Spatiotemporal Modelling of Atmospheric Pollution: A Computational Approach with Advection-Diffusion Equation

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ABSTRACT

Atmospheric pollution presents a serious environmental and public health challenge globally. Accurate modelling and simulation of pollutant dispersion is essential for effective air quality management. This paper employs the Advection-Diffusion Equation to simulate the spatiotemporal behaviour of pollutant concentrations in the atmosphere. The study incorporates various atmospheric parameters including wind velocity, diffusion coefficients, and source term dynamics. Using both analytical and numerical techniques—specifically finite difference methods—we simulate pollutant spread from a continuous ground-level source. The simulation results reveal that pollutant concentration is initially localized near the emission point, with peak levels exceeding 2.5 units at 100 seconds. Over time, the pollutant plume elongates and disperses predominantly in the wind direction (x-axis), with concentration gradients smoothing out due to diffusion. By 1000 seconds, the plume extends significantly across the domain, showing a marked decrease in peak concentration and an increased spatial spread. These results highlight the critical role of wind advection and atmospheric diffusion in shaping pollutant transport. The modeling approach provides valuable insights for environmental monitoring, public health risk assessment, and urban air quality management strategies.

Keywords: Advection-Diffusion Equation, Atmospheric Pollution, Pollutant Dispersion, Numerical Simulation, Finite Difference Method, Air Quality Modelling, Environmental Modelling, Wind-Driven Transport, Spatiotemporal Analysis, Point Source Emission.

INTRODUCTION

Air pollution has emerged as one of the most pressing global challenges, with significant implications for both human health and environmental sustainability [12,26]. The World Health Organization (WHO) identifies air pollution as a major environmental risk to health, contributing to respiratory diseases, cardiovascular disorders, and premature mortality. Moreover, pollutants such as carbon monoxide (CO), sulfur dioxide (SO_2), nitrogen oxides (NO_x), and particulate matter (PM) not only deteriorate air quality but also exacerbate climate change by interacting with atmospheric processes [16, 20]. With the rapid pace of urbanization, industrial expansion, and increasing vehicular emissions, understanding the mechanisms behind pollutant dispersion in the atmosphere has become an urgent priority. In recent decades, mathematical modelling has played a pivotal role in analysing the transport and fate of atmospheric pollutants [3,27]. These models enable researchers and policymakers to simulate pollutant dynamics under varying meteorological and emission conditions, thereby providing a predictive framework for environmental management and planning. Among the array of modelling techniques, partial differential equations (PDEs) have proven particularly effective due to their capacity to describe complex physical processes with high fidelity. The Advection-Diffusion Equation (ADE) stands out as one of the most widely employed mathematical tools for studying pollutant dispersion. It encapsulates the dual processes of advection, representing the bulk transport of pollutants by wind, and diffusion, which accounts for the spreading of pollutants due to atmospheric turbulence and molecular motion [10,17]. The flexibility and analytical strength of the ADE make it suitable for a range of applications, from local emission analysis to regional-scale air quality forecasting. This paper presents a comprehensive study on the development and simulation of a two-dimensional ADE-based model aimed at characterizing the concentration distribution of atmospheric pollutants originating from a localized emission source [6,19,22]. The model incorporates realistic boundary conditions and wind profiles to reflect actual environmental scenarios. Through numerical simulation, the study seeks to capture the spatial and temporal evolution of pollutant concentrations, offering insights into dispersion patterns under various atmospheric conditions. The outcomes of this research are expected to have practical relevance in several domains, including environmental risk assessment, urban planning, and policy formulation [11,24]. By improving our understanding of pollutant behaviour in the atmosphere, this work contributes to the broader effort of mitigating air pollution and protecting public health and the environment [21,30].

