A Bayesian Estimation Procedure for One-Parameter Exponential Survival Distributions Facilitated by an Inverted Gamma Prior

John Effiong Usen*; Emmanuel Emmanuel Asuk; Godswill Itobo Egede

University of Cross River State, Calabar, Cross River State, Nigeria

***Corresponding Author**

DOI: https://doi.org/10.51584/IJRIAS.2025.100500079

Received: 05 May 2025; Accepted: 09 May 2025; Published: 14 June 2025

ABSTRACT

A Bayesian estimation procedure for one-parameter exponential survival distributions facilitated by an inverted gamma prior was performed in this study using data obtained from data obtained from the University of Calabar Teaching Hospital (UCTH). The exponential survival distributions are just one amongst a number of distributions adopted for tackling problems in survival analysis, and may occur either in one parameter or two parameters under uncensored or censored conditions. The review of literature exposed the absence of studies addressing the need for an alternative procedure that carefully considered the peculiarities of the exponential survival distribution; hence, this study aimed to address this gap. Based on this, a Bayesian estimation technique was employed to estimate only the parameter of the one-parameter exponential survival distribution under uncensored and censored circumstances, with the survivorship and hazard functions deduced, thereafter. The results obtained showed that the parameter of the exponential survival distribution (λ) existed both for the one-parameter uncensored and censored cases of the exponential survival distribution, with known expressions. Both the MLE and Bayesian estimation results were simulated using real-life data, and the results showed near-convergence of the MLE and Bayesian estimation for unit values of the parameters of the inverted gamma prior used in the study.

Keywords: Bayesian Estimation Procedure, One-Parameter Exponential Survival Distributions, Inverted Gamma Prior

INTRODUCTION

Background of the study

In industries, the interest of process engineers may often be the estimation of the time available until a machine fails, while the interest of specialists in the clinical sciences may be the estimation of the time available: until a tumor reoccurs, until a cardiovascular death after some treatment intervention, until AIDS overwhelms an HIV infected person, etc. [1; 2]. Without the adoption of appropriate statistical know-how, these professionals may rely solely on intuition or experience for estimating such time [2]. The need to estimate such times non-intuitively, with high level of precision, has birthed the concept of survival analysis in statistics; and this technique has proven to be efficient [1; 2].

The area of survival analysis uses a host of techniques and statistical routines [3; 4]. As a statistical concept, survival analysis estimates the expected time duration until one or more events (such as death in biological organisms, and failure in mechanical systems) happen [5; 6]. Since its initiation in the early parts of 1940s, as a very useful bio-statistical tool, several statistical distributions like: exponential, Weibull, log-normal, gamma, generalized gamma, and log-logistic distributions, have all been adopted for studies on survival analysis, with the exponential survival distribution being the most commonly used distribution out of the lot [14; 15; 16; 17; 18]. More so, all these used distributions, as evidenced in literature, have had their parameters estimated via the general maximum likelihood estimation (MLE) routine, and this has no doubt produced formidable results;

and with the parameter estimates for these distributions, their survival and hazard functions have been obtained [19; 20; 20; 22].

In recent times, to allow for advancements in statistical theory along the direction of exploring alternative estimation routines to the MLE, researchers have begun to attempt the adoption of Bayesian routine for achieving all (and even more) of what the MLE has been used to achieve with the afore-mentioned survival distributions [23; 24; 25; 26]. But one encountered problem, among several, has remained the nature of prior (informative or non-informative), and the suitability of prior (be it the: gamma, inverted gamma, Rayleigh, etc.) to be used [28; 29; 31]. In this study, an informative prior (the inverted gamma distribution) has been adopted, not only for the aim of obtaining Bayesian estimates of the one-parameter exponential distribution, but also for making inference about their survivorship and hazard functions under censored and uncensored circumstances.

Statement of the problem

One of the most commonly used distributions in survival analysis is the exponential survival distribution, be it for the one-parameter case or two-parameter case (with or without covariates). As evidenced in literature, Maximum Likelihood Estimation (MLE) has been used a lot for estimating parameters of one-parameter and two-parameter exponential survival distributions, and for obtaining their survival and hazard functions. And, although the MLE procedure has remained very efficient for achieving this, its usage yet restricts survival analysts to just one choice of estimation procedure. The absence of an attempt, in literature, to proffer the use of the Bayesian alternative to the MLE for achieving the same results, with a suitable informative prior, limits statistical theoretical advancement, going forward. Thus, this study is a foremost attempt in this regard.

Aim and objectives of the study

This study aims to perform a Bayesian estimation for one-parameter exponential survival distributions facilitated by an inverted gamma prior. In line with achieving the stated aim, the objectives of the study are to: (i) review the MLE procedure for obtaining the survivorship and hazard functions under (uncensored and censored conditions), (ii) adopt Bayesian estimation procedure for obtaining the survivorship and hazard functions (under uncensored and censored conditions), and (iii) simulate the MLE and Bayesian alternative on real-life data.

MATERIALS AND METHODS

MLE for data with right-censored observations

Suppose that persons were followed to death or censored in a study. Let $t_1, t_2, \ldots, t_r,\ t_{r+1}^+, t_{r+2}^+, \ldots, t_n^+$ be the survival times observed from the n individuals, with r exact times and $(n-r)$ right-censored times. Assume that the survival times follow a distribution with density function $f(t, \boldsymbol{b})$, and survivorship function $S(t, \boldsymbol{b})$, where $\boldsymbol{b} = (b_1, \ldots, b_p)$ denotes unknown p parameters $b_1, \ldots, b_p$ in the distribution. If the survival time is discrete (that is, it is observed at discrete time only), then $f(t, \boldsymbol{b})$ represents the probability of observing t, and $S(t, \boldsymbol{b})$ represents the probability that the survival or event time is greater than t [7]. In other words, $f(t, \boldsymbol{b})$ and $S(t, \boldsymbol{b})$ represent the information that can be obtained respectively from an observed uncensored survival time and observed right-censored survival time [8]. Therefore, the product $\prod_{i=1}^{n} f(t_i,\ \boldsymbol{b})$ represents the joint probability of observing the uncensored survival times, and $\prod_{i=r+1}^{n} S(t_i^+,\ \boldsymbol{b})$ represents the joint probability of those right-censored survival times. The product of these two probabilities, denoted by $L(\boldsymbol{b})$,

$$L(\boldsymbol{b}) = \prod_{i=1}^{n} f(t_i,\ \boldsymbol{b}) \prod_{i=r+1}^{n} S(t_i^+,\ \boldsymbol{b})$$

represents the joint probability of observing $t_1, t_2, \ldots, t_r, t_{r+1}^+, t_{r+2}^+, \ldots, t_n^+$ [7].

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

A similar interpretation applies to continuous survival $L(\boldsymbol{b})$, called the likelihood function of $\boldsymbol{b}$, which can also be interpreted as a measure of the likelihood of observing a specific set of survival times, namely: $t_1, t_2, \dots, t_r$, $t_{r+1}^+, t_{r+2}^+, \dots, t_n^+$, given a specific set of parameters $\boldsymbol{b}$ [7; 9]. The method of the MLE is to find an estimator of $\boldsymbol{b}$ that maximizes $L(\boldsymbol{b})$ (or which is most likely to have produced the observed data $t_1, t_2, \dots, t_r$, $t_{r+1}^+, t_{r+2}^+, \dots, t_n^+$) [8].

Taking the logarithm of $L(\boldsymbol{b})$, and denoting it by $l(\boldsymbol{b})$, gives:

$$l(\boldsymbol{b}) = log\, L\,(\boldsymbol{b}) = \sum_{i=1}^{r} log[f(t_i, \boldsymbol{b})] + \sum_{i=r+1}^{r} log[S(t_i^+, \boldsymbol{b})] \qquad (1)$$

Then the MLE $\widehat{\boldsymbol{b}}$ is a $\boldsymbol{b}$ in the set of $\hat{b}_1, \hat{b}_2, \dots, \hat{b}_p$ that maximizes $l(\boldsymbol{b})$:

$$l(\widehat{\boldsymbol{b}}) = \max_{allb}(l(\boldsymbol{b}))$$

It is clear that $\widehat{\boldsymbol{b}}$ is a solution of the following simultaneous equations, which are obtained by taking the derivative of $l(\boldsymbol{b})$ with respect to each b_j:

$$\frac{\partial l(\boldsymbol{b})}{\partial b_j} = 0 \qquad j = 1,2,\dots,p \qquad (2)$$

To obtain the MLE $\widehat{\boldsymbol{b}}$, one can use a numerical method. A commonly used numerical method is the Newton-Raphson iterative procedure, which can be summarized as follows [10; 12].

i. Let the initial values $b_1, \dots, b_p$ be zero; that is, let

$$\boldsymbol{b}^{(0)} = 0$$

ii. The changes for $\boldsymbol{b}$ at each subsequent step, denoted by $\Delta^{(j)}$, is obtained by taking the second derivative of the log-likelihood function:

$$\Delta^{(j)} = \left[-\frac{\partial^2 l\left(\boldsymbol{b}^{(j-1)}\right)}{\partial \boldsymbol{b} \partial \boldsymbol{b}'}\right]^{-1} \frac{\partial l\left(\boldsymbol{b}^{(j-1)}\right)}{\partial \boldsymbol{b}} \qquad (3)$$

iii. Using $\Delta^{(j)}$, the value of $\boldsymbol{b}^{(j)}$ at j^{th} step is

$$\boldsymbol{b}^{(j)} = \boldsymbol{b}^{(j-1)} + \Delta^{(j)} \qquad j = 1,2,3,\dots$$

