Mathematical Modeling of the Biomass Waste to Energy Conversion Technology

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ABSTRACT

This paper presents a mathematical modeling framework for a biomass waste-to-energy conversion technology, focusing on the anaerobic digestion of manure to produce biogas. The core of the model is the Anaerobic Digestion Model No. 1 (ADM1), which simulates the biochemical conversion processes within the digester. The ADM1 model was adapted to dairy manure by modifying its kinetic parameters, allowing for accurate prediction of biogas production based on waste characteristics such as chemical composition, dry matter content, and organic fractions. The biogas output is subsequently used to generate electricity and heat through a technology comprising an internal combustion engine, an induction generator, a heat exchanger, and a dual-fuel boiler. Each component is mathematically modeled to estimate performance outputs, including torque, electric power, and thermal energy. The integration of these models enables a detailed energy balance analysis and supports technology optimization. Limitations of the current model include assumptions of steady-state operation and limited adaptability to varied waste streams. Future research will focus on improving model flexibility, incorporating dynamic behaviors, and enhancing real-time control capabilities for broader application in waste-to-energy technology.

Keywords: Anaerobic Digestion, ADM1 Model, Biomass Waste-to-Energy, Mathematical Modeling, Renewable Energy

INTRODUCTION

The increasing global demand for sustainable energy has brought renewed focus on the conversion of organic waste into renewable energy through technologies such as anaerobic digestion [1-3]. Anaerobic digestion not only reduces the environmental burden of waste disposal but also produces biogas, a versatile energy source that can be used for heat, electricity, or upgraded for other applications [4-6]. However, the complexity of biochemical processes involved in anaerobic digestion necessitates robust modeling tools that can accurately simulate technology performance and optimize outcomes under varying conditions.

One of the key challenges in waste-to-energy technology lies in understanding how the physical and chemical properties of different waste streams influence biogas yield and technology behavior [7-9]. Agricultural residues, animal manure, food waste, and municipal solid waste all present different compositional characteristics and require careful characterization for effective treatment and energy recovery [10,11]. Accurate modeling enables stakeholders to make informed decisions about digester design, operational strategies, and resource allocation.

This paper addresses these challenges by focusing on the development of a modeling approach that integrates waste characterization, energy output estimation, and process economics. Specifically, it explores how measurable waste properties, commonly available through standard laboratory tests, can be transformed into actionable inputs for dynamic simulation of anaerobic digestion processes. The study aims to bridge the gap between laboratory measurements and computational modeling, enhancing the applicability of simulation tools in practical biomass-to-energy applications.

The ADM1 Model for Biomass Waste-to-Energy Conversion

The Anaerobic Digestion Model No. 1 (ADM1), forms the core of the modeling framework for this biomass

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waste-to-energy conversion technology. ADM1 is a comprehensive computer-based model that simulates the anaerobic digestion of organic waste and estimates energy generation outputs, process costs, and biogas production [12-14]. In this study, ADM1 is applied specifically to the treatment of manure waste.

ADM1 represents the anaerobic digestion process as a sequence of biochemical and physico-chemical reactions [15-17]. The model employs a modular approach, wherein each stage of waste treatment, hydrolysis, acidogenesis, acetogenesis, and methanogenesis, is treated distinctly, with its own set of kinetic and stoichiometric parameters [13, 18]. The input to the ADM1 model is defined primarily by the mass flow rate of the biomass waste, along with detailed waste characterisation parameters, including density, dry matter content, and the mass fractions of carbon (C), hydrogen (H), oxygen (O), nitrogen (N), sulfur (S), phosphorus (P), potassium (K), and ash content. The ADM1 software calculates key technology outputs such as the volume flow rate of biogas, electricity output, heat generation, and process costs, using a mass balance approach [19,20]. This involves writing balanced chemical equations based on the elemental composition of the input waste. Unlike some simplified empirical models, ADM1 provides a granular representation of the anaerobic digestion process by tracking individual components and microbial groups across the digestion stages.

A major contribution of this study is the methodology developed to transform measurable waste characteristics such as total solids, volatile solids, Chemical Oxygen Demand (COD), Volatile Fatty Acids (VFAs), total nitrogen, ammonia content, phosphorus, and orthophosphates, into the characterisation parameters required by ADM1. These conversions are essential to enhance model input accuracy and to make the ADM1 model applicable across a broader range of real-world waste streams. By incorporating such measurable data, the model becomes more adaptable to site-specific conditions and facilitates practical deployment.

A review of existing models that simulate anaerobic digestion reveals multiple modeling paradigms: the mass balance approach [21,22], fuzzy logic, statistical and neural network models [23-25], and knowledge-based models [12,26]. While mass balance models (e.g., Contois-based models) are computationally simple, they often lack accuracy and applicability across diverse technology configurations. Fuzzy and neural network models provide empirical predictions but require extensive datasets and offer limited insight into process mechanisms. Knowledge-based models such as ADM1, stand out due to their detailed representation of biological pathways and use of kinetic and stoichiometric modeling. In contrast to models that estimate biogas output solely from proximate analysis or empirical correlations, ADM1 integrates both proximate and ultimate analyses to develop a complete stoichiometric and kinetic model. This enables a more accurate prediction of biogas yield by accounting for the biochemical composition of the waste and the environmental conditions under which digestion occurs.

