-delta effect was considered in structures with the presence of stiffness irregularities, there is a variation in stiffness of up to 59.85 % and therefore concluded that p-delta effect produces a greater degradation of the overall stiffness of structures. Rosman (2003) in his study on stability and dynamics of shear-wall frame structures, applied the shear-cantilever concept to describe the deflection-load-relationship of the frames, buckling and free vibrations of shear wall frame structures, with both plane and torional problem considered. He derived some formulae for the buckling loads and free vibration periods. Numerical values of stability and dynamic coefficients are determined which enable rapid stability and dynamic analyses of shear-wall-frame structures, and that knowledge of building loads allows consideration of p-delta effect to be evaluated. The p-delta effect is a second order (nonlinear) effect that occurs when an axial load acts upon a lateral displacement generating additional overturning moment that

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normally are not accounted in static and dynamic analysis.. Abbood, Jasim and Weli (2021), in their investigative study of high rise buildings, design, analysis ans safety identified that high-rise buildings are very complicated due to the high number of structural components and elements, and similarly demand high structural stability for safety unlike the low-rise buildings. Consequently, a general overview is presented that covers the behavior of various structural systems for different heights by implementinf a number of nonlinear static procedure analysis. Finally, a critical review of available simplified model and seismic energy base design are also presented, and further expressed that the paper is intended to assist in development and application of construction systems for high-rise buildings

METHODOLOGY

Structural stability involves simulation and modeling of the structures against dynamical effect and ensuring that all criteria for static equilibrium are established and satisfactory throughout the loading period. Structural modeling is an important aspect of building construction process and involves series of algorithmic processes and procedures to execute by a software program. The software used for the case study design is Prota-structure, which is an innovative BIM (building information model) solution that model, analyze and design buildings accurately and quickly while creating high quality design documents. The software utilizes advanced 3D finite element analysis (frame and shell), rigid zones, detailed diaphragm modeling, staged construction and diverse analysis approaches for simple and complex structures

3.1: The method involved software modeling ad design including structural stability and assessment of equilibrium (displacement) with specified permissible limits, with a case study is considered for the purpose

Case study: structural design of a proposed 12-story "multi-purpose complex at Sango Ota, Ogun state, Nigeria, using Prota-structure softwares.

3.2.1: Input data: Structural systems' configuration, storey height and numbers, applied loads, self weight, material specifications, soil bearing capacity, foundation type, etc

3.2.2: Execute the design

Design checks for serviceability limits with respect to the design codes (BS 8110) and EN standards

Checks for stability and equilibrium state of the structural system involve,

Lateral displacement limit defined as $\delta_a \leq \delta_p$ where δ_p is between H/500 and H/600 according to most design codes.

The limits ensure that the structure is stable and safe during load application, even during dynamical condition.

DISCUSSION OF RESULTS

Pre-Analysis Checks

Building data

No of storey = 12

Storey height = 3.5 (floors 2 to 12th) and 1.2m (1 to 2^{nd} floor) to allow for maintenance service lines

Structures height = 3.5x11 + 1.2 = 39.7m

Analysis Type: Static Analysis

Storey DOF = X, Y and Torsion

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Rigidity at joints = Permits rotation

Load combinations: dead + live + wind, with appropriate factor of safety according to the software analysis protocol

Materials characteristics for the structural components

Concrete f_{cu} = 25 N/mm^2

Steel f_y = 410 N/mm^2

Allowable soil Pressure = 200 kN/mm^2

Post Analysis Checks

Relative Storey Drift Check

Table 4.1: Definition of terms

Terms	Definitions
h_t	Total height up to particulate storey (m)
h	Storey height (m)
δ_{max}	Maximum absolute storey drift (mm)
Δ	Relative storey drift (mm) ($\delta_{top\ column} - \delta_{bottom\ column}$
δ_{eff}	Effective relative storey drift (mm)