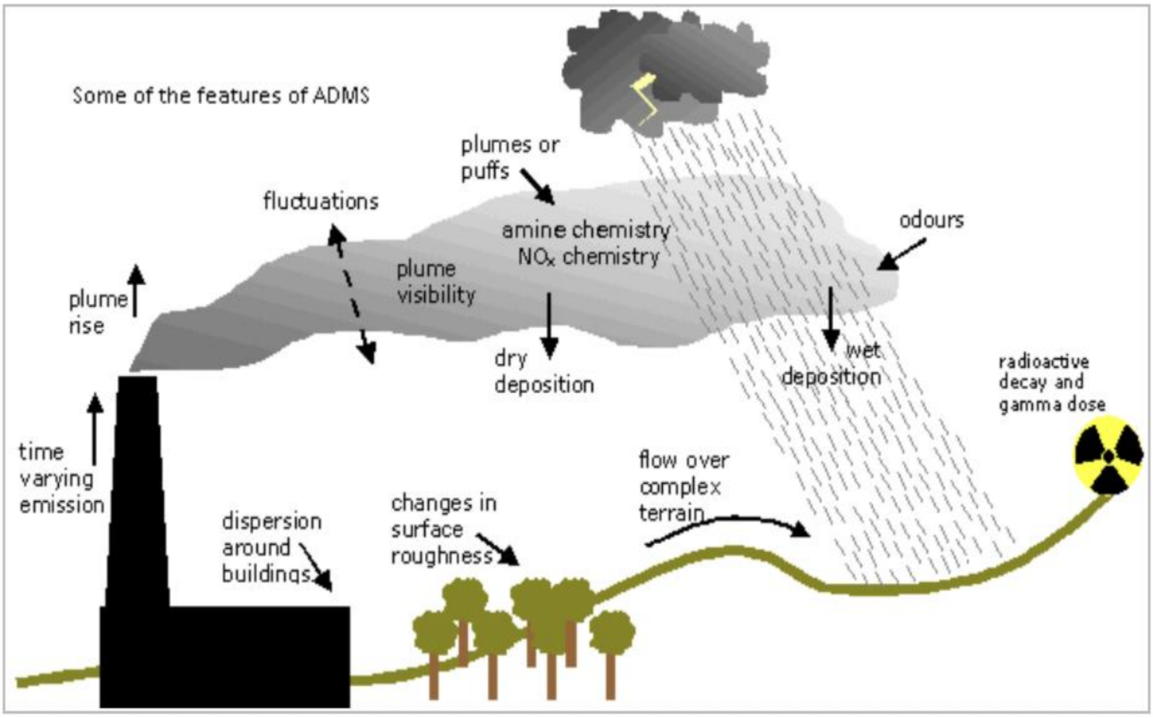

Figure (1): Mechanisms of Atmospheric Pollutant Dispersion from an Industrial Stack

Mathematical Formulation: The general two-dimensional Advection-Diffusion Equation is given by:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_x\frac{\partial^2 C}{\partial x^2} + D_y\frac{\partial^2 C}{\partial y^2} + \mathrm{S(x, y, t)} \qquad (1)$$

where C(x,y,t) represents the concentration pollutants in the atmosphere, $u,$ and v, are the wind velocities in x, and y directions. D_x and D_y represents the diffusion coefficients in x and y dirctions, S(x,y,t) represents the source term and t represents time. The terrain is considered flat and homogeneous, ensuring uniformity in surface characteristics. The pollutant is assumed to be non-reactive and does not undergo chemical transformation or settling, maintaining its concentration throughout the dispersion process [2,7,18,23]. Wind velocity is taken as constant and horizontal, which aids in steady-state modeling and avoids complications from vertical or variable airflow. Furthermore, the emissions originate from a continuous point source located at ground level, providing a consistent and concentrated input of pollutants into the atmosphere.

3. Numerical Solution and Discussion: To simulate the model, we use the finite difference method (FDM) and Matlab to find out the results [5,29,31]. The spatial domain is discretized into a grid with spacing Δx and Δy, and time into steps of Δt. The concentration update at each grid point is computed using an explicit scheme:

$$C_{i,j}^{n+1} = C_{i,j}^{n} - \frac{u\Delta t}{2\Delta x}(C_{i+1,j}^{n} - C_{i-1,j}^{n}) - \frac{v\Delta t}{2\Delta y}(C_{i,j+1}^{n} - C_{i,j-1}^{n}) + \frac{D_x\Delta t}{\Delta x^2}(C_{i+1,j}^{n} - 2C_{i,j}^{n} + C_{i-1,j}^{n}) + \frac{D_y\Delta t}{\Delta y^2}(C_{i,j+1}^{n} - 2C_{i,j}^{n} + C_{i,j-1}^{n} + \Delta \mathrm{t} S_{i,j}^{n} \qquad (2)$$

This scheme ensures stability when the Courant–Friedrichs–Lewy (CFL) condition is satisfied [4,16,21].