While the model provides robust predictions for biogas generation, opportunities for enhancement remain. One such extension is the integration of pH modeling, which is essential for assessing microbial activity and technology stability during digestion. Currently, ADM1 assumes standard pH conditions [27,28], but incorporating pH as a variable would allow more dynamic simulation of acidogenesis and methanogenesis phases. Moreover, ADM1 supports greenhouse gas emission reduction assessments [29,30], making it suitable for evaluating the environmental impact of biogas technologies. In this regard, it can be aligned with sustainability objectives, such as reducing carbon footprints and promoting circular economy principles in agriculture and waste management.

ADM1 stands out as a powerful and flexible modeling framework for the simulation and optimization of anaerobic digestion technologies. Its detailed representation of biological processes, integration of real-world data, and adaptability to various waste streams make it particularly valuable in the context of energy and environmental engineering. As global efforts to decarbonize energy technologies and manage organic waste intensify, models like ADM1 are critical for supporting the design, operation, and policy planning of sustainable bioenergy solutions.

Modeling of the technology components.

The input to the biomass waste to energy conversion technology is volume flow rate of manure waste. This

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goes into the digester. The output of the biomass waste to energy conversion technology is heat and electricity. The core of the conversion system is the anaerobic digester, where manure undergoes biological decomposition to produce biogas. The digestion process is mathematically represented using the ADM1, a widely accepted framework initially developed for wastewater treatment applications [12,17,31]. For this study, the ADM1 kinetic parameters have been adapted to better represent the anaerobic digestion of dairy manure, capturing the mass flow rate, air-fuel ratio, biogas density, and lower heating value (LHV). These outputs are crucial for subsequent components, particularly the internal combustion engine.

Digester

The ADM1 is used to model the digester. The ADM1 was formulated as a tool for modeling waste water treatment [12,17]. The kinetic parameters of the ADM1 were modified to simulate the anaerobic digestion of dairy manure. The manure from the lagoon undergoes anaerobic digestion, in the digester, to produce biogas. The anaerobic digestion process is modeled and the mass flow rate, the air-fuel ratio, the density and the LHV (Lower Heating Value) of biogas are calculated. These values are required by the internal combustion engine model to calculate torque output. The stages of the anaerobic digestion process are shown in Figure 1. The first stage of the anaerobic digestion process is hydrolysis, where bacteria break down organic matter to sugars, fatty acids and amino acids. This is followed by the acid digestion stages, acidogenesis and acetogenesis. During acid digestion the molecules from hydrolysis are absorbed by the acid forming bacteria, producing short chain fatty acids, carbon dioxide and hydrogen. The final stage of the anaerobic digestion process is gas digestion.

