Effects of Electron Charge Fluctuations on Linear Ion-Acoustic Waves

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ABSTRACT

The linear propagation of the ion-acoustic waves (IAWs) in a dusty plasma consisting of Boltzmann-distributed electrons and ions, mobile charge fluctuating negative dust and charge fluctuating stationary negative dust has been theoretically investigated. It has been shown that the electron charge fluctuation is a source of dissipation, and is responsible for forming IA shock waves in such a dusty plasma. The basic features of the IA shock waves have been identified in this investigation which could be useful in understanding the properties of localized space dusty plasmas. It has been proposed to design a new laboratory experiment, which will be able to identify the basic features of the ion-acoustic shock waves predicted in this theoretical investigation.

Keywords: Charge fluctuation, linear ion-acoustic waves, dusty plasmas, shock waves.

INTRODUCTION

The wave propagation in plasmas has received much attention in recent years because of its vital role in understanding different types of collective processes in space environments, namely, lower and upper mesosphere, cometary tails, planetary rings, planetary magnetosphere, interplanetary spaces, interstellar media, etc. [1]-[6]. The dusty plasmas have also noticeable applications in laboratory devices [7]-[10]. The consideration of charge grains in plasmas does not only modify the existing plasma wave spectra [11]-[13], but also introduces a number of novel eigenmodes, such as the ion-acoustic (IA) waves, the lower-hybrid (LH) waves, the lattice waves, etc [14]-[18]. Most of the studies in dusty plasmas have been confined in considering the dust as negatively charged grains in addition to electrons and positively charged ions as the plasma species [4]-[6], [19]-[24]. It has been found that there are some plasma systems, particularly in space plasma environments, namely, cometary tails [1]-[3], [25], [26], upper mesosphere [27], Jupiter's magnetosphere [28], etc. where positively charged dust grains play significant roles. There are three mechanisms by which the dust grains in the plasma systems mentioned above can be positively charged. These mechanisms are the following: (i) photoemission in the presence of a flux of ultraviolet (UV) photons; (ii) thermionic emission induced by radiative heating; and (iii) secondary emission of electrons from the surface of the dust grains. In this paper, we have considered a dusty plasma containing mobile charge fluctuating negative dust, charge fluctuating stationary negative dust, Boltzmann-distributed electrons, and ions, and have studied the linear propagation of DA waves.

This paper is organized as follows. The basic equations describing our dusty plasma model are presented in Section II. I have derived the dispersion relation in Section III.I have analyzed numerically the dispersion properties of the DA wave mode in Section IV, where I have seen that negative dust-charge fluctuation is a source of linear growth instability. Finally, a brief discussion is given in Section V.

Governing Equations:

We consider an unmagnetized collisionless dusty plasma system consisting of charge fluctuating positively charged mobile dust, charge fluctuating stationary negative dust and Boltzmann-distributed electrons and ions. Thus at equilibrium , we have $n_{eo} + z^{-}_{do} n^{-}_{do} = n_{io} + z^{+}_{do} n^{+}_{do}$ where n_{eo} (n_{io}) is the equilibrium electron (ion) number density, n_{do}^{-} is the negative dust number density, n_{do}^{+} is the positive dust number density, z_{do}^{+} is the equilibrium charge state of the positive dust component, z_{do}^{-} is the equilibrium charge state of the negative dust component. The dynamics of the DA waves of such a dusty plasma system in one-dimensional form is given by

$$\frac{\partial n_d^+}{\partial t} + \frac{\partial}{\partial x}(n_d^+ u_d^+) = 0 \qquad (1)$$
$$\frac{\partial u_d^+}{\partial t} + u_d^+ \frac{\partial u_d^+}{\partial x} = -\frac{z_d^+ e}{m_d}\frac{\partial \phi}{\partial x} \qquad (2)$$
$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e[n_e - n_i - z_d^+ n_d^+ + z_d^- n_d^-] \qquad (3)$$

where is the number density of the plasma species j (j equals i for ions, e for electrons), $n_d^+(n_d^-)$ is the number density of positive (negative) dust. $u_d^+(u_d^-)$ is the positive (negative) dust fluid speed. $z_d^+(z_d^-)$ is the charge state of the positive (negative) dust component. ϕ is the electrostatic wave potential. The electron and the ion densities are assumed to follow the Boltzmann distribution:

$$n_e = n_{eo}\, exp\left(\frac{e\phi}{k_B T_e}\right) (4)$$
$$n_i = n_{io}\, exp\left(-\frac{e\phi}{k_B T_i}\right) (5)$$

where k_B is the Boltzmann constant and T_e is the electron temperature, T_i is the ion temperature. We assume that dust is charged by photo-emission current(I_P^+), the thermionic emission current $(I_t^+)$and the electron absorption current(I^-), the electron current for negative charge dust(I_e), the ion current for negative charge dust (I_i) only. All other charging processes are neglected. The charge state z_d^+ component is not constant, but varies according to the following equations:

$$\frac{\partial z_d^+}{\partial t} + u_d^+ \frac{\partial z_d^+}{\partial t} = \frac{I_P^+ + I_t^+ + I^-}{e} \quad (6)$$
$$\frac{\partial z_d^-}{\partial t} = -\left(\frac{I_e + I_i}{e}\right) \quad (7)$$

where

$$I_p^+ = \pi r_d^2 eJY\, exp\left(-\frac{z_d^+ e^2}{k_B r_d T_{ph}}\right) \qquad (8)$$

$$I_t^+ = 2\pi r_d^2 e\left(\frac{m_e k_B T_P}{2\pi h^2}\right)^{\frac{3}{2}}\left(\frac{8k_B T_P}{\pi m_e}\right)^{\frac{1}{2}}\left(1 + \frac{z_d^+ e^2}{k_B r_d T_p}\right) \times exp\left(-\frac{z_d^+ e^2}{k_B r_d T_p} - \frac{W_e}{k_B T_P}\right) \qquad (9)$$

$$I^- = -\pi r_d^2 e n_{eo} e^{\frac{e\phi}{k_B T_P}}\left(\frac{8k_B T_P}{\pi m_e}\right)^{\frac{1}{2}}\left(1 + \frac{z_d^+ e^2}{k_B r_d T_p}\right) \qquad (10)$$

$$I_e = -4\pi r_d^2 n_{eo} e^{\frac{e\phi}{k_B T_e}} e\left(\frac{k_B T_e}{2\pi m_e}\right)^{\frac{1}{2}} exp\left(-\frac{z_d^- e^2}{k_B r_d T_e}\right) \qquad (11)$$

$$I_i = 4\pi r_d^2 n_{io} e^{-\frac{e\phi}{k_B T_i}} e\left(\frac{k_B T_i}{2\pi m_i}\right)^{\frac{1}{2}}\left(1 + \frac{z_d^- e^2}{k_B r_d T_i}\right) \qquad (12)$$

where $\hbar$ is the Planck's constant,T_{ph} is the photon temperature,W_e is the work function , J is the UV photon flux, Y is the yield of photons (typical values of W_e, J and Y are 2.2 eV, 5.0×10^{14} photons/cm^2/s, and 0.1, respectively), and r_d is the dust radius. Now, using $q_d = +z_d^+ e$ (where the charge state z_d is the number of electrons residing on the dust grain space). Introducing the following normalized variables:

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$N_d^+ = n_d^+/n_{do}^+$, $U_d^+ = u_d^+/C_d$, $\Phi = e\varphi/k_B T_e$, $Z_d^- = z_d^-/z_{do}^-$ $Z_d^+ = z_d^+/z_d^+$, $Z_d^- = z_d^-/z_{do}^-$, $X = x/\lambda_{Dd}$, $T = t\omega_{pd}$, $\lambda_{Dd} = \left(k_B T_e/4\pi z_{do}^+ n_{do}^+ e^2\right)^{1/2}$, $C_d = (z_{do}^+ k_B T_e/m_d)^{1/2}$ and $\omega_{pd} = \left(4\pi {z_{do}^+}^2 n_{do}^+ e^2/m_d\right)^{1/2}$.