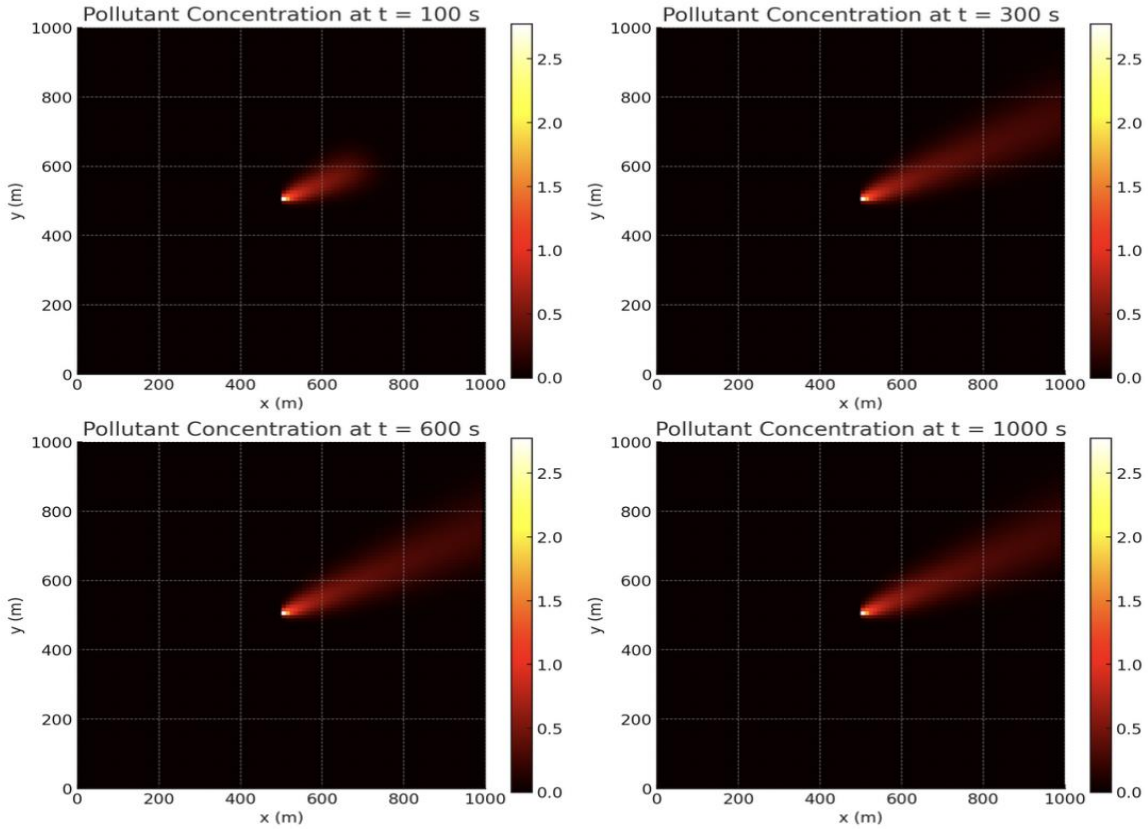

Figure (2): Pollutant Concentration distribution at different time snapshots (100s, 300s, 600s, 1000s)

The spatiotemporal evolution of pollutant concentration due to a continuous point source emission is depicted across four-time instances: 100 seconds, 300 seconds, 600 seconds, and 1000 seconds. The simulation domain covers an area of 1000 m × 1000 m, and the pollutant dispersion is visualized through heatmaps with a common colour bar indicating concentration levels, ranging from 0.0 to over 2.5 units. At t = 100 s, the pollutant plume is highly localized near the source, with peak concentrations exceeding 2.5 units. The dispersion pattern is sharply defined, and the pollutant remains concentrated in a compact region extending slightly along the x-axis, suggesting the initial phase of advection with limited diffusion. By t = 300 s, the plume has extended further along the x-axis and has slightly spread in the y-direction, indicating enhanced dispersion [4,25]. The concentration peak has slightly diminished, and the pollutant begins to exhibit a more defined tail due to the combined effects of advection and diffusion. The leading edge of the plume reaches approximately x = 800 m, with the main body concentrated between x = 500–750 m. At t = 600 s, the pollutant shows significant downstream movement and lateral spreading. The shape of the plume becomes more diffused, and the intensity reduces gradually, indicating dilution over time. A longer and broader tail forms behind the main plume body, suggesting the impact of atmospheric turbulence or diffusion mechanisms that enhance lateral transport. The concentration peak, although still noticeable, starts to decrease more evidently. By t = 1000 s, the plume extends even further in both x and y directions [8,14].