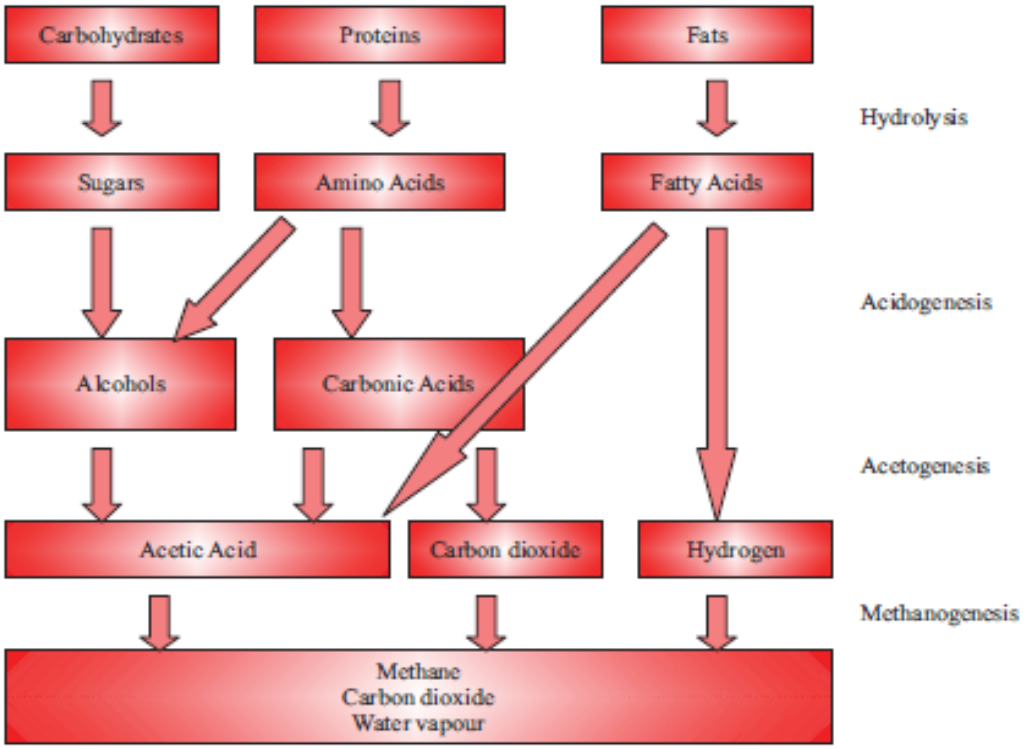

Figure 1: Anaerobic Digestion Process

During gas digestion methane forming bacteria attack the fatty acids to form methane, carbon dioxide and water vapour. The ADM1 model groups the anaerobic digestion processes into biochemical and physico-chemical processes. Biochemical processes are catalysed by intracellular or extracellular enzymes and act on a pool of organic material. Physico-chemical processes are not biologically mediated and involve association or dissociation, transfer between gas and liquid phases and precipitation. The ADM1 does not model precipitation. The ADM1 model is based on a completely stirred reactor with a single input and output waste stream and a constant liquid volume with a gas above it (Figure 2). The waste stream and the gas are categorised into components, designated by *i*. The waste stream components comprise of substrates and active biomass. There are 12 substrates and 12 active biomass components whose concentrations are defined by:

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$S_{in,i}$	for i= 1,2,3,…,12,		(1)
$S_{liq,i}$	for i= 1,2,3,…,12,		(2)
$X_{in,\ i}$	for i= 12,13,14,…,24,		(3)
X_{liq},i	for i= 12,13,14,…,24,		(4)

Where S_{in} is the concentration of the substrate in the input waster stream, S_{liq} is the concentration of the substrate in the liquid phase of the waste stream, X_{in} is the concentration of the active biomass in the input waste stream. The gas above the reactor has 3 components. The concentration of the substrates in the gas components and their partial pressures are defined by:

$S_{gas,i}$	for i= 1,2,3,		(5)
$p_{\ gas,i}$	for i= 1,2,3,		(6)

where S_{gas} is the concentration of the substrate in the gas component and p_{gas} is the partial pressure of the gas component. A mass balance of the components is carried out. The mass balance is the rate of mass change of the components. The mass change occurs as a result of the biochemical and physico-chemical reactions. The structure used for modeling the biochemical reactions in the ADM1 is shown in Figure 3. The structure has two extracellular steps: disintegration and hydrolysis, and three intra-cellular steps: acidogenesis, acetogenesis and methanogenesis. The arrows in Figure 3 show the process flow, with hydrolysis, acidogenesis and acetogenesis having a number of parallel reactions.

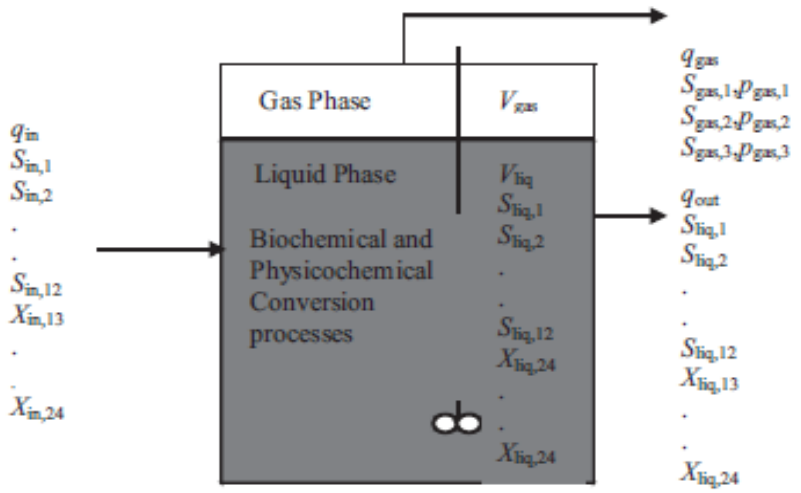

Figure 2: ADM1 Reactor

There are 19 biochemical reactions modeled by ADM1, designated by j. Reactions j = 1, 2, 3, 4, are disintegration and hydrolysis reactions and j = 5, 6, 7, ..., 19, are acidogenesis, acetogenesis and methanogenesis reactions. The mass balance of the substrates in the liquid phase [32,33] is calculated by:

$\frac{dS_{liq,i}}{dt} = \frac{q_{in}\, s_{in,i} - q_{out} S_{liq,i}}{V_{liq}} + \sum_{j=1}^{19} p_j v_{i.j}$ kgCOD/m^3/day, for i= 1,2,3,…12,	(7)

Where S_{liq} is the concentration of the component in the digester, q_{in} is the volume flow rate of manure going into the digester, Sin is the concentration of the component going into the digester, q_{out} is the volume flow rate of the effluent leaving the digester, V_{liq} is the volume of the digester, ρ is the kinetic rate of the reaction and v

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is the stoichiometric coefficient of the reaction. kgCOD/m^3 is the chemical component base unit used to model the anaerobic digestion process. COD (Chemical Oxygen Demand) is the mass of oxygen required to completely oxidise a given organic compound. The calculation of the stoichiometric coefficients v of the different reactions is detailed in [32,34]. The kinetic rate ρ depends on the type of reaction. The kinetic rate of the disintegration and hydrolysis reactions [32,34] is calculated by:

$\rho_j = k_j X_i$ for i= 13,14,15,16 and j =1,2,3,4,	kgCOD/m^3/day,	(8)

where ρ is the kinetic rate of the reaction, k is the first order rate coefficient of the reaction and X is the concentration of the active biomass component.