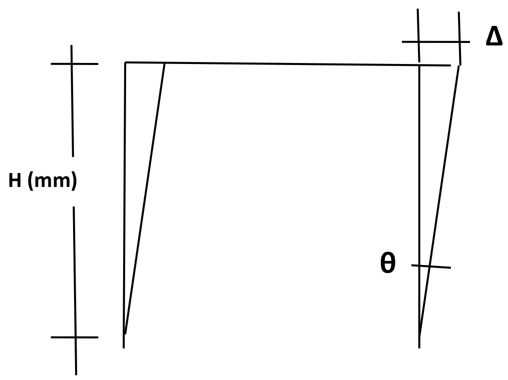

Figure 4.1: Typical frame system showing lateral displacement and rotation

Table 4.2: Load Case 1 : N_X – loading direction from X 0.00 Deg

Storey	h_t (m)	δ_{max} (m)	δ_{max}/h	Δ_{max} (m)	Δ_{max}/h
Storey 12	39.70	0.007839	0.00020	0.000619	0.00020
Storey 11	36.20	0.007220	0.00020	0.000722	0.00021

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Storey 10	32.70	0.006497	0.00020	0.000761	0.00022
Storey 9	29.20	0.005736	0.00020	0.000790	0.00023
Storey 8	25.70	0.004946	0.00019	0.000814	0.00023
Storey 7	22.20	0.004123	0.00019	0.000825	0.00024
Storey 6	18.70	0.003307	0.00018	0.000813	0.00023
Storey 5	15.20	0.002494	0.00016	0.000774	0.00022
Storey 4	11.70	0.001720	0.00015	0.000697	0.00020
Storey 3	8.20	0.001023	0.00012	0.000575	0/00016
Storey 2	4.70	0.000448	0.00010	0.000394	0.00011
Storey 1	1.20	0.000054	0.00005	0.000054	0.00005

$\theta_\delta = \delta_{max}/h = 0.0002 \leq 0.0010$ (the permissible absolute storey drift value)

$\theta_\Delta = \Delta_{max}/h = 0.00020 \leq 0.002$ (the permissible relative storey drift value)

From the analysis, it was found that the storey drifts satisfies the limits

Table 4.3: Load Case 2: N_y – loading direction at 90.00 Deg to X

Storey	h_t (m)	δ_{max} (m)	δ_{max}/h	Δ_{max} (m)	Δ_{max}/h
Storey 12	39.70	0.003127	0.00008	0.000275	0.00009
Storey 11	36.20	0.002853	0.00008	0.000319	0.00009
Storey 10	32.70	0.002533	0.00008	0.000327	0.00009
Storey 9	29.20	0.002206	0.00008	0.000331	0.00009
Storey 8	25.70	0.001875	0.00007	0.000331	0.00009
Storey 7	22.20	0.001544	0.00007	0.000324	0.00009
Storey 6	18.70	0.001220	0.00007	0.000310	0.00009
Storey 5	15.20	0.000910	0.00006	0.000287	0.00008
Storey 4	11.70	0.000622	0.00005	0.000253	0.00007
Storey 3	8.20	0.000369	0.00005	0.000206	0.00006
Storey 2	4.70	0.000163	0.00003	0.000141	0.00004
Storey 1	1.20	0.000022	0.00002	0.000022	0.00002

$\theta_\delta = \delta_{max}/h = 0.0002 \leq 0.0010$ (the permissible absolute storey drift value)

$\theta_\Delta = \Delta_{max}/h = 0.00020 \leq 0.002$ (the permissible relative storey drift value)

From the analysis, it was found that the storey drifts satisfies the limits

CONCLUSION AND RECOMMENDATION

Conclusion

Followings were deduced from the softwares analysis and results,

Maximum lateral displacement (δ) from story drift analysis is within permissible limit for both N_x and N_y , as shown in tables 4.2 and 4.3

Maximum rotation (θ) is found to be within the permissible limit as indicated in tables 4.2 and 4.3 respectively

Recommendation

Structural design provide basic information and specification required for construction of real-ife building, hence if the construction is implemented as specified including enforcing quality work according to appropriate engineering standard to ensure safety and reliability

Lateral displacement is an expected situation in multistory building due to configuration and load intensity, which creates P-delta effect and instability (ie, non static equilibrium state). Therefore if all anticipated forces are accurately evaluated and inputted correctly, the design results wi be very reliable

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