One can reduce equation (1) to (7) as

$$\frac{\partial N_d^+}{\partial T} + \frac{\partial}{\partial X}(N_d^+ U_d^+) = 0 \quad (13)$$

$$\frac{\partial U_d^+}{\partial T} + U_d^+ \frac{\partial U_d^+}{\partial X} = -Z_d^+ \frac{\partial \Phi}{\partial X} \quad (14)$$

$$\frac{\partial^2 \Phi}{\partial X^2} = (1 + \mu_i - \gamma)e^{\Phi} - \mu_i e^{-\sigma\Phi} - Z_d^+ N_d^+ + \gamma Z_d^- \quad (15)$$

$$\frac{\partial Z_d^+}{\partial T} + U_d^+ \frac{\partial Z_d^+}{\partial X} = \mu^+\left[P e^{-\alpha Z_d^+} + Q(1 + \beta Z_d^+)e^{-\beta Z_d^+} - R e^{\Phi}(1 + \beta Z_d^+)\right] \quad (16)$$

$$\frac{\partial Z_d^-}{\partial T} = \mu^-\left[X_e e^{\Phi - \alpha_e Z_d^-} - X_i(1 + \alpha_i Z_d^-)e^{-\sigma\Phi}\right] \quad (17)$$

Where $\mu^- = \frac{4\pi r_d^2}{z_{do}^- \omega_{pd}}$ $\mu^+ = \frac{\pi r_d^2}{z_{do}^+ \omega_{pd}}$, $\alpha_e = \frac{z_{do}^- e^2}{r_d k_B T_e}$, $\alpha_i = \frac{z_{do}^- e^2}{r_d k_B T_i}$

$\alpha = \frac{z_{do}^+ e^2}{r_d k_B T_{ph}}$, $\beta = \frac{z_{do}^+ e^2}{r_d k_B T_p}$, $\gamma = \frac{z_{do}^- n_{do}^-}{z_{do}^+ n_{do}^+}$, $\mu_e = \frac{n_{eo}}{z_{do}^+ n_{do}^+}$, $\mu_i = \frac{n_{io}}{z_{do}^+ n_{do}^+}$,

$\sigma = \frac{T_e}{T_i}$, $X_e = n_{eo}\left(\frac{k_B T_e}{2\pi m_e}\right)^{1/2}$, $X_i = n_{io}\left(\frac{k_B T_i}{2\pi m_i}\right)^{1/2}$, P = JY,

$Q = 2\left(\frac{m_e k_B T_P}{2\pi\hbar^2}\right)^{3/2}\left(\frac{8 k_B T_P}{\pi m_e}\right)^{1/2} e^{\frac{-W_e}{k_B T_P}}$, $R = n_{eo}\left(\frac{8 k_B T_p}{\pi m_e}\right)^{1/2}$.

Derivation of the Linear Dispersion Relation

To derive a dynamical equation for the linear propagation of the DA shock waves in a dusty plasma, I first express our dependent variables N_d^+, U_d^+, Φ, Z_d^+, and Z_d^- in terms of their equilibrium and perturbed parts as

$$N_d^+ = 1 + \epsilon N_d^{+(1)} + \epsilon^2 N_d^{+(2)} + --------- \quad (18)$$

$$U_d^+ = \epsilon U_d^{+(1)} + \epsilon^2 U_d^{+(2)} + --------- ---- \quad (19)$$

$$Z_d^+ = 1 + \epsilon Z_d^{+(1)} + \epsilon^2 Z_d^{+(2)} + ---------- -- \quad (20)$$

$$Z_d^- = 1 + \epsilon Z_d^{+(1)} + \epsilon^2 Z_d^{+(2)} + ---------- -- \quad (21)$$

$$\Phi = \epsilon \Phi^{(1)} + \epsilon^2 \Phi^{(2)} + ------------- \quad (22)$$

Now, substituting (18)-(22) into (13)-(17), we develop equations in various power of we have

$$\frac{\partial N_d^{+(1)}}{\partial T} + \frac{\partial U_d^{+(1)}}{\partial X} = 0 \quad (23)$$

$$\frac{\partial U_d^{+(1)}}{\partial T} = -\frac{\partial \Phi^{(1)}}{\partial X} \quad (24)$$

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$$\frac{\partial^2 \Phi}{\partial X^2} = (1+\mu_i-\gamma)\Phi^{(1)} + \mu_i\sigma\Phi^{(1)} - N_d^{+(1)} - Z_d^{+(1)} + \gamma Z_d^{-(1)} \quad (25)$$

$$\frac{\partial Z_d^{+(1)}}{\partial T} = \mu^+[(Q\beta^2 - P\alpha)Z_d^{+(1)} - R(1+\beta)\Phi^{(1)}] \quad (26)$$

$$\frac{\partial Z_d^{-(1)}}{\partial T} = \mu^-[(-\alpha_e X_e - \alpha_i X_i)Z_d^{-(1)} - (X_e + X_e\alpha_e - X_i\sigma - X_i\sigma\alpha_i)\Phi^{(1)}] \quad (27)$$