The kinetic rate of the acidogenesis, acetogenesis and methanogenesis reactions is

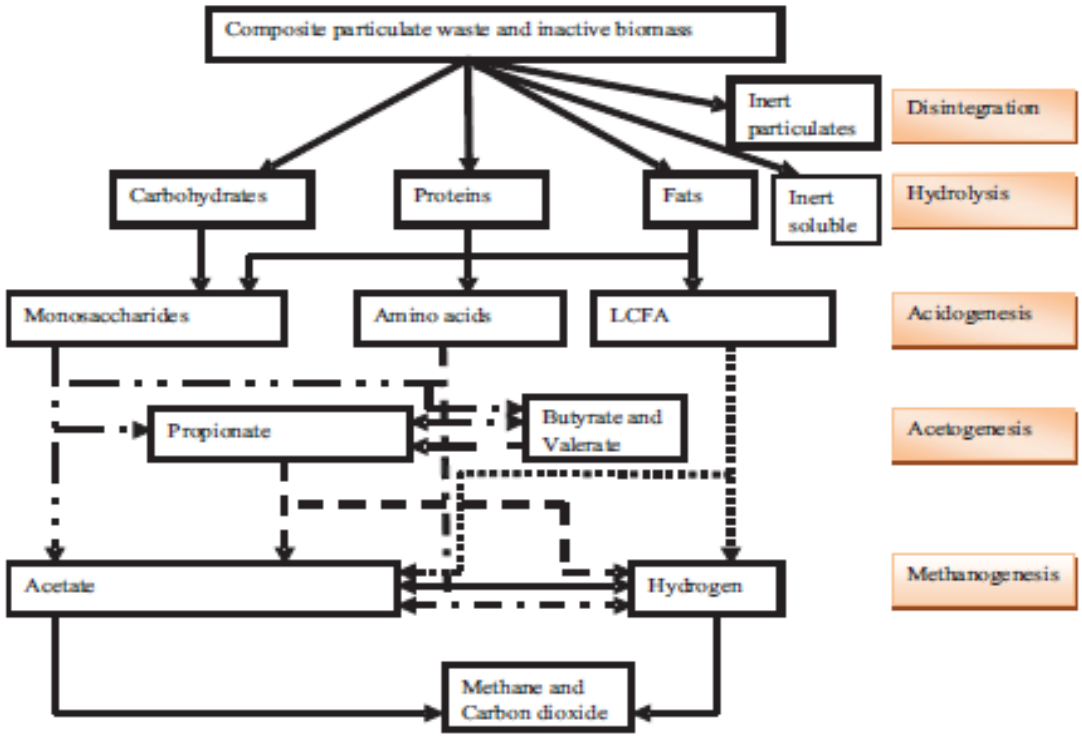

Figure 3: Modeling of Biochemical Reactions in ADM1

calculated by:

$\rho_j = \left(k_{m,j} S_i/(K_i + S_i)\right) X I_1 I_2 I_3$ for i = 1,2,3,…12, and j =5,6,7,…12,	kgCOD/m^3/day,	(9)
$X = X_i$ *for* i = 17,18,19,…23	kgCOD/m^3,	(10)

where ρ is the kinetic rate of the reaction, k_m is the maximum specific rate of substrate utilisation, S is the concentration of the waste component, K is the concentration giving half the maximum rate of utilisation of the component, X is the concentration of active biomass in the component, I_1 is hydrogen inhibition, I_2 is free ammonia inhibition and I_3 is pH inhibition.

Hydrogen and free ammonia inhibitions are calculated by:

$$I = 1/(1+S_I/K_I) \quad (11)$$

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VI June 2025

where I is inhibition, S_1 is the concentration of the inhibitory component I and K_1 is an inhibition constant. pH inhibition is calculated by:

$$I = \begin{cases} \exp(-3((\mathrm{pH} - \mathrm{pH_{UL}})/(\mathrm{pH_{UL}} - \mathrm{pH_{LL}}))^2) \text{ if } \mathrm{pH} < \mathrm{pH_{UL}} \\ \left(1 + 2 \times 10^{0.5(\mathrm{pH_{LL}} - \mathrm{pH_{UL}})}\right)/\left(1 + 10^{(\mathrm{pH} - \mathrm{pH_{UL}})} + 10^{(\mathrm{pH_{LL}} - \mathrm{pH})}\right) \text{ if } \mathrm{pH} \geq \mathrm{pH_{UL}}, \end{cases} \quad (12)$$

where I is inhibition, $\mathrm{pH_{LL}}$ is the lower limit of pH and $\mathrm{pH_{UL}}$ is the upper limit of pH. The mass balance equation [32] for the gas phase is:

$$\frac{dS_{gas,i}}{dt} = -\frac{q_{gas}S_{gas,i}}{V_{gas}} + \rho_{\mathrm{T},i}\frac{V_{liq}}{V_{gas}} \text{ for } i = 1,2,3, \quad \text{kgCOD/m}^3\text{/day,} \quad (13)$$

where S_{gas} is the concentration of the biogas component, q_{gas} is the volume flow rate of biogas from the digester, V_{gas} is the volume of the gas headspace in the digester, ρ_{T} is the kinetic rate of the liquid-gas transfer reaction of the biogas component and V_{liq} is the volume of the digester. The kinetic rates of the liquid-gas transfer reactions for hydrogen, methane and carbon dioxide are calculated by:

$$\rho_{T,H_2} = k_L\alpha\left(S_{liq,H_2} - 16K_{\mathrm{H,H_2}}\,\mathrm{p_{gas,H_2}}\right) \quad \text{kgCOD/m}^3\text{,} \quad (14)$$

$$\rho_{T,CH_4} = k_L\alpha\left(S_{liq,CH_4} - 64K_{\mathrm{H,CH_4}}\mathrm{p_{gas,CH_4}}\right) \quad \text{kgCOD/m}^3\text{,} \quad (15)$$

$$\rho_{T,CO_2} = k_L\alpha\left(S_{liq,CO_2} - K_{\mathrm{H,CO_2}}\mathrm{p_{gas,CO_2}}\right) \quad \text{kgCOD/m}^3\text{,} \quad (16)$$

where ρ_{T,H_2}, ρ_{T,CH_4} and ρ_{T,CO_2} are the kinetic rates of the liquid-gas transfer reactions of hydrogen, methane and carbon dioxide respectively, k_{L} is the overall mass transfer coefficient, a is the specific transfer area, S_{liq,H_2}, S_{liq,CH_4} and S_{liq,CO_2} are the concentrations of hydrogen, methane and carbon dioxide respectively, $K_{\mathrm{H,H_2}}$, $K_{\mathrm{H,CH_4}}$ and $K_{\mathrm{H,CO_2}}$ are the Henry's law coefficients of hydrogen, methane and carbon dioxide respectively and $\mathrm{p_{gas,}H_2}$, p_{gas},$\mathrm{CH_4}$ and p_{gas},$\mathrm{CO_2}$ are the partial pressures of hydrogen, methane and carbon dioxide respectively. The mass balance equation of the gas phase calculates the volume flow rate of biogas produced. The internal combustion engine model requires the mass flow rate of biogas, the air-fuel ratio of biogas and the LHV of biogas in order to calculate torque output. The volume flow rate of biogas, q_{gas} is required to solve the differential equation (13). This is calculated by:

$$q_{\mathrm{gas}} = k_{\mathrm{p}}\left(P_{\mathrm{gas}} - P_{\mathrm{atm}}\right) \quad \text{m}^3\text{/day,} \quad (17)$$

where q_{gas} is the volume flow rate of biogas, k_{p} is a pipe resistance coefficient, P_{gas} is the pressure of biogas and P_{atm} is atmospheric pressure. The mass flow rate of biogas is calculated from the density and the volume flow rate of biogas. The density of biogas is calculated by:

$$\rho_{\mathrm{gas}} = M_{\mathrm{gas}}P_{\mathrm{gas}}/RT_{\mathrm{biogas}} \quad \text{kg/m}^3\text{,} \quad (18)$$

where ρ_{gas} is the density of biogas, M_{gas} is the molar mass of biogas, P_{gas} is the pressure of biogas, R is the universal perfect gas constant and T_{biogas} is the temperature of biogas. The pressure of biogas is the sum of the partial pressures of hydrogen, methane, carbon dioxide and water vapour, which are calculated by:

$$p_{\mathrm{gas},H_2} = S_{\mathrm{gas,H_2}}RT_{\mathrm{biogas}} \quad \text{bar,} \quad (19)$$

$$p_{\mathrm{gas},CH_4} = S_{\mathrm{gas,CH_4}}RT_{\mathrm{biogas}} \quad \text{bar,} \quad (20)$$

$$p_{\mathrm{gas},CO_2} = S_{\mathrm{gas,CO_2}}RT_{\mathrm{biogas}} \quad \text{bar,} \quad (21)$$

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue VI June 2025

where p_{gas,H_2}, p_{gas,CH_4} and p_{gas,CO_2} are the partial pressures of hydrogen, methane and carbon dioxide respectively, $S_{\text{gas},\text{H}_2}$, $S_{\text{gas},\text{CH}_4}$ and $S_{\text{gas},\text{CO}_2}$ are the concentrations of hydrogen, methane and carbon dioxide respectively, R is the universal perfect gas constant and T_{biogas} is the temperature of the biogas. The partial pressure of watervapour is calculated by:

$$p_{\text{gas},\text{H}_2\text{O}} = 0.0313\exp\left(T_{\text{biogas}} - 298/298T_{\text{biogas}}\right) \quad \text{bar,} \quad (22)$$

where $p_{\text{gas},\text{H}_2\text{O}}$ is the partial pressure of water vapour and T_{biogas} is the temperature of the biogas. The molar mass of biogas (M_{gas}) is required to calculated the density of biogas and is given by:

$$M_{\text{gas}} = \left(M_{CH_4}p_{\text{gas},\text{CH}_4} + M_{CO_2}p_{\text{gas},CO_2} + M_{H_2}p_{\text{gas},\text{H}_2} + M_{H_2O}p_{\text{gas},\text{H}_2\text{O}}\right)/P_{\text{gas}} \quad \text{kg/mol,} \quad (23)$$

where M_{gas} is the molar mass of biogas, M_{CH_4}, M_{CO_2}, M_{H_2} and M_{H_2O} are the molar masses of methane, carbon dioxide, hydrogen and water vapour respectively, $p_{\text{gas},\text{CH}_4}$, p_{gas,CO_2}, $p_{\text{gas},\text{H}_2}$ and $p_{\text{gas},\text{H}_2\text{O}}$ are the partial pressures of methane, carbon dioxide, hydrogen and water vapour respectively, and Pgas is the pressure of the biogas. The second input required for calculation of the torque output is the air-fuel ratio of biogas. This is calculated by:

$$AF = 2.38\left(4p_{\text{gas},\text{CH}_4} + p_{\text{gas},\text{H}_2}\right)M_{\text{air}}/P_{\text{gas}}M_{\text{gas}} \quad (24)$$

where AF is the air-fuel ratio of biogas, $p_{\text{gas},\text{CH}_4}$ is the partial pressure of methane, $p_{\text{gas},\text{H}_2}$ is the partial pressure of hydrogen, M_{air} is the molar mass of a standard composition of dry air, P_{gas} is the pressure of biogas and M_{gas} is the molar mass of biogas. The third input required for the calculation of the output torque, the LHV of the biogas is determined from the heat of combustion of the reactants in the digester:

$$LHV_{\text{gas}} = \left(hrpo + \Delta H_{\text{p}} - \Delta H_{gas} - \Delta H_{air}\right)/M_{\text{gas}} \quad \text{kJ/kg,} \quad (25)$$

where LHV_{gas} is the Lower Heating Value of the biogas, $hrpo$ is the total heat of combustion of the gases at standard conditions, ΔH_{p} is the enthalpy change of the manure from standard temperature to the operating temperature of the digester, ΔH_{gas} is the enthalpy change of the biogas from standard temperature to the temperature of the biogas, ΔH_{air} is the enthalpy change of air from standard temperature to the operating temperature of the digester and M_{gas} is the molar mass of the biogas. The total heat of combustion of the gases at standard conditions, $hrpo$ is given by:

$$hrpo = \left(\left(p_{\text{gas},\text{CH}_4} + p_{\text{gas},\text{CO}_2}\right)hfo_{CO_2} + \left(2p_{\text{gas},CH_4} + p_{\text{gas},\text{H}_2o} + p_{\text{gas},\text{H}_2}\right)hfo_{H_2o} - \left(p_{\text{gas},\text{CH}_4}hfo_{CH_4} + \right. \quad \text{kJ/mol,} \quad (26)$$

where $hrpo$ is the total heat of combustion of the gases at standard conditions, $p_{\text{gas},\text{CH}_4}$, $p_{\text{gas},\text{CO}_2}$, $p_{\text{gas},\text{H}_2o}$ and $p_{\text{gas},\text{H}_2}$ are the partial pressures of methane, carbon dioxide, water vapour and hydrogen respectively, hfo_{CO_2}, hfo_{H_2o} and hfo_{CH_4} are the heats of combustion of carbon dioxide, water vapour and methane respectively, and P_{gas} is the pressure of biogas.

Internal Combustion Engine

An engine-generator set comprises of an internal combustion engine coupled to an induction machine. The internal combustion engine produces a torque as a result of combustion of biogas. The torque is applied to the induction machine to generate electric power. The power rating of the induction machine has to be matched to that of the internal combustion engine. The internal combustion engine model is obtained from the ADVISOR software [35-37]. The internal combustion engine model used is based on the Advanced Vehicle Simulator fuel converter for the John Deere natural gas fuelled engine. The software has fuel use maps obtained from experimental work. The software uses the Newton-Raphson method and a two dimensional linear interpolation

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function, to calculate the torque output of the internal combustion engine:

$T_{L,n+1} = T_{L,n} - \frac{f^{ICE}(T_{L,n})}{f^{ICE\prime}(T_{L,n})}$ Nm,	(27)
$f^{ICE}(T_{L,n}) = \frac{m_{\text{gas}} LHV_{\text{gas}}}{\omega_{mech}} f^{\text{interp}}(fc_{\text{map\_trq}}, fc_{\text{map\_spd}}, fc_{\text{map\_bte}}, T_{L,n}, \omega_{mech})$ Nm,	(28)

where TL is the torque output of the internal combustion engine, $f^{ICE'}$ is the derivative of the function (28), m_{gas} is the mass flow rate of biogas, LHV_{gas} is the LHV of biogas, ω_{mech} is the speed of the internal combustion engine, $fc_{\text{map\_trq}}$ is the torque range of the internal combustion engine, $fc_{\text{map\_spd}}$ is the speed range of the internal combustion engine and $fc_{\text{map\_bte}}$ is the fuel use map of the internal combustion engine in terms of brake thermal efficiency. ADVISOR sofware specifies the maximum torque for different engine speeds for a given engine rating. To match the power rating of the internal combustion engine to that of the induction machine, the maximum torque specified in the ADVISOR software is changed. It is changed to the torque required to produce the rated power of the induction machine. The ADVISOR software then uses interpolation to redefine the torque scale based on the new maximum torque specified. Torque output is then obtained from interpolation of mass flow rate, LHV, air-fuel ratio of biogas and engine speed, on the redefined torque scale. The torque output is used by the induction machine model to calculate the electricity output.

