INTERNATIONAL JOURNAL OF RESEARCH AND INNOVATION IN APPLIED SCIENCE (IJRIAS)
ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

Tangible Mathematics in Grade 7 Classrooms: Effects on Learner Understanding, Problem-Solving Skills, and Engagement in Rational Number Operations

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DOI: https://doi.org/10.51584/IJRIAS.2025.1005000118

Received: 05 June 2025; Accepted: 07 June 2025; Published: 21 June 2025

ABSTRACT

This study investigated the effects of tangible mathematics instruction on grade 7 learners' engagement, understanding, and problem-solving skills in rational number operations. Using a mixed-methods approach with a quasi-experimental design, the research involved 109 grade 7 learners (58 in the experimental group using tangible mathematics manipulatives and 51 in the control group receiving conventional instruction) over an eight-week intervention period. Quantitative analysis revealed significantly higher post-test scores in the experimental group (M = 64.17, SD = 27.22) compared to the control group (M = 33.55, SD = 12.85), with a large effect size (r = 0.844, 95% CI [0.781, 0.907]). The Number Needed to Treat of 2.37 indicated that for approximately every 2-3 learners receiving the intervention, one additional learner achieved mastery compared to conventional instruction. Qualitative analysis demonstrated that learners using tangible mathematics developed more visual representation strategies and stronger conceptual understanding, particularly in bridging concrete and abstract mathematical concepts. Learner engagement metrics showed substantial improvements, with active participation increasing from 3.12 to 3.84 and confidence levels rising from 2.86 to 3.59. Thematic analysis of learner feedback revealed enhanced understanding, increased enjoyment, appreciation for visual and physical learning supports, and improved mathematical connections. The findings suggest that tangible mathematics instruction effectively enhances learners' mathematical understanding and problem-solving abilities while significantly improving engagement and confidence, offering valuable implications for mathematics instruction in grade 7 classrooms.

Keywords: tangible mathematics, manipulatives, rational numbers, mathematics education, engagement, problem-solving, mathematical understanding, grade 7, mixed-methods research

INTRODUCTION

Mathematics education, particularly in the domain of rational numbers, presents significant challenges for grade 7 learners transitioning from concrete to abstract mathematical thinking. The gap between conceptual understanding and procedural fluency in mathematics has been well-documented in educational research, with rational number operations consistently identified as an area of difficulty. Traditional instructional approaches often emphasize procedural knowledge without adequately developing the conceptual foundations necessary for deep mathematical understanding. This research addresses this challenge by investigating the effectiveness of tangible mathematics instruction—an approach that incorporates physical manipulatives and visual representations—in enhancing grade 7 learners' engagement, understanding, and problem-solving abilities in rational number operations.

The significance of this research lies in its potential to bridge the gap between concrete and abstract mathematical thinking at a critical developmental stage. Grade 7 represents a pivotal transition point where learners are expected to move from basic arithmetic to more complex algebraic thinking. Challenges in understanding rational numbers at this stage can create persistent difficulties in higher-level mathematics. By examining how tangible approaches influence learner outcomes, this study contributes valuable insights into effective instructional strategies for this crucial mathematical domain.

The study aims to address two primary research questions: (1) What is the effect of tangible mathematics instruction on Grade 7 learners' mathematical understanding and problem-solving skills? and (2) How does tangible mathematics instruction influence grade 7 learners' engagement in mathematics learning? Through a comprehensive mixed-methods investigation, this research seeks to provide empirical evidence for the effectiveness of tangible mathematics approaches while offering practical guidance for classroom implementation.

LITERATURE REVIEW

Research on mathematical understanding and problem-solving provides crucial insights for mathematics instruction. Skemp's (1976) foundational work on relational versus instrumental understanding established the critical distinction between knowing "how" and knowing "why" in mathematics. This distinction is particularly relevant when considering approaches to teaching rational numbers, where procedural knowledge often overshadows conceptual understanding.