The iteration terminates at, say, the m^{th} step if $\left\|\Delta^{(m)}\right\| < \delta$, where δ is a given precision, usually a very small value, 10^{-4} or 10^{-5} [11]. Then the MLE $\widehat{\boldsymbol{b}}$ is defined as

$$\widehat{\boldsymbol{b}} = \boldsymbol{b}^{(m-1)} \qquad (4)$$

The estimated covariance matrix of the MLE $\widehat{\boldsymbol{b}}$ is given by

$$\overset{\Lambda}{var}(\widehat{\boldsymbol{b}}) = \overset{\Lambda}{cov}(\widehat{\boldsymbol{b}}) = \left[-\frac{\partial^2 l(\widehat{\boldsymbol{b}})}{\partial \boldsymbol{b} \partial \boldsymbol{b}'}\right]^{-1} \qquad (5)$$

One of the good properties of a MLE is that if $\widehat{\boldsymbol{b}}$ is the MLE of $\boldsymbol{b}$, then $g(\widehat{\boldsymbol{b}})$ is the MLE of $g(\boldsymbol{b})$ if $g(\boldsymbol{b})$ is a finite function and need not be one-to-one [10; 30].

The estimated $100(1-\alpha)\%$ confidence interval for any parameter b_i is

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

$$\left(\hat{\boldsymbol{b}}_i - \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\hat{\boldsymbol{b}}_i + \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\right) \tag{6}$$

where v_{ii} is the i^{th} diagonal element of $\hat{V}(\hat{\boldsymbol{b}})$ and $Z_{\alpha/2}$ is the $100(1-\alpha/2)$ percentile point of the standard normal distribution $\left[P(Z > Z_{\alpha/2}) = \alpha/2\right]$. For a finite function $g(b_i)$ of b_i, the estimated $100(1-\alpha)\%$ confidence interval for $g(\boldsymbol{b_i})$ is its respective range R on the confidence interval in equation (6) [10; 30], that is,

$$R = \left\{g(\boldsymbol{b_i}) : \boldsymbol{b_i} \in \left(\hat{\boldsymbol{b}}_i - \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\hat{\boldsymbol{b}}_i + \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\right)\right\} \tag{7}$$

In case $g(b_i)$ is monotone in b_i, the estimated $100(1-\alpha)\%$ confidence interval for $g(b_i)$ is

$$R = \left\{g\left(\hat{\boldsymbol{b}}_i - \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\right),\ g\left(\hat{\boldsymbol{b}}_i + \boldsymbol{Z}_{\alpha/2}\sqrt{\boldsymbol{v}_{ii}}\right)\right\} \tag{8}$$

MLE for data with right-, left-, and interval-censored observations

If the survival times $t_1, t_2, \dots, t_n$ observed for the n persons consist of uncensored left-, right-, and interval-censored observations, then the estimation procedures are similar [10; 30]. Assume that the survival times follow a distribution with density function $f(t, \boldsymbol{b})$ and the survivorship function $S(t, \boldsymbol{b})$, where $\boldsymbol{b}$ denotes all unknown parameters of the distribution. Then the log-likelihood function is given by:

$$\left.\begin{aligned} l(\boldsymbol{b}) = log\,L\,(\boldsymbol{b}) = \sum log[f(t_i, \boldsymbol{b})] + \sum log[S(t_i, \boldsymbol{b})] \\ + \sum log[1 - S(t_i, \boldsymbol{b})] + \sum log[S(v_i, \boldsymbol{b}) - S(t_i, \boldsymbol{b})] \end{aligned}\right\} \tag{9}$$

where the first sum is over the uncensored observations, the second sum is over the right-censored observations, the third sum is over the left-censored observations, and the last sum is over the interval-censored observations, with v_i as the lower end of a censoring interval [10; 30]. The other steps for obtaining the MLE $\hat{\boldsymbol{b}}$ of $\boldsymbol{b}$ are similar to the steps shown in section (3.1) by substituting for the log-likelihood function defined in equation (1) with the log-likelihood function in equation (9).

MLE on the one-parameter exponential survival distribution

The one-parameter exponential distribution has the following function;

$$f(t) = \lambda e^{-\lambda t} \tag{10}$$

survivorship function;

$$S(t) = e^{-\lambda t} \tag{11}$$

and hazard function;

$$h(t) = \lambda \tag{12}$$

where $t \geq 0$, $\lambda > 0$. Obviously, the exponential distribution is characterized by one parameter, λ. The estimation of λ by maximum likelihood methods for data without censored observations will be given first followed by the case with censored observations.

Maximum likelihood estimation of λ for data without censored observations

Suppose that there are n persons in the study and everyone is followed to death or failure. Let $t_1, t_2, t_3, \dots, t_n$ be the exact survival times of the n people. The likelihood function, using (10) and (1), is:

$$L = \prod_{i=1}^{n} \lambda e^{-\lambda t_i}$$

and the log-likelihood function is

$$l(\lambda) = n \, log \, \lambda - \lambda \sum_{i=1}^{n} t_i \quad (13)$$

From equation (2), the MLE of λ is

$$\lambda = \frac{n}{\sum_{t=1}^{n} t_i} \quad (14)$$

Since the mean μ of the exponential distribution is $1/\lambda$ and a MLE is invariant under a one-to-one transformation, the MLE of μ is

$$\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{\sum_{t=1}^{n} t_i}{n} = \bar{t} \quad (15)$$

It can be shown that $2n\hat{\mu}/\mu$ has an exact chi-square distribution with $2n$ degrees of freedom. Since $\lambda = 1/\mu$ and $\hat{\lambda} = 1/\hat{\mu}$ an exact $100(1-\alpha)\%$ confidence interval for λ is

$$\frac{\hat{\lambda}\chi^2_{2n,1-\alpha/2}}{2n} < \lambda < \frac{\hat{\lambda}\chi^2_{2n,\alpha/2}}{2n} \quad (16)$$

where $\chi^2_{2n,\alpha}$ is the 100α percentage point of the chi-square distribution with $2n$ degrees of freedom; that is, $P\left(\chi^2_{2n} > \chi^2_{2n,\alpha}\right) = \alpha$ [10; 12]. When n is large (say, $n \geq 25$), $\hat{\lambda}$ is approximately normally distributed with mean λ and variance λ^2/n [10; 12]. Thus, an approximate $100(1-\alpha)\%$ confidence interval for λ is

$$\hat{\lambda} - \frac{\hat{\lambda} Z_{\alpha/2}}{\sqrt{n}} < \lambda < \hat{\lambda} + \frac{\hat{\lambda} Z_{\alpha/2}}{\sqrt{n}} \quad (17)$$

where $Z_{\alpha/2}$ is the $100\alpha/2$ percentage point, $P\left(Z > Z_{\alpha/2}\right) = \alpha/2$, of the standard normal distribution [10; 12]. Since $2n\hat{\mu}/\mu$ has an exact chi-square distribution with $2n$ degrees of freedom, an exact $100(1-\alpha)\%$ confidence interval for the mean survival time is

$$\frac{2n\hat{\mu}}{\chi^2_{2n,\alpha/2}} < \mu < \frac{2n\hat{\mu}}{\chi^2_{2n,1-\alpha/2}} \quad (18)$$

Maximum likelihood estimation of λ for data with censored observations

We first consider singly censored and then progressively censored data. Suppose that without loss of generality, the study or experiment begins at time zero with a total of n subjects. Survival times are recorded and the data become available when the subjects die one after the other in such a way that the shortest time comes first, the second shortest time comes second, and so on [12; 27]. Suppose that the investigator has decided to terminate the study after r out of the n subjects have and to sacrifice the remaining $n-r$ subjects at that time. Then the survival times for the n subjects are

$$t_{(1)} \leq t_{(2)} \leq t_{(3)} \leq \cdots \leq t_{(r)} = t^+_{(r+1)} = \cdots = t^+_{(n)}$$

where a superscript plus indicates a sacrificed subject, and thus $t^+_{(i)}$ is a censored observation. In this case, n and r are fixed values and all of the $n-r$ censored observations are equal [12; 27].