Exhaust gases are generated as a result of combustion of biogas in the internal combustion engine. The heat from the exhaust gases is captured by a heat exchanger and contributes to the total heat output of the biomass waste to energy conversion technology. The mass flow rate and temperature of the exhaust gases are required to calculate the heat captured by the heat exchanger. The mass flow rate and the temperature of the exhaust gases are calculated by:

$m_{exh} = m_{\text{gas}}(1 + AF)$	kg/s,	(29)
$T_{\text{exh}} = (m_{\text{gas}} LHV_{\text{gas}} - T_L \omega_{mech})/m_{\text{exh}} cp_{\text{exh}} + T_{\text{amb}}$	K,	(30)

where m_{exh} is the mass flow rate of the exhaust gases, m_{gas} is the mass flow rate of the biogas, AF is the air-fuel ratio of the biogas. T_{exh} is the temperature of the exhaust gases, LHV_{gas} is the LHV of the biogas, TL is the output torque, ω_{mech} is the speed of the internal combustion engine cp_{exh} is the specific heat capacity of the exhaust gases and T_{amb} is the ambient temperature.

Induction Machine

The induction machine was modeled in the dq (direct-quadrature) synchronous reference frame [38,39] and was based on the transient model of the induction machine shown in Figure 4. The dq currents i_{sd}, i_{sq}, i_{rd} and i_{rq} are used as state variables and the

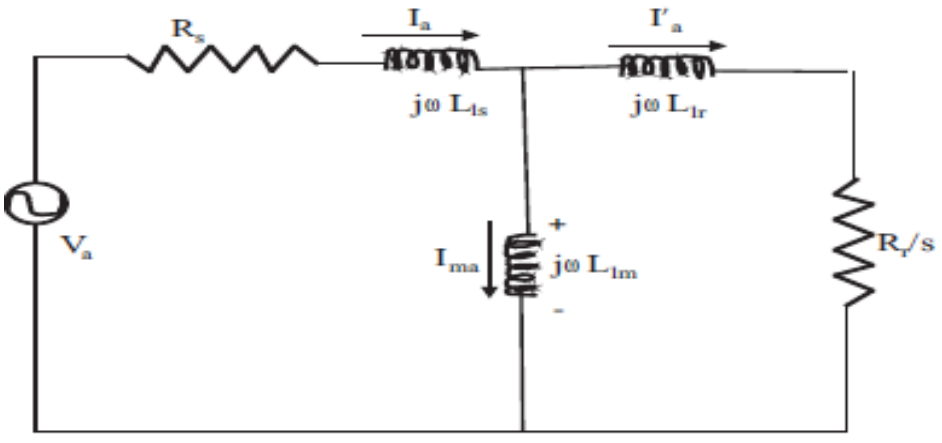

Figure 4: Induction Machine Equivalent Circuit Model

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flux linkages are expressed in terms of these state variables. Power input is calculated by:

$v_{sd} = R_s i_{sd} - \omega_d\left(L_s i_{sq} + L_m i_{rq}\right) + L_m \frac{di_{rd}}{dt} + L_s \frac{di_{sd}}{dt}$	V,	(31)
$v_{sq} = R_s i_{sq} - \omega_d(L_s i_{sd} + L_m i_{rd}) + L_m \frac{di_{rq}}{dt} + L_s \frac{di_{sq}}{dt}$	V,	(32)
$v_{rd} = R_r i_{rd} - \omega_{dA}\left(L_m i_{sq} + L_r i_{rq}\right) + L_m \frac{di_{sd}}{dt} + L_r \frac{di_{rq}}{dt}$	V,	(33)
$v_{rq} = R_r i_{rq} - \omega_{dA}(L_m i_{sd} + L_r i_{rd}) + L_m \frac{di_{sq}}{dt} + L_r \frac{di_{rq}}{dt}$	V,	(34)
$L_s = L_{ls} + L_m$	H,	(35)
$L_r = L_{lr} + L_m$	H,	(36)
$\omega_{dA} = \omega_d - \omega_m$	rad/s,	(37)
$\omega_m = (P/2)\omega_{mech}$	rad/s,	(38)
$T_{em} = (3/2)(P/2)L_m\left(i_{sq} i_{rd} - i_{sd} i_{rq}\right)$	Nm,	(39)
$\frac{d\omega_{mech}}{dt} = (T_{em} - T_L)/J_{eq}$	rad/s^2,	(40)
$P_{mech} = v_{sd} i_{ds} + v_{sq} i_{sq}$	W,	(41)