Now assuming that all perturbed quantities are proportional to $exp(-i\omega T + ikX)$, i.e. taking $\partial/\partial T \rightarrow -i\omega$ and $\partial/\partial X \rightarrow ik$, where ω and k are the wave angular frequency and the propagation constant respectively in Eqs. (23)-(27), we obtain

Now using (30)-(34), one can eliminate $N_d^{+(2)}, U_d^{+(2)}, Z_d^{+(2)}, Z_d^{-(2)}$ and $\Phi^{(2)}$, and can finally obtain the following equation:

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + A\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi} = C\frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} \quad (35)$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$\mathrm{A} = \frac{B^{/}}{A^{/}} \quad (36)$$

$$\mathrm{C} = -\frac{C^{/}}{A^{/}} \quad (37)$$

$$A^{/} = -\frac{2}{V_0^3} \quad (38)$$

$$B^{/} = (1+\mu_i-\gamma) - \sigma^2\mu_i - \frac{3f}{V_0^2} + \frac{f^2\left(P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3\right)}{P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta} - \frac{2R\beta f + R + R\beta}{P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta} - \frac{3}{V_0^4} + \frac{2\gamma\left(\frac{1}{2}X_e - \frac{1}{2}X_e\alpha_e - X_e\alpha_e\rho + \frac{1}{2}\alpha_e^2 X_e\rho^2 + \alpha_e^2 X_e\rho - \frac{1}{2}\sigma^2 X_i + \frac{1}{4}\alpha_e^2 X_e - \frac{1}{2}\sigma^2 X_i\alpha_i + \sigma^2 X_i\alpha_i\right)}{\alpha_i X_i - X_e\alpha_e^2 + X_e\alpha_e} \quad (39)$$

$$C^{/} = \frac{V_0 f}{\mu^+\left(P\alpha^2 - Q\beta^2 + \frac{3}{2}Q\beta^3 - P\alpha - R\beta\right)} + \frac{V_0\rho\gamma}{\mu^-(\alpha_i X_i - X_e\alpha_e^2 + X_e\alpha_e)}, \quad (40)$$

Equation (35) is the well-known Burgers equation describing the nonlinear propagation of the DA shock waves in the dusty plasma under consideration. It is obvious from (35) and (37) that the dissipative term, i.e. the right-hand side of (35) is due to the presence of the charge fluctuating dust.

Numerical Analysis:

We are now interested in looking for the stationary shock wave solution of (35) by introducing the variables $\varsigma = \xi - U_0\tau^{/}$ and $\tau^{/} = \tau$, where is the shock wave speed (in the reference frame) normalized by C_d , ς is normalized by λ_{Dd}, and τ is normalized by ω_{pd}^{-1}. This leads us to write (35), under the steady state condition $(\partial/\partial\tau = 0)$, as

$$-U_0\frac{\partial \Phi^{(1)}}{\partial \varsigma} + A\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \varsigma} = C\frac{\partial^2 \Phi^{(1)}}{\partial \varsigma^2} \quad (41)$$

It can be easily shown that [36], [37] that (41) describes shock waves whose speed U_0 (in the reference frame) is related to the extreme values $\Phi^{(1)}(-\infty)$and $\Phi^{(1)}(\infty)$by $\Phi^{(1)}(-\infty)$-$\Phi^{(1)}(\infty)$=$2U_0/A$. Thus, under the condition that $\Phi^{(1)}$ is bounded at$\varsigma = \pm\infty$, the shock wave solution of (41) can be written as

$$\Phi^{(1)} = \Phi_0[1 - tanh(\varsigma/\Delta)] \quad (42)$$

where

$$\Phi_0 = U_0/A \quad (43)$$

$$\Delta = 2C/U_0 \quad (44)$$

are respectively, the height and thickness of the shock waves moving with the speed U_0. It is obvious from (41) to (44) that the shock waves are due to the presence of the charge fluctuating dust, and the shock structures are associated with the negative potential $(A < 0)$ as well as with positive potential $(A > 0)$.To find the parametric regimes for which positive and negative shock wave (potential) profiles exist, we have numerically analyzed A and obtain A = 0 (2-D) curves for $\gamma = 0.2 \text{to} 0.6$ and $\mu_i = 0 \text{to} 3.8$. The A = 0 curve is shown in Fig.1. It shows that we can have positive shock wave (potential) profiles for the parameters whose values lie above A = 0 curve and negative shock wave (potential) profiles for the parameters whose value lie below the A = 0 curve. These are shown in Figs.2-3. Figs 2 and 3 show the positive and negative shock potential profiles respectively.