The likelihood function, using equations (1), (10) and (11), is

$$L = \frac{n!}{(n-r)!} \prod_{i=1}^{n} \lambda e^{-\lambda t_{(i)}} \left(e^{-\lambda t_{(r)}}\right)^{n-r}$$

and from equation (2), the MLE of λ is

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}} \qquad (19)$$

The mean survival time $\mu = 1/\lambda$ can then be estimated by

$$\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}{r} \qquad (20)$$

It has been shown that $2r\lambda/\hat{\lambda}$ has a chi-square distribution with $2r$ degrees of freedom [12; 27]. The mean and variance of $\hat{\lambda}$ are $r\lambda/(r-1)$ and $\lambda^2/(r-1)$ respectively. The $100(1-\alpha)\%$ confidence interval for λ is

$$\frac{\hat{\lambda}\chi^2_{2r,1-\alpha/2}}{2r} < \lambda < \frac{\hat{\lambda}\chi^2_{2r,\alpha/2}}{2r} \qquad (21)$$

When n is large, the distribution of $\hat{\lambda}$ is approximately normal with mean λ and variance $\lambda^2/(r-1)$ [12; 27]. An approximate $100(1-\alpha)\%$ confidence interval for λ is then

$$\hat{\lambda} - \frac{\hat{\lambda} Z_{\alpha/2}}{\sqrt{r-1}} < \lambda < \hat{\lambda} + \frac{\hat{\lambda} Z_{\alpha/2}}{\sqrt{r-1}} \qquad (22)$$

It has also been shown that $2r\hat{\mu}/\mu$ has a chi-square distribution with $\boldsymbol{2r}$ degrees of freedom [12; 27]. Thus, a $100(1-\alpha)\%$ confidence interval for μ is

$$\frac{2r\hat{\mu}}{\chi^2_{2r,\alpha/2}} < \mu < \frac{2r\hat{\mu}}{\chi^2_{2r,1-\alpha/2}} \qquad (23)$$

They also develop test procedures for the hypothesis $H_0{:}\,\mu = \mu_0$ against the alternative $H_1{:}\,\mu < \mu_0$. One of their rules of action is to accept H_0 if $\hat{\mu} > c$ and reject H_0 if $\hat{\mu} < c$, where $c = \left(\mu_0 \chi^2_{2r,\alpha}\right)/2r$ and α is the significance level [12]. Or if the estimated mean survival time calculated from equation (20) is greater than c, the hypothesis H_0 is rejected at the α level of significance [27].

A slightly different situation may arise in laboratory experiments. Instead of terminating the study after the r^{th} death, the experimenter may stop after a period of time T, which may be six months or a year [27]. If we denote the number of deaths between 0 and T as r, the survival data may look as follows:

$$t_{(1)} \le t_{(2)} \le t_{(3)} \le \cdots \le t_{(r)} \le t_{(r+1)}^{+} = \cdots = t_{(n)}^{+} = T$$

Mathematical derivations of the MLE of λ and μ are exactly the same and equation (19) can still be used [27].

Progressively censored data come more frequently from clinical studies where patients are entered at different times and the study lasts a predetermined period of time [13]. Suppose that the study begins at time 0 and terminates at time T and there are a total of n people entered [12; 27]. Let r be the number of patients who die before or at time T and $n-r$ the number of patients who are lost to follow-up during the study period or remain alive at time T. The data look as follows: $t_1, t_2, \ldots, t_r\ t_{r+1}^{+}, t_{r+2}^{+}, \ldots, t_n^{+}$. Ordering the r uncensored observations according to their magnitude, we have:

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

$$t_{(1)} \leq t_{(2)} \leq t_{(3)} \leq \cdots \leq t_r, t_{r+1}^+, t_{r+2}^+, \ldots, t_n^+$$

The likelihood function, using equations (1), (10) and (11), is

$$L = \prod_{i=1}^{n} \lambda e^{-\lambda t_{(i)}} \prod_{i=r+1}^{n} \lambda e^{-\lambda t_i^+}$$

and the log-likelihood function is

$$l(\hat{\lambda}) = n\lambda - \lambda \sum_{i=1}^{r} t_i - \lambda \sum_{i=r+1}^{n} t_i^+ \qquad (24)$$

and from equation (2), the MLE of the parameter λ is

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_i^+} \qquad (25)$$

Consequently,

$$\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_i^+}{r} \qquad (26)$$

is the MLE of the mean survival time [12; 27]. The sum of all of the observations, censored and uncensored, divided by the number of uncensored observations, gives the MLE of the mean survival time [12; 27]. To overcome the mathematical difficulties arising when all of the observations are censored $(r = 0)$. [7] defines

$$\hat{\mu} = \sum_{i=1}^{n} t_i^+ \qquad (27)$$

In practice, this estimate has little value.

The distribution of $\hat{\lambda}$ for large n is approximately normal with mean λ and variance:

$$var(\hat{\lambda}) = \frac{\lambda^2}{\sum_{i=1}^{n}(1 - e^{-\lambda T_i})} \qquad (28)$$

where T_i is the time that the i^{th} person is under observations. In other words, T_i is computed from the time i^{th} person enters the study to the end of the study [12; 27]. If the observations times T_i are not known, the following quick estimate of $var(\hat{\lambda})$ can be used [7].

$$\hat{var}(\hat{\lambda}) = \frac{\hat{\lambda}^2}{r} \qquad (29)$$

Thus, an approximate $100(1 - \lambda)\%$ confidence interval for λ is, by (6),

$$\hat{\lambda} - Z_{\alpha/2}\sqrt{\hat{var}(\hat{\lambda})} < \lambda < \hat{\lambda} + Z_{\alpha/2}\sqrt{\hat{var}(\hat{\lambda})} \qquad (30)$$

The distribution of $\hat{\mu}$ is approximately normal with mean μ and variance:

$$var(\hat{\mu}) = \frac{\mu^2}{\sum_{i=1}^{n}(1 - e^{-\lambda T_i})} \qquad (31)$$

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

Again, a quick estimate is

$$\overset{\Lambda}{var}(\hat{\mu}) = \frac{\hat{\mu}^2}{r} \tag{32}$$

An approximate $100(1-\lambda)\%$ confidence interval for μ is then, by (6),

$$\hat{\mu} - Z_{\alpha/2}\sqrt{\overset{\Lambda}{var}(\hat{\mu})} < \mu < \hat{\mu} + Z_{\alpha/2}\sqrt{\overset{\Lambda}{var}(\hat{\mu})} \tag{33}$$

Informative and non-informative priors

Let $g(\lambda^*)$ be a prior distribution for the parameter λ^* of a distribution. Then, according to [16; 17],

i. The distribution is an informative prior, if it biases the parameter towards particular values (i.e., an informative prior expresses specific, definite information about a variable);

ii. The distribution is a non-informative prior, if it does not influence the posterior hyperparameters (i.e., an uninformative prior, or diffuse prior, expresses unclear or general information about a variable).

The inverted gamma prior

If a random variable λ has the gamma distribution $\lambda \sim G(\alpha, \beta)$, then the random variable $\lambda^* = \frac{1}{\lambda}$ has the inverted gamma distribution $\lambda^* \sim IG(\alpha, \beta)$ with the density function [16; 17].

$$g(\lambda^*|\alpha,\beta) = \begin{cases} \frac{\alpha^\beta}{\Gamma(\beta)}\left(\frac{1}{\lambda^*}\right)^{\beta+1} e^{-\frac{\alpha}{\lambda^*}}; & for\ \alpha > 0;\ \beta > 0;\ 0 < \lambda^* < \infty \\ 0, & for\ the\ remaining\ values\ of\ \lambda^* \end{cases}$$

For $\lambda^* \sim IG(\alpha, \beta)$, we obtain

$$E(\lambda^*) = \frac{\alpha}{\beta - 1};\ \ for\ \beta > 1$$

And,

$$var(\lambda^*) = \frac{\alpha^2}{(\beta-1)^2(\beta-2)};\ \ for\ \beta > 2$$

The Bayesian estimation procedure

Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from the density $f(x;\lambda)$. Before taking the sample, the distribution of λ, $g(\lambda)$ (called a prior distribution), is assumed known. The objective in Bayesian estimation of the parameter λ is to determine the distribution $f(\lambda|x)$ (known as the posterior distribution) after taking the sample.

Let us consider the conditional distribution

$$f(x|\lambda) = \frac{f(x;\lambda)}{g(\lambda)}$$

$$\Rightarrow f(x;\lambda) = f(x|\lambda)g(\lambda) \tag{34}$$

$$\Rightarrow f(\lambda|x) = \frac{f(x;\lambda)}{h(x)} \tag{35}$$

Substituting for equation (34) in equation (35) gives,

$$\Rightarrow f(\lambda|x) = \frac{f(x|\lambda)g(\lambda)}{h(x)} \tag{36}$$

But $\int_{\Omega} f(\lambda|x)\, d\lambda = 1$

Therefore,

$$\int_{\Omega} f(\lambda|x)\, d\lambda = \int_{\Omega} \frac{f(x|\lambda)g(\lambda)}{h(x)} d\lambda = 1$$

$$\Rightarrow 1 = \frac{1}{h(x)} \int_{\Omega} f(x|\lambda)g(\lambda)\, d\lambda$$

$$\Rightarrow h(x) = \int_{\Omega} f(x|\lambda)g(\lambda)\, d\lambda \tag{37}$$

Putting equation (37) into equation (36) gives

$$\Rightarrow f(\lambda|x) = \frac{f(x|\lambda)g(\lambda)}{\int_{\Omega} f(x|\lambda)g(\lambda)d\lambda} \tag{38}$$

Since we are taking a random sample of this distribution

$$f(x|\lambda) = L(x|\lambda) = \prod_{i=1}^{n} f(x_i|\lambda)$$

Hence, equation (38) becomes:

$$\Rightarrow f(\lambda|x) = \frac{L(x|\lambda)g(\lambda)}{\int_{\Omega} L(x|\lambda)g(\lambda)d\lambda} \tag{39}$$

The above equation (39) gives $f(\lambda|x)$ as the posterior Bayes distribution with respect to the prior distribution $g(\lambda)$.