where v_{sd}, vs_q, v_{rd} and v_{rq} are dq voltages, i_{sd}, i_{sq}, i_{rd} and i_{rq} are dq currents, ω_d is the instantaneous speed of the dq winding, ω_{dA} is the instantaneous speed of the dq winding with respect to the rotor axis, ω_m is the rotor speed, ω_{mech} is the mechanical speed of the induction machine, P is the number of poles of the induction machine, T_{em} is the electromagnetic torque, T_L is the load torque, P_{mech} is the input power of the induction machine, R_s is the stator winding resistance, R_r is the rotor winding resistance, L_{ls} is the stator leakage inductance, L_{lr} is the rotor leakage inductance, L_m is stator magnetizing reactance and J_{eq} is the rotor inertia. The load torque T_L is the torque output of the internal combustion engine.

Heat Exchanger

The exhaust heat captured by the heat exchanger is calculated by [40,41]:

$$Q_{\text{HEX}} = \eta_{\text{HEX}} m_{\text{exh}} cp_{\text{exh}} (T_{\text{exh}} - T_{\text{water}}) \quad \text{W,} \quad (42)$$

where Q_{HEX} is the heat from the heat exchanger, η_{HEX} is the efficiency of the heat exchanger, m_{exh} is the mass flow rate of the exhaust gases, cp_{exh} is the specific heat capacity of the exhaust gases, T_{exh} is the temperature of the exhaust gases and T_{water} is the temperature of the water in the heat exchanger.

Boiler

It is assumed that a dual fuel boiler is used. The heat output of the boiler is obtained by [42]:

$$Q_{\text{boiler}} = \left(m_{\text{propane}} LHV_{\text{propane}} + m_{\text{gas}} LHV_{\text{gas}}\right)\eta_{\text{boiler}} \quad \text{W,} \quad (43)$$

where Q_{boiler} is the heat output of the boiler, m_{propane} is the mass flow rate of propane, LHV_{propane} is the LHV of propane, m_{gas} is the mass flow rate of biogas, LHV_{gas} is the LHV of biogas and η_{boiler} is the efficiency of the

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boiler.

The boiler rating is calculated by:

$$b_r = \max_{dh}(d_h^m) - Q_{\mathrm{HEX}}^m + \delta_{\mathrm{b}} \text{ for } m \in M \qquad \text{W,} \qquad (44)$$

where b_{r} is the boiler rating, d_h^m is the heating demand, Q_{HEX}^m is the heat exchanger output, δ_{b} is an allowance for the boiler rating and M is a set of months comprising the optimisation.

Limitations and Future Research Directions

Limited Scope of Waste Stream Variability: The ADM1 model, as implemented, is calibrated primarily for specific waste types such as dairy manure. This restricts its applicability across a broader range of waste streams (e.g., food waste, agricultural residues, municipal solid waste) without significant recalibration. Future research should focus on expanding the model's flexibility by incorporating machine learning techniques or adaptive parameter estimation methods to generalize it for diverse waste compositions.

Simplified Engine and Generator Modeling: The internal combustion engine model relies on predefined fuel maps and uses interpolation for torque estimation, which may not account for real-time fluctuations in biogas composition or engine wear over time. Additionally, the induction generator model assumes idealized operating conditions. Future work could explore integrating real-time control algorithms and dynamic response models that adjust for gas quality variations and aging effects of mechanical components.

Static Operating Conditions and Absence of Control Technologies: The model assumes steady-state or quasi-steady operation for components like the digester, engine, and heat exchanger, which may not reflect real-world operating scenarios involving disturbances, startup/shutdown cycles, or environmental fluctuations (e.g., temperature). Improving model robustness through the incorporation of dynamic control strategies, fault detection algorithms, and seasonal variability modeling would enhance its predictive accuracy and reliability under varying field conditions.

CONCLUSION

This study presented a comprehensive mathematical modeling framework for a biomass waste-to-energy conversion technology, focusing on the anaerobic digestion of manure using the ADM1 model. The model accurately simulates the biochemical processes involved in anaerobic digestion and integrates key technology components, including the internal combustion engine, induction generator, heat exchanger, and boiler. By linking biogas production to energy outputs such as electricity and heat, the model provides valuable insights into technology performance and energy recovery potential.

The ADM1 model, enhanced for dairy manure applications, serves as a robust foundation for predicting biogas yields based on the chemical and physical characteristics of the waste. Additionally, the integration of engine and generator models facilitates the estimation of mechanical and electrical outputs, contributing to technology design and optimization efforts.

However, the modeling approach assumes steady-state conditions, specific waste inputs, and idealized equipment behavior. Future research should aim to improve model adaptability to different waste types, incorporate dynamic operating conditions, and implement real-time control strategies for enhanced technology reliability and efficiency.

In conclusion, this work lays the groundwork for advanced simulations and optimization of waste-to-energy technology, promoting sustainable energy recovery from organic waste streams.

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