CONCLUSION

We have studied the linear propagation of IA waves in an unmagnetized dusty plasma containing Boltzmann-distributed electrons and ions, mobile charge fluctuating negative dust, and charge fluctuating stationary negative dust. We have shown here how the nonlinear IA waves' basic features are modified by the charge fluctuating dust in dusty plasmas. The results, which have been obtained from this investigation, can be summarized as follows:

The dust charge fluctuation is a source of dissipation and is responsible for forming IA shock waves in the dusty plasma. The shock structures are associated with the negative potential (A<0) as well as the positive potential (A<0). It is shown that the height (normalized by $k_B T_e/\text{e}$) of the potential structures in the form of the shock waves is directly proportional to the shock speed U_0, and it is also found that the thickness (normalized by λ_{Dd}) of these shock structures is inversely proportional to the shock speed U_0.

The parametric regimes for the existence of positive as well as negative shock structures are shown in Fig.1. Figs. 2 and 3 show the positive and negative shock potential profiles of shock waves respectively.

It is to be mentioned here that the parameters we have chosen in our numerical analysis are very much relevant to the plasma in the mesosphere [27]. We stress that the results of the present investigation could be useful in understanding the properties of localized DA waves of dusty plasmas in the mesosphere.

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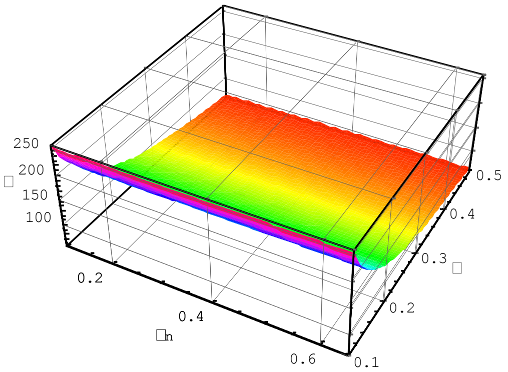

Fig. 1. Showing $A = 0$ (Φvs. β) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} .

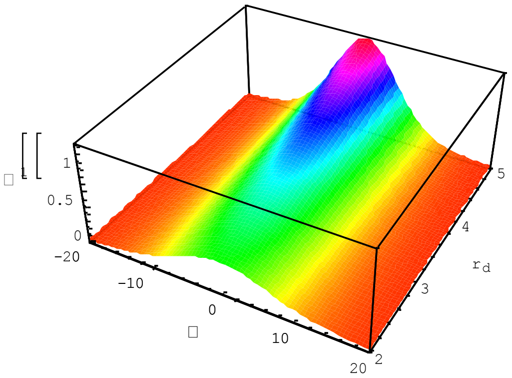

Fig. 2. Showing positive potential (Φ vs. ζ) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} with $\beta = 38.6$,

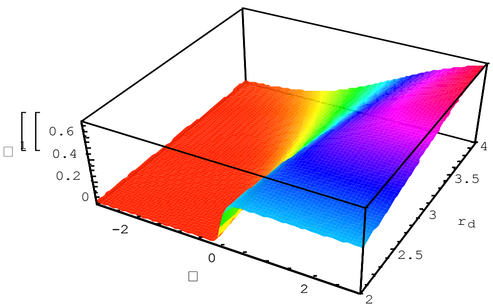

Fig. 3. Showing negative potential (Φ vs. ζ) curves for the parameters $P = 5.0 \times 10^{13}$ cm^{-2}, $Q = 1.93 \times 10^{28}$ cm^{-2}s^{-1}, $R = 2.48 \times 10^{28}$ cm^{-2}s^{-1} with $\alpha_i = 4.77$, $\beta = 38.6$.