Hence,

$$E[\tau(\lambda)] = \int_{\Omega} \tau(\lambda) f(\lambda|x) d\lambda \tag{40}$$

is called the posterior Bayes estimator with respect to the prior distribution $g(\lambda)$; where $\tau(\lambda)$ is any function of λ.

An outline of the research estimation procedure

The proposed Bayesian alternative will be implemented with the procedure below.

Step 1: Determine an appropriate prior $\pi(\lambda)$.

In this case, the appropriate prior for the one-parameter exponential distribution is the inverted gamma distribution with parameters α and β. That is,

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

$$\pi(\lambda) = \frac{\alpha^{\beta}}{\Gamma(\beta)}\left(\frac{1}{\lambda^{*}}\right)^{\beta+1} e^{-\frac{\alpha}{\lambda^{*}}} = \frac{\alpha^{\beta}}{\Gamma(\beta)}\lambda^{\beta+1}e^{-\lambda\alpha}$$

Step 2: Obtain the Bayesian estimate of λ for data without censored observations.

Here the procedure involved requires that we obtain $\hat{\lambda}$ thus:

$$\hat{\lambda} = E[\lambda|t] = \frac{\int_{-\infty}^{\infty} \lambda l(t|\lambda)\pi(\lambda)d\lambda}{\int_{-\infty}^{\infty} l(t|\lambda)\pi(\lambda)d\lambda}$$

where, $l(t|\lambda)$ denotes the likelihood function. That is,

$$l(t|\lambda) = \lambda^{n} e^{-\lambda\sum_{i=1}^{n} t_{(i)}}$$

Step 3: Obtain the Bayesian estimate of λ for data with censored observations.

Here the procedure involved requires that we obtain $\hat{\lambda}$ thus:

$$\hat{\lambda} = E[\lambda|t] = \frac{\int_{-\infty}^{\infty} \lambda l(t|\lambda)\pi(\lambda)d\lambda}{\int_{-\infty}^{\infty} l(t|\lambda)\pi(\lambda)d\lambda}$$

where, $l(t|\lambda)$ denotes the likelihood function. That is,

$$l(t|\lambda) = \prod_{i=1}^{r} \lambda e^{-\lambda t_{(i)}} \prod_{i=r+1}^{n} \lambda e^{-\lambda t_{(i)}^{+}}$$

RESULTS

Bayesian estimation for the one-parameter uncensored case

Theorem 1

If t is an uncensored exponential random variable with parameter λ, and the prior density of λ is the inverted gamma with parameters α and β, then the estimate of λ provided by the posterior is $\hat{\lambda}$, given by:

$$\hat{\lambda} = \frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}}$$

Proof 1:

Now, $f(t|\lambda) = \lambda e^{-\lambda t}$; $0 < t < \infty$ is the posterior distribution of the parameter λ. Let $l(t|\lambda)$ be the likelihood function. Then,

$$\Rightarrow l(t|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda t_{(i)}}$$

$$\Rightarrow l(t|\lambda) = \lambda^{n} e^{-\lambda\sum_{i=1}^{n} t_{(i)}}$$

Hence,

$$\hat{\lambda} = E[\lambda|t] = \frac{\int_{-\infty}^{\infty} \lambda l(t|\lambda)\pi(\lambda)d\lambda}{\int_{-\infty}^{\infty} l(t|\lambda)\pi(\lambda)d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda\lambda^n e^{-\lambda\sum_{i=1}^n t_{(i)}} \frac{\alpha^\beta}{\Gamma(\beta)} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^n e^{-\lambda\sum_{i=1}^n t_{(i)}} \frac{\alpha^\beta}{\Gamma(\beta)} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\frac{\alpha^\beta}{\Gamma(\beta)} \int_0^{\infty} \lambda\lambda^n \lambda^{\beta+1} e^{-\lambda\sum_{i=1}^n t_{(i)}} e^{-\lambda\alpha} d\lambda}{\frac{\alpha^\beta}{\Gamma(\beta)} \int_0^{\infty} \lambda^n \lambda^{\beta+1} e^{-\lambda\sum_{i=1}^n t_{(i)}} e^{-\lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda^{1+n+\beta+1} e^{-\lambda\sum_{i=1}^n t_{(i)} - \lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^{n+\beta+1} e^{-\lambda\sum_{i=1}^n t_{(i)} - \lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda^{n+\beta+2} e^{-\lambda\sum_{i=1}^n t_{(i)} - \lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^{n+\beta+1} e^{-\lambda\sum_{i=1}^n t_{(i)} - \lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda^{(n+\beta+3)-1} e^{-\lambda\left(\sum_{i=1}^n t_{(i)} + \alpha\right)} d\lambda}{\int_0^{\infty} \lambda^{(n+\beta+2)-1} e^{-\lambda\left(\sum_{i=1}^n t_{(i)} + \alpha\right)} d\lambda}$$

Let $u = \lambda\left(\sum_{i=1}^n t_{(i)} + \alpha\right) \Rightarrow \lambda = \frac{u}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)} \Rightarrow d\lambda = \frac{du}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \left(\frac{u}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}\right)^{(n+\beta+3)-1} e^{-u} \frac{du}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}}{\int_0^{\infty} \left(\frac{u}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}\right)^{(n+\beta+2)-1} e^{-u} \frac{du}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}}$$

$$\Rightarrow \hat{\lambda} = \frac{\left(\frac{1}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}\right)^{n+\beta+3} \int_0^{\infty} u^{(n+\beta+3)-1} e^{-u} du}{\left(\frac{1}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)}\right)^{n+\beta+2} \int_0^{\infty} u^{(n+\beta+2)-1} e^{-u} du}$$

$$\Rightarrow \hat{\lambda} = \frac{1}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)} \frac{\Gamma(n+\beta+3)}{\Gamma(n+\beta+2)}$$

$$\Rightarrow \hat{\lambda} = \frac{1}{\left(\sum_{i=1}^n t_{(i)} + \alpha\right)} \frac{(n+\beta+2)\Gamma(n+\beta+2)}{\Gamma(n+\beta+2)}$$

$$\Rightarrow \hat{\lambda} = \frac{\beta+n+2}{\alpha + \sum_{i=1}^n t_{(i)}}$$

Since the mean μ of the exponential distribution is $\frac{1}{\lambda}$ and Bayesian estimation is invariant under one-to-one transformation, then the Bayesian estimate of μ is given by:

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

$$\hat{\mu} = \bar{t} = \frac{1}{\hat{\lambda}}$$

$$\Rightarrow \hat{\mu} = \frac{1}{\frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}}}$$

$$\Rightarrow \hat{\mu} = \frac{\alpha + \sum_{i=1}^{n} t_{(i)}}{\beta + n + 2}$$

Therefore, the following axiom is established:

Axiom 1:

(a) The estimate of the survivorship function is given as:

$$\hat{S}(t) = e^{-\hat{\lambda} t} = e^{-\left(\frac{\beta+n+2}{\alpha+\sum_{i=1}^{n} t_{(i)}}\right)t}$$

(b) The estimate of the hazard function is given as:

$$\Rightarrow \hat{h}(t) = \hat{\lambda} = \frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}}$$

Bayesian estimation for the one-parameter censored case

Theorem 2:

If t is a censored exponential random variable with parameter λ, and the prior density of λ is the inverted gamma with parameters α and β, then the estimate of λ provided by the posterior is $\hat{\lambda}$, given by:

$$\hat{\lambda} = \frac{\beta + n + 2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$$

Proof 2:

Now, $f(t|\lambda) = \lambda e^{-\lambda t}$; $0 < t < \infty$ is the posterior distribution of the parameter λ. Let $l(t|\lambda)$ be the likelihood function. Then,

$$\Rightarrow l(t|\lambda) = \prod_{i=1}^{r} \lambda e^{-\lambda t_{(i)}} \prod_{i=r+1}^{n} \lambda e^{-\lambda t_{(i)}^{+}}$$

$$\Rightarrow l(t|\lambda) = \lambda^{r} e^{-\lambda \sum_{i=1}^{r} t_{(i)}} \lambda^{n-r} e^{-\lambda \sum_{i=1+r}^{n} t_{(i)}^{+}}$$

$$\Rightarrow l(t|\lambda) = \lambda^{r} \lambda^{n-r} e^{-\lambda \sum_{i=1}^{r} t_{(i)}} e^{-\lambda \sum_{i=1+r}^{n} t_{(i)}^{+}}$$

$$\Rightarrow l(t|\lambda) = \lambda^{r+n-r} e^{-\lambda \sum_{i=1}^{r} t_{(i)} - \lambda \sum_{i=1+r}^{n} t_{(i)}^{+}}$$

$$\Rightarrow l(t|\lambda) = \lambda^{n} e^{-\lambda\left(\sum_{i=1}^{r} t_{(i)} + \sum_{i=1+r}^{n} t_{(i)}^{+}\right)}$$