The concentration field demonstrates a more elongated and smeared pattern with further reduction in peak concentration levels. The downstream drift of the plume is most apparent at this stage, with the head of the plume nearing x = 1000 m, while the trailing concentration remains visible close to the source. This indicates that the pollutant continues to spread spatially over time, with noticeable diffusion along both axes, suggesting a mature dispersion regime. Throughout all time instances, the plume displays a dominant directional bias along the x-axis, implying that wind or bulk advection governs the primary transport direction. The relatively symmetric widening of the plume along the y-axis over time suggests isotropic diffusion in the lateral direction [9,15]. The location of the source, near x = 500 m and y = 500 m, serves as the origin for this consistent spread pattern. The colour gradient, ranging from dark red to yellowish-white, effectively captures the declining concentration values over time and distance from the source. The white-to-yellow regions mark the high-concentration zones, which gradually transition into red and black, denoting lower concentrations and areas not significantly affected by the pollutant. The progressive decline in peak concentration values and the increasing spatial spread reflect the natural diffusion and advection behaviour of pollutants in an open environment. In practical scenarios, such dispersion dynamics are critical in assessing the impact of pollutant sources on surrounding ecosystems and human health. The data illustrates the importance of early-time intervention near the source, as the highest concentration levels and potential harm occur shortly after emission [1,13,28]. Furthermore, the increasing reach of the pollutant over time emphasizes the need for predictive models in environmental monitoring and control. The results are consistent with solutions of the advection-diffusion equation in two dimensions, reinforcing the role of wind velocity, diffusion coefficients, and source strength in shaping pollutant transport profiles.

The results of this study provide critical insights that can inform effective environmental policy-making. Given the significant role of wind and diffusion in spreading pollutants over time and distance, it is essential for urban planning authorities to enforce zoning regulations that ensure adequate separation between industrial emission sources and vulnerable areas such as residential neighbourhoods, schools, and hospitals. Urban layouts should be designed with prevailing wind patterns in mind to enhance natural ventilation and minimize pollutant accumulation. Moreover, the incorporation of real-time air quality monitoring systems across cities can support dynamic forecasting and timely public advisories during high pollution events. Policymakers should mandate the use of advanced dispersion models—such as the one applied in this study—in environmental impact assessments (EIAs) for new infrastructure and industrial projects. Emission thresholds should be reviewed and adjusted periodically using localized simulation data to ensure they reflect actual dispersion potential under varying atmospheric conditions. In addition, investment in green buffers and vegetative corridors should be prioritized, particularly in high-risk zones, as they can serve as natural barriers against the spread of pollutants. Finally, public health preparedness strategies must include education campaigns and early warning systems to alert communities about potential exposure risks, enabling them to take preventive measures during critical time windows. Collectively, these policy directions can greatly enhance urban air quality management and protect public health in rapidly growing urban environments.

CONCLUSION

This study successfully modelled the spatiotemporal dispersion of atmospheric pollutants using the Advection-Diffusion Equation, incorporating key environmental parameters such as wind velocity, diffusion coefficients, and a continuous ground-level source. Through numerical simulation using finite difference methods, we captured the dynamic evolution of pollutant concentration over time. The results clearly indicate that pollutant dispersion is strongly influenced by advection in the direction of prevailing wind, while diffusion governs the lateral spread and dilution of the plume. Initially, the pollutant remains highly concentrated near the source; however, as time progresses, the plume elongates and diffuses over a broader area, with noticeable decreases in peak concentration levels. By 1000 seconds, the plume covers a substantial region, underscoring the potential for widespread exposure if emissions are not effectively managed. These findings emphasize the importance of incorporating both advection and diffusion processes in atmospheric pollution models for realistic predictions. The model provides a valuable framework for urban planners, environmental agencies, and public health authorities to assess the impact of pollutant sources, optimize monitoring strategies, and implement timely mitigation measures.

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