The development of algebraic thinking represents a crucial dimension of mathematical understanding that bridges numerical operations and abstract reasoning. Kriegler (2008) provides a comprehensive framework for algebraic thinking, identifying two key strands: the development of mathematical thinking tools, including problem-solving skills, representation abilities, and reasoning capabilities; and the study of fundamental algebraic ideas, including generalization, linearity, equality, variable, function, and modeling. This framework offers valuable guidance for developing instructional approaches that systematically build algebraic reasoning through carefully structured learning progressions.

Research on rational number development provides important insights for bridging number and algebraic understanding. Siegler et al. (2011) proposed an integrated theory of whole number and fractions development that highlights how understanding rational numbers requires extending rather than replacing whole number knowledge. Their theoretical framework explains many difficulties students encounter with fractions and provides guidance for instructional approaches that support conceptual integration rather than compartmentalization.

Recent empirical studies have further illuminated the relationship between conceptual understanding and mathematical performance. Andamon and Tan (2018) conducted a comprehensive investigation of Grade 7 learners' conceptual understanding, attitudes, and performance in mathematics. Their research revealed significant correlations between conceptual understanding and overall mathematical achievement, demonstrating that learners with stronger grasp of fundamental concepts demonstrated greater success in applying those concepts to novel situations.

A substantial body of research has examined the effects of manipulative use on mathematical understanding and achievement. Domino (2010) conducted a comprehensive meta-analysis on the effects of physical manipulatives on achievement in mathematics in grades K-6, finding consistent positive effects across diverse educational contexts. Similarly, Hawkins (2007) documented significant achievement gains associated with systematic manipulative use, particularly for concepts requiring spatial reasoning and abstract representation.

Tangible approaches to developing mathematical understanding have received significant attention. Scarlatos (2006) explored the concept of "tangible math," demonstrating how physical manipulation of mathematical objects supports the development of abstract concepts through concrete experiences. Their research documents how learners develop more robust mental models through hands-on interaction with mathematical principles. Kelly (2006) extended this understanding through performance-based analysis of manipulative use in mathematical problem-solving, revealing that strategic use of manipulatives enhanced learners' problem-solving capabilities by providing external representations that supported working memory and facilitated solution strategy development.

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

The integration of digital and physical approaches has emerged as a promising direction. Fokides and Alatzas (2023) investigated the use of digitally enhanced tangible materials for teaching fractions, demonstrating significant improvements in learner comprehension and application of fractional concepts compared to traditional approaches. Similarly, Pires et al. (2019) examined how building blocks of mathematical learning differ between virtual and tangible manipulatives, revealing that these different approaches led to distinct strategies in number composition.

METHODOLOGY

This study employed a mixed-methods approach with a quasi-experimental design to investigate the effects of tangible mathematics instruction on grade 7 learners' understanding, problem-solving skills, and engagement. The research design included both quantitative measures to assess learning outcomes and qualitative methods to explore learner experiences and problem-solving approaches.

Research Design

The implementation phase focused on evaluating the effectiveness of tangible mathematics materials through a quasi-experimental design. The developed manipulatives, "Tangible Mathematics," consist of two main component types: fraction-based manipulatives and operational tiles, each designed with specific features to support mathematical learning. The fraction-based manipulatives include three distinct forms: fraction circles, fraction bars, and fraction tiles. The fraction circles were manufactured with a consistent 5-inch diameter, while both the fraction bars and fraction tiles were produced with dimensions of 18.25 inches in length and 1.5 inches in width. Each of these manipulatives incorporates comprehensive fractional divisions, ranging from one whole to twelfths, allowing learners to explore and understand various fractional relationships.

Using a pre-test/post-test control group approach, this phase examined how tangible mathematics instruction affected learner learning outcomes. The implementation involved 109 grade 7 learners, with 58 learners in the experimental group receiving the tangible mathematics intervention and 51 learners in the control group continuing with conventional instruction. This eight-week intervention period provided sufficient time to observe and measure the effects of tangible mathematics on learner engagement, understanding, and problem-solving skills.

Participants

The study participants shown in Table 1 included 109 grade 7 learners from the regular basic curriculum through cluster sampling using intact classes. The participants were divided into an experimental group of 58 learners and a control group of 51 learners. Initial