Hence,

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

$$\hat{\lambda} = E[\lambda|t] = \frac{\int_{-\infty}^{\infty} \lambda l(t|\lambda)\pi(\lambda)d\lambda}{\int_{-\infty}^{\infty} l(t|\lambda)\pi(\lambda)d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda\lambda^n e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} \frac{\alpha^{\beta}}{\Gamma(\beta)} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^n e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} \frac{\alpha^{\beta}}{\Gamma(\beta)} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\frac{\alpha^{\beta}}{\Gamma(\beta)} \int_0^{\infty} \lambda\lambda^n e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}{\frac{\alpha^{\beta}}{\Gamma(\beta)} \int_0^{\infty} \lambda^n e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} \lambda^{\beta+1} e^{-\lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda\lambda^n \lambda^{\beta+1} e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} e^{-\lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^n \lambda^{\beta+1} e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]} e^{-\lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda^{1+n+\beta+1} e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] - \lambda\alpha} d\lambda}{\int_0^{\infty} \lambda^{n+\beta+1} e^{-\lambda\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] - \lambda\alpha} d\lambda}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \lambda^{n+\beta+2} e^{-\lambda\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}} d\lambda}{\int_0^{\infty} \lambda^{n+\beta+1} e^{-\lambda\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}} d\lambda}$$

Let $u = \lambda\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\} \Rightarrow \lambda = \frac{u}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$

$$\Rightarrow d\lambda = \frac{du}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$$

$$\Rightarrow \hat{\lambda} = \frac{\int_0^{\infty} \left(\frac{u}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}\right)^{(n+\beta+3)-1} e^{-u} \frac{du}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}}{\int_0^{\infty} \left(\frac{u}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}\right)^{(n+\beta+2)-1} e^{-u} \frac{du}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}}$$

$$\Rightarrow \hat{\lambda} = \frac{\left(\frac{1}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}\right)^{n+\beta+3} \int_0^{\infty} u^{(n+\beta+3)-1} e^{-u} du}{\left(\frac{1}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}\right)^{n+\beta+2} \int_0^{\infty} u^{(n+\beta+2)-1} e^{-u} du}$$

$$\Rightarrow \hat{\lambda} = \frac{1}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}} \frac{\Gamma(n+\beta+3)}{\Gamma(n+\beta+2)}$$

$$\Rightarrow \hat{\lambda} = \frac{1}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}} \frac{(n+\beta+2)\Gamma(n+\beta+2)}{\Gamma(n+\beta+2)}$$

$$\Rightarrow \hat{\lambda} = \frac{\beta+n+2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$$

Again, since the mean μ of the exponential distribution is $\frac{1}{\lambda}$ and Bayesian estimation is invariant under one-to-one transformation, then the Bayesian estimate of μ is given by:

$$\hat{\mu} = \bar{t} = \frac{1}{\hat{\lambda}}$$

$$\Rightarrow \hat{\mu} = \frac{1}{\frac{\beta + n + 2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}}$$

$$\Rightarrow \hat{\mu} = \frac{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}{\beta + n + 2}$$

Therefore, the following axiom is established:

Axiom 2:

(a) The estimate of the survivorship function is given as:

$$\hat{S}(t) = e^{-\hat{\lambda}t} = e^{-\left(\frac{\beta+n+2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right]+\alpha\right\}}\right)t}$$

(b) The estimate of the hazard function is given as:

$$\Rightarrow \hat{h}(t) = \hat{\lambda} = \frac{\beta + n + 2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$$

Simulation

One-parameter exponential distribution without censored observations

Given the remission times in months for 734 HIV/AIDS patients (Appendix 1). Assuming that remission duration follows the exponential distribution, we can estimate the parameter λ as follows.

Simulating the one-parameter uncensored case, via the MLE, the relapse rate, λ, is

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i} = \frac{734}{97691} \cong 0.008 \text{ per week}$$

The mean remission time μ is then $\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{n}{\sum_{i=1}^{n} t_i} = \bar{t} = \frac{97691}{734} \cong 133.094$ weeks.

Using the analytical procedures given above, confidence intervals for λ and μ can also be obtained.

A 95% confidence interval for the relapse rate λ, following $\frac{\hat{\lambda}\chi^2_{2n,1-\frac{\alpha}{2}}}{2n} < \lambda < \frac{\hat{\lambda}\chi^2_{2n,\frac{\alpha}{2}}}{2n}$ is approximately

$$\frac{(0.008)(74.222)}{1468} < \lambda < \frac{(0.008)(129.561)}{1468} = (0.0004, 0.0007)$$

A 95% confidence interval for the mean remission time following $\frac{2n}{\hat{\lambda}\chi^2_{2n,\frac{\alpha}{2}}} < \mu < \frac{2n}{\hat{\lambda}\chi^2_{2n,1-\frac{\alpha}{2}}}$ is

$$\frac{(1468)(133.094)}{129.561} < \mu < \frac{(1468)(133.094)}{74.222} = (1508.031, 2632.400)$$

Once the parameter λ is estimated, other estimates can be obtained. For example, the probability of staying in remission for at least 20 months, can be estimated from:

$$S(t) = e^{-\lambda t} \Longrightarrow S(20) = e^{-0.008(20)} = 0.852$$

However, simulating the one-parameter uncensored case by Bayesian alternative, we have a Bayesian estimate of the relapse rate, λ, at $\alpha = 1$; $\beta = 1$, gave results consistent with those which were obtained from the uncensored case (via existing method). And this is based on the convergence of the values. That is,

$$\hat{\lambda} = \frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}} = \frac{n+3}{1 + \sum_{i=1}^{n} t_{(i)}} = \frac{737}{1 + 97691} = \frac{737}{97692} \cong 0.008$$

One-parameter exponential distribution with censored observations

A study is carried out on 312 AIDS patients. The study is terminated after half (156) of the AIDS are dead with the other half sacrificed at that time. The survival data of the 312 AIDS patients are shown in Appendix 2. Assuming that the failure of these AIDS patients follows an exponential distribution, the survival rate λ and mean survival time μ via the MLE method are estimated, respectively, according to

$$\hat{\lambda} = \frac{r}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}$$

And

$$\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}{r}$$

By

$$\hat{\lambda} = \frac{156}{8569+15500} = 0.00648 \text{ per week}$$

and $\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{1}{0.00648} = 154.321$ weeks. A 95% confidence interval for λ by $\frac{\hat{\lambda}\chi^2_{2r,1-\frac{\alpha}{2}}}{2r} < \lambda < \frac{\hat{\lambda}\chi^2_{2r,\frac{\alpha}{2}}}{2r}$ is

$$\frac{(0.00648)(74.222)}{(2)(156)} < \lambda < \frac{(0.00648)(129.561)}{(2)(156)} = (0.002, 0.003)$$

A 95% confidence interval for μ following $\hat{\lambda} - \frac{\hat{\lambda} Z_{\frac{\alpha}{2}}}{\sqrt{r-1}} < \lambda < \hat{\lambda} + \frac{\hat{\lambda} Z_{\frac{\alpha}{2}}}{\sqrt{r-1}}$ is

$$\frac{(2)(156)(154.321)}{(129.561)} < \lambda < \frac{(2)(156)(154.321)}{(74.222)} = (371.625, 648.706)$$

The probability of surviving a given time for the patients can be estimated from $S(t) = e^{-\lambda t}$. For example, the probability that a patient exposed to AIDS will survive longer than 122 months is

$$\hat{S}(122) = exp[-0.00648(122)] \cong 0.454$$

The probability of dying in 122 months is then $1 - 0.454 = 0.546$.

However, assuming that the failure of these AIDS patients follows an exponential distribution, the survival rate λ and mean survival time μ via the Bayesian method are estimated, respectively, at $\alpha = 1$; $\beta = 1$ according to

$$\hat{\lambda} = \frac{\beta + n + 2}{\{[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}] + \alpha\}} = \frac{1 + 312 + 2}{\{8569 + 15500 + 1\}} = \frac{315}{24070} \cong 0.013$$

$$\hat{\mu} = \frac{1}{\hat{\lambda}} = \frac{\{[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}] + \alpha\}}{\beta + n + 2} = \frac{24070}{315} \cong 76.4$$

Hence, $\hat{\lambda} = 0.013$ per week and $\hat{\mu} = 76.4$ weeks. A 95% confidence interval for λ by $\frac{\hat{\lambda}\chi^2_{2r,1-\frac{\alpha}{2}}}{2r} < \lambda < \frac{\hat{\lambda}\chi^2_{2r,\frac{\alpha}{2}}}{2r}$ is

$$\frac{(0.013)(74.222)}{(2)(156)} < \lambda < \frac{(0.013)(129.561)}{(2)(156)} = (0.003, 0.005)$$

A 95% confidence interval for μ following $\hat{\lambda} - \frac{\hat{\lambda}z_{\frac{\alpha}{2}}}{\sqrt{r-1}} < \lambda < \hat{\lambda} + \frac{\hat{\lambda}z_{\frac{\alpha}{2}}}{\sqrt{r-1}}$ is

$$\frac{(2)(156)(76.4)}{(129.561)} < \lambda < \frac{(2)(156)(76.4)}{(74.222)} = (183.981, 321.155)$$

Similarly, the probability of surviving a given time for the patients can be estimated from $S(t) = e^{-\lambda t}$. For example, the probability that a patient exposed to AIDS will survive longer than 122 months will now be:

$$\hat{S}(122) = exp[-0.013(122)] \cong 0.205$$

The probability of dying in 122 months is then $1 - 0.205 = 0.795$.

DISCUSSION

The results of this study are summarized in Table 1 and Table 2. These two tables respectively show the parameter estimates of λ using the maximum likelihood estimation and Bayesian estimation procedures under uncensored and censored circumstances. Here, Table 1 and Table 2 show that the parameter estimates of λ exists and are non-zero. Consequently, this implies the existence of the survivorship and hazard functions.

For the one-parameter exponential distribution without censored observations, we note that at $\alpha = 1$; $\beta = 1$, using the Bayesian approach gave results consistent with those which were obtained from the uncensored case through the MLE procedure as confirmed by the convergence of the $\hat{\lambda}$, $\hat{\mu}$, interval and $S(t)$ values.

For the one-parameter exponential distribution with censored observations, we see that at $\alpha = 1$; $\beta = 1$, using the Bayesian approach gave results with significant divergence from those which were obtained from the uncensored case based on the MLE method as confirmed by the divergence of the $\hat{\lambda}$, $\hat{\mu}$, and interval $S(t)$ values.

CONCLUSION

In conclusion, this study has attempted a Bayesian estimation alternative to the MLE for estimating parameters of exponential survival distributions (in one parameter and two parameters) under uncensored and censored scenarios. Two axioms were deduced on what the survivorship and hazard functions are, for each of the estimated parameters. The results obtained from the one-parameter cases showed that known parameter estimates of λ, as well as the survivorship and hazard function, exists, and are similar to the results obtained via the MLE under certain values of the parameters of the prior used (in this case, the inverted gamma distribution).

Table 1 Maximum likelihood estimates for one-parameter exponential survival distribution

ET		Maximum Likelihood		
Sur. Dist.	Case	One parameter		
		$\hat{\lambda}$	$\hat{S}(t)$	$\hat{H}(t)$
$\lambda e^{-\lambda t}$	U	$\frac{n}{\sum_{i=1}^{n} t_{(i)}}$	$e^{-\left(\frac{n}{\sum_{i=1}^{n} t_{(i)}}\right)t}$	$\frac{n}{\sum_{i=1}^{n} t_{(i)}}$
	C	$\frac{n}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}$	$e^{-\left(\frac{n}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}\right)t}$	$\frac{n}{\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}}$

ET = Estimation Technique; Sur. Dist. = Survival Distribution; U = Uncensored; C = Censored

Table 2 Bayesian estimates for one-parameter exponential survival distribution

ET		Bayesian		
Sur. Dist.	Case	One parameter		
		$\hat{\lambda}$	$\hat{S}(t)$	$\hat{H}(t)$
$\lambda e^{-\lambda t}$	U	$\frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}}$	$e^{-\left(\frac{\beta+n+2}{\alpha+\sum_{i=1}^{n} t_{(i)}}\right)t}$	$\frac{\beta + n + 2}{\alpha + \sum_{i=1}^{n} t_{(i)}}$
	C	$\frac{\beta + n + 2}{\left\{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha\right\}}$	$e^{-\left(\frac{\beta+n+2}{\left\{\left[\sum_{i=1}^{r} t_{(i)}+\sum_{i=r+1}^{n} t_{(i)}^{+}\right]+\alpha\right\}}\right)t}$	$\frac{\beta + n + 2}{\left[\sum_{i=1}^{r} t_{(i)} + \sum_{i=r+1}^{n} t_{(i)}^{+}\right] + \alpha}$

ET = Estimation Technique; Sur. Dist. = Survival Distribution; U = Uncensored; C = Censored

REFERENCES

1. Abujarad, M. H., Abujarad, E. S. A. & Khan, A. A. (2019). Bayesian survival analysis of type I general exponential distributions. Annals of Data Science.
2. Aelen, O. O. (1994). Effects of frailty in survival analysis. Statistical Methods in Medical Research, 12(5), 34-56.
3. Ahmed, O. M. (2021). Bayesian estimations of exponential distributions based on interval-censored data with cure fraction. Hindawi Journal of Mathematics, Article ID: 9822870, 1-11.
4. Aliyu, Y. & Usman, U. (2023). On bivariate Nadarajah-Haghighi distribution derived from Farlie-Gumbel-Morgenstern copula in the presence of covariates. Journal of the Nigerian Society of Physical Sciences, 5(2), 871.
5. Al-Kutubi, H. S. & Ibrahim, N. A. (2009). Bayes estimator for exponential distribution with extension of Jeffrey's prior information. Malaysian Journal of Mathematical Sciences, 3(2), 297-313.
6. Amico, M. & Keilegom, I. V. (2018). Cure models in survival analysis. Annual Review of Statistics and its Applications, 5, 311-342.
7. Bartholomew, D. J. (1957). A problem in life testing. Journal of the American Statistical Association, 52, 350-355.
8. Belaghi, R. A., Asl, M. N., Alma, O. G., Singh, S. & Vasfi, M. (2019). Estimation and prediction for the Poisson-exponential distribution based on type-II censored data. American Journal of Mathematical and Management Sciences, 38(1), 96-115.

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

9. Clark, T. G., Bradburn, M. J., Love, S. B. & Altman, D. G. (2003). Survival analysis part 1: Basic concepts and first analyses. British Journal of Cancer, 89, 232-238.
10. Epstein, B. (1960). Estimation of the parameters of two-parameter exponential distribution from censored samples. Technometrics, 2, 403-406.
11. Garcia, V., Marcel-Escobar, M., & Varquez-Polo, F. J. (2020). Generalizing exponential distributions using an extended Marshall-Olkin procedure. MDPI, 12(464), 1-15.
12. Gehan, E. A. (1970). Unpublished notes on survival time studies. The University of Texas, M. D. Anderson Cancer Center, Houston, Texas.
13. George, B. Seals, S. & Aban, I. (2014). Survival analysis and regression models. Journal of Nuclear Cardiology, 21(4), 686-694.
14. Iddrisu, A., Alhassan, A. & Amidu, N. (2019). Survival analysis of birth defect infants and children with Pneumonia mortality in Ghana. Hidawi – Advances in Public Health, 1-7.
15. Lin, J., Sinha, D., Lipsitz, S. & Polpo, A. (2012). Semiparametric Bayesian survival analysis using models with log-linear median. Biometrics, 68(14), 1136-1145.
16. Kaushik, A. (2017). Bayesian estimation of the parameters of exponentiated exponential distribution under progressive interval type-1 censoring scheme with binomial removals. Journal of Statistics Applications and Processes, 6(1), 65-79.
17. Khan, N. & Khan, A. A. (2018). Bayesian analysis of Topp-Leone generalized exponential distribution. Austrian Journal of Statistics, 47, 1-15.
18. Kharazmi, O., Hamedani, G. G. & Cordeiro, G. M. (2023). Log-mean distribution: applications to medical data, survival regression, Bayesian and non-Bayesian discussion with MCMC algorithm. Journal of Applied Statistics, 50(5), 1152-1177.
19. Khumar, D. (2014). Moment generating functions of generalized order statistics from extended type II generalized logistic distribution. Journal of Statistical Theory and Applications, 13(4), 273-288.
20. Kim, J. (2016). Survival analysis. Research and Statistics, 33(4), 172-174.
21. Kishan, R. & Sangal, P. K. (2022). Classical and Bayesian estimation in exponential power distribution under type-1 progressive hybrid censoring with binomial removals. Malaysian Journal of Mathematical Sciences, 16(3), 537-558.
22. Okwuokenye, M. & Peace, K. E. (2015). Size and power of tests of hypotheses on survival parameters from the Lindley distribution with covariates. Austin Biometrics and Biostatistics, 2(4), 1026.
23. Omer, M. E., Abubakar, M. R., Adam, M. B. & Mustafa, S. M. (2020). Cure models with exponentiated Weibull exponential distribution for the analysis of Melanoma patients. MDPI Mathematics, 8, 1-15.
24. Ramadan, A. T., Tolba, A. H. & El-Desouky, B. S. (2021). Generalized power Akshaya distribution and its applications. Open Journal of Modeling and Simulation, 9(4), 323-338.
25. Raqab, M. Z. & Madi, M. T. (2005). Bayesian inference for the generalized exponential distribution. Journal of Statistical Computation and Simulation, 75(10), 841-852.
26. Rashwan, N. I. (2016). A note on Kumaraswamy exponentiated Rayleigh distribution. Journal of Statistical Theory and Applications, 15(3), 286-295.
27. Shim, J. Y. (1997). Discount survival distributions for no covariate case. The Korean Communications in Statistics, 4(2), 491-496.
28. Sundaram, N. (2011). Exponentiated exponential models for survival data. Indian Journal of Science and Technology, 4, 923-930.
29. Yari, G. & Tondpour, Z. (2017). Estimation of the Burr XII-exponential distribution parameters. Applications and Applied Mathematics: An International Journal, 13(1), 47-66.
30. Zelen, M. (1966). Applications of exponential methods to problems in cancer research. Journal of the Royal Statistical Society, Series A, 129, 368-398.
31. Zhou, W. & Zhang, G. (2013). The generalized Kumaraswamy exponential distribution with application in survival analysis. Journal of Zhejiang University, 40(5).

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

APPENDICES

Appendix 1

Uncensored Data for Remission Times of HIV Patients

S/N	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT
1	126	**36**	130	**71**	130	**106**	131	**141**	128	**176**	134	**211**	139	**246**	135
2	130	**37**	135	**72**	130	**107**	135	**142**	133	**177**	134	**212**	132	**247**	137
3	138	**38**	130	**73**	128	**108**	132	**143**	134	**178**	127	**213**	128	**248**	134
4	131	**39**	135	**74**	137	**109**	138	**144**	129	**179**	132	**214**	135	**249**	133
5	139	**40**	135	**75**	135	**110**	138	**145**	134	**180**	130	**215**	127	**250**	129
6	132	**41**	132	**76**	132	**111**	137	**146**	137	**181**	134	**216.**	135	**251**	140
7	127	**42**	139	**77**	127	**112**	135	**147**	134	**182**	138	**217**	133	**252**	132
8	138	**43**	137	**78**	127	**113**	139	**148**	127	**183**	126	**218**	130	**253**	136
9	132	**44**	132	**79**	130	**114**	129	**149**	139	**184**	125	**219**	128	**254**	138
10	130	**45**	138	**80**	129	**115**	131	**150**	134	**185**	134	**220**	131	**255**	128
11	134	**46**	129	**81**	137	**116**	139	**151**	136	**186**	129	**221**	136	**256**	134
12	138	**47**	136	**82**	131	**117**	131	**152**	133	**187**	126	**222**	135	**257**	135
13	127	**48**	130	**83**	130	**118**	129	**153**	131	**188**	131	**223**	135	**258**	139
14	136	**49**	133	**84**	134	**119**	131	**154**	130	**189**	131	**224**	131	**259**	137
15	134	**50**	133	**85**	128	**120**	140	**155**	139	**190**	135	**225**	126	**260**	135
16	130	**51**	133	**86**	139	**121**	126	**156**	137	**191**	135	**226**	128	**261**	132
17	138	**52**	137	**87**	131	**122**	140	**157**	135	**192**	130	**227**	132	**262**	132
18	132	**53**	129	**88**	135	**123**	134	**158**	136	**193**	132	**228**	135	**263**	133
19	130	**54**	135	**89**	127	**124**	133	**159**	133	**194**	133	**229**	127	**264**	136
20	133	**55**	125	**90**	135	**125**	141	**160**	133	**195**	133	**230**	133	**265**	133
21	135	**56**	133	**91**	136	**126**	133	**161**	126	**196**	131	**231**	137	**266**	137
22	136	**57**	136	**92**	132	**127**	134	**162**	131	**197**	132	**232**	133	**267**	136
23	130	**58**	130	**93**	134	**128**	131	**163**	131	**198**	137	**233**	130	**268**	140
24	131	**59**	129	**94**	137	**129**	135	**164**	136	**199**	133	**234**	134	**269**	130

25	138	**60**	132	**95**	130	**130**	129	**165**	138	**200**	131	**235**	129	**270**	138
26	130	**61**	127	**96**	140	**131**	133	**166**	137	**201**	139	**236**	135	**271**	130
27	140	**62**	137	**97**	128	**132**	140	**167**	129	**202**	136	**237**	129	**272**	135
28	137	**63**	140	**98**	128	**133**	134	**168**	139	**203**	137	**238**	132	**273**	125
29	135	**64**	139	**99**	136	**134**	132	**169**	133	**204**	139	**239**	134	**274**	132
30	134	**65**	135	**100**	127	**135**	130	**170**	137	**205**	127	**240**	137	**275**	130
31	131	**66**	133	**101**	133	**136**	139	**171**	131	**206**	134	**241**	132	**276**	133
32	137	**67**	130	**102**	139	**137**	133	**172**	130	**207**	137	**242**	132	**277**	136
33	138	**68**	136	**103**	127	**138**	130	**173**	132	**208**	135	**243**	133	**278**	133
34	126	**69**	128	**104**	133	**139**	137	**174**	137	**209**	133	**244**	140	**279**	127
35	136	**70**	129	**105**	134	**140**	135	**175**	139	**210**	137	**245**	129	**280**	131

S/N = Serial Numbers; RT = Remission Times

Data Obtained from University of Calabar Teaching Hospital (UCTH)

S/N	RT	S/N	RT	S/N	RT	S/N	RT	S/N	RT	S/N	RT	S/N	RT	S/N	RT
281	131	**316**	137	**351**	136	**386**	131	**421**	128	**456**	139	**491**	135	**526**	137
282	134	**317**	132	**352**	127	**387**	134	**422**	134	**457**	130	**492**	139	**527**	131
283	127	**318**	134	**353**	128	**388**	131	**423**	133	**458**	127	**493**	129	**528**	131
284	139	**319**	133	**354**	132	**389**	135	**424**	134	**459**	134	**494**	124	**529**	140
285	126	**320**	133	**355**	138	**390**	131	**425**	135	**460**	136	**495**	139	**530**	137
286	132	**321**	138	**356**	133	**391**	127	**426**	140	**461**	133	**496**	136	**531**	131
287	140	**322**	132	**357**	129	**392**	133	**427**	138	**462**	135	**497**	139	**532**	130
288	135	**323**	136	**358**	129	**393**	126	**428**	131	**463**	135	**498**	141	**533**	133
289	133	**324**	140	**359**	135	**394**	131	**429**	134	**464**	127	**499**	136	**534**	134
290	134	**325**	138	**360**	133	**395**	131	**430**	130	**465**	131	**500**	141	**535**	127
291	139	**326**	131	**361**	131	**396**	135	**431**	132	**466**	139	**501**	131	**536**	130
292	129	**327**	136	**362**	130	**397**	138	**432**	129	**467**	135	**502**	132	**537**	133
293	128	**328**	133	**363**	127	**398**	130	**433**	132	**468**	135	**503**	132	**538**	128
294	137	**329**	138	**364**	133	**399**	131	**434**	132	**469**	129	**504**	130	**539**	129

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

295	135	**330**	137	**365**	131	**400**	131	**435**	130	**470**	139	**505**	127	**540**	132
296	131	**331**	132	**366**	132	**401**	140	**436**	131	**471**	134	**506**	131	**541**	131
297	141	**332**	130	**367**	129	**402**	136	**437**	140	**472**	136	**507**	131	**542**	129
298	130	**333**	127	**368**	128	**403**	129	**438**	130	**473**	140	**508**	133	**543**	133
299	133	**334**	138	**369**	135	**404**	131	**439**	132	**474**	133	**509**	126	**544**	133
300	134	**335**	138	**370**	135	**405**	137	**440**	133	**475**	135	**510**	128	**545**	137
301	127	**336**	130	**371**	135	**406**	135	**441**	132	**476**	128	**511**	135	**546**	135
302	134	**337**	130	**372**	125	**407**	130	**442**	133	**477**	130	**512**	131	**547**	137
303	132	**338**	133	**373**	137	**408**	136	**443**	137	**478**	131	**513**	125	**548**	130
304	124	**339**	141	**374**	134	**409**	138	**444**	125	**479**	133	**514**	134	**549**	135
305	132	**340**	128	**375**	133	**410**	133	**445**	135	**480**	128	**515**	137	**550**	133
306	137	**341**	137	**376**	137	**411**	128	**446**	133	**481**	136	**516**	132	**551**	134
307	136	**342**	132	**377**	136	**412**	129	**447**	135	**482**	129	**517**	129	**552**	130
308	134	**343**	126	**378**	135	**413**	131	**448**	135	**483**	128	**518**	126	**553**	125
309	134	**344**	128	**379**	137	**414**	137	**449**	130	**484**	129	**519**	136	**554**	136
310	135	**345**	136	**380**	133	**415**	131	**450**	135	**485**	137	**520**	132	**555**	134
311	133	**346**	134	**381**	130	**416**	129	**451**	135	**486**	135	**521**	130	**556**	140
312	133	**347**	136	**382**	131	**417**	129	**452**	133	**487**	127	**522**	139	**557**	131
313	135	**348**	128	**383**	133	**418**	132	**453**	125	**488**	131	**523**	131	**558**	132
314	128	**349**	139	**384**	129	**419**	134	**454**	134	**489**	135	**524**	133	**559**	137
315	130	**350**	133	**385**	131	**420**	126	**455**	132	**490**	128	**525**	136	**560**	139

S/N = Serial Numbers; RT = Remission Times

Data Obtained from University of Calabar Teaching Hospital (UCTH)

S/N	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT	**S/N**	RT
561	126	**596**	140	**631**	137	**666**	135	**701**	133	**736**	-	**771**	-	**806**	-
562	130	**597**	134	**632**	137	**667**	136	**702**	132	**737**	-	**772**	-	**807**	-
563	126	**598**	134	**633**	127	**668**	138	**703**	130	**738**	-	**773**	-	**808**	-
564	127	**599**	127	**634**	133	**669**	130	**704**	137	**739**	-	**774**	-	**809**	-

INTERNATIONAL JOURNAL OF RESEARCH AND INNOVATION IN APPLIED SCIENCE (IJRIAS)
ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

565	125	**600**	138	**635**	133	**670**	127	**705**	137	**740**	-	**775**	-	**810**	-
566	135	**601**	135	**636**	131	**671**	136	**706**	129	**741**	-	**776**	-	**811**	-
567	133	**602**	136	**637**	127	**672**	139	**707**	137	**742**	-	**777**	-	**812**	-
568	140	**603**	140	**638**	139	**673**	134	**708**	134	**743**	-	**778**	-	**813**	-
569	136	**604**	129	**639**	133	**674**	134	**709**	137	**744**	-	**779**	-	**814**	-
570	135	**605**	134	**640**	136	**675**	132	**710**	130	**745**	-	**780**	-	**815**	-
571	132	**606**	137	**641**	138	**676**	134	**711**	134	**746**	-	**781**	-	**816**	-
572	135	**607**	130	**642**	134	**677**	136	**712**	127	**747**	-	**782**	-	**817**	-
573	129	**608**	130	**643**	132	**678**	130	**713**	133	**748**	-	**783**	-	**818**	-
574	139	**609**	134	**644**	137	**679**	133	**714**	131	**749**	-	**784**	-	**819**	-
575	128	**610**	133	**645**	132	**680**	138	**715**	128	**750**	-	**785**	-	**820**	-
576	132	**611**	140	**646**	132	**681**	140	**716**	128	**751**	-	**786**	-	**821**	-
577	133	**612**	135	**647**	130	**682**	135	**717**	130	**752**	-	**787**	-	**822**	-
578	130	**613**	140	**648**	139	**683**	139	**718**	129	**753**	-	**788**	-	**823**	-
579	139	**614**	133	**649**	129	**684**	129	**719**	133	**754**	-	**789**	-	**824**	-
580	133	**615**	135	**650**	139	**685**	131	**720**	130	**755**	-	**790**	-	**825**	-
581	130	**616**	136	**651**	134	**686**	126	**721**	133	**756**	-	**791**	-	**826**	-
582	128	**617**	134	**652**	140	**687**	133	**722**	138	**757**	-	**792**	-	**827**	-
583	136	**618**	130	**653**	132	**688**	132	**723**	133	**758**	-	**793**	-	**828**	-
584	137	**619**	138	**654**	133	**689**	130	**724**	137	**759**	-	**794**	-	**829**	-
585	132	**620**	139	**655**	140	**690**	128	**725**	137	**760**	-	**795**	-	**830**	-
586	126	**621**	138	**656**	140	**691**	139	**726**	139	**761**	-	**796**	-	**831**	-
587	135	**622**	139	**657**	137	**692**	137	**727**	133	**762**	-	**797**	-	**832**	-
588	133	**623**	137	**658**	128	**693**	133	**728**	134	**763**	-	**798**	-	**833**	-
589	139	**624**	130	**659**	139	**694**	132	**729**	135	**764**	-	**799**	-	**834**	-
590	139	**625**	136	**660**	130	**695**	135	**730**	140	**765**	-	**800**	-	**835**	-
591	135	**626**	132	**661**	137	**696**	132	**731**	139	**766**	-	**801**	-	**836**	-
592	134	**627**	135	**662**	132	**697**	131	**732**	139	**767**	-	**802**	-	**837**	-

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

593	135	**628**	134	**663**	130	**698**	137	**733**	125	**768**	-	**803**	-	**838**	-
594	136	**629**	131	**664**	130	**699**	129	**734**	129	**769**	-	**804**	-	**839**	-
595	132	**630**	127	**665**	132	**700**	134	**735**	-	**770**	-	**805**	-	**840**	-

S/N = Serial Numbers; RT = Remission Times

Data Obtained from University of Calabar Teaching Hospital (UCTH)

Appendix 2

Censored Data for AIDS Patients

S/N	ST	S/N	ST	S/N	ST	S/N	ST	S/N	ST	S/N	ST	S/N	ST	S/N	ST
1	113	**36**	72	**71**	103	**106**	80	**141**	8	**176**	125+	**211**	125+	**246**	125+
2	5	**37**	125	**72**	71	**107**	6	**142**	34	**177**	125+	**212**	125+	**247**	125+
3	53	**38**	5	**73**	92	**108**	91	**143**	2	**178**	125+	**213**	125+	**248**	125+
4	6	**39**	1	**74**	60	**109**	51	**144**	115	**179**	125+	**214**	125+	**249**	125+
5	14	**40**	85	**75**	65	**110**	66	**145**	73	**180**	125+	**215**	125+	**250**	125+
6	102	**41**	40	**76**	90	**111**	116	**146**	43	**181**	125+	**216**	125+	**251**	125+
7	75	**42**	107	**77**	65	**112**	98	**147**	5	**182**	125+	**217**	125+	**252**	125+
8	3	**43**	5	**78**	82	**113**	81	**148**	42	**183**	125+	**218**	125+	**253**	125+
9	61	**44**	66	**79**	49	**114**	69	**149**	5	**184**	125+	**219**	125+	**254**	125+
10	23	**45**	102	**80**	23	**115**	33	**150**	12	**185**	125+	**220**	125+	**255**	125+
11	39	**46**	45	**81**	87	**116**	60	**151**	4	**186**	125+	**221**	125+	**256**	125+
12	8	**47**	80	**82**	60	**117**	83	**152**	121	**187**	125+	**222**	125+	**257**	125+
13	25	**48**	74	**83**	25	**118**	119	**153**	100	**188**	125+	**223**	125+	**258**	125+
14	1	**49**	57	**84**	3	**119**	27	**154**	75	**189**	125+	**224**	125+	**259**	125+
15	59	**50**	62	**85**	11	**120**	122	**155**	60	**190**	125+	**225**	125+	**260**	125+
16	1	**51**	89	**86**	36	**121**	79	**156**	1	**191**	125+	**226**	125+	**261**	125+
17	80	**52**	109	**87**	84	**122**	0	**157**	125+	**192**	125+	**227**	125+	**262**	125+
18	85	**53**	12	**88**	70	**123**	54	**158**	125+	**193**	125+	**228**	125+	**263**	125+
19	1	**54**	100	**89**	84	**124**	115	**159**	125+	**194**	125+	**229**	125+	**264**	125+
20	10	**55**	104	**90**	5	**125**	119	**160**	125+	**195**	125+	**230**	125+	**265**	125+

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

21	110	**56**	59	**91**	96	**126**	0	**161**	125+	**196**	125+	**231**	125+	**266**	125+
22	55	**57**	101	**92**	54	**127**	16	**162**	125+	**197**	125+	**232**	125+	**267**	125+
23	1	**58**	75	**93**	65	**128**	11	**163**	125+	**198**	125+	**233**	125+	**268**	125+
24	89	**59**	94	**94**	25	**129**	10	**164**	125+	**199**	125+	**234**	125+	**269**	125+
25	33	**60**	102	**95**	46	**130**	16	**165**	125+	**200**	125+	**235**	125+	**270**	125+
26	57	**61**	55	**96**	6	**131**	7	**166**	125+	**201**	125+	**236**	125+	**271**	125+
27	5	**62**	71	**97**	25	**132**	95	**167**	125+	**202**	125+	**237**	125+	**272**	125+
28	106	**63**	105	**98**	10	**133**	19	**168**	125+	**203**	125+	**238**	125+	**273**	125+
29	29	**64**	99	**99**	47	**134**	1	**169**	125+	**204**	125+	**239**	125+	**274**	125+
30	9	**65**	75	**100**	18	**135**	86	**170**	125+	**205**	125+	**240**	125+	**275**	125+
31	46	**66**	6	**101**	83	**136**	5	**171**	125+	**206**	125+	**241**	125+	**276**	125+
32	5	**67**	99	**102**	91	**137**	8	**172**	125+	**207**	125+	**242**	125+	**277**	125+
33	115	**68**	61	**103**	55	**138**	63	**173**	125+	**208**	125+	**243**	125+	**278**	125+
34	51	**69**	28	**104**	72	**139**	83	**174**	125+	**209**	125+	**244**	125+	**279**	125+
35	68	**70**	106	**105**	97	**140**	9	**175**	125+	**210**	125+	**245**	125+	**280**	125+

S/N = Serial Numbers; ST = Survival Times

Data Obtained from University of Calabar Teaching Hospital (UCTH)

S/N	ST	**S/N**	ST	**S/N**	ST	**S/N**	ST	**S/N**	ST	**S/N**	ST	**S/N**	ST	**S/N**	ST
281	125+														
282	125+														
283	125+														
284	125+														
285	125+														
286	125+														
287	125+														
288	125+														
289	125+														
290	125+														

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

291	125+														
292	125+														
293	125+														
294	125+														
295	125+														
296	125+														
297	125+														
298	125+														
299	125+														
300	125+														
301	125+														
302	125+														
303	125+														
304	125+														
305	125+														
306	125+														
307	125+														
308	125+														
309	125+														
310	125+														
311	125+														
312	125+														
313	-														
314	-														
315	-														

S/N = Serial Numbers; ST = Survival Times

Data Obtained from University of Calabar Teaching Hospital (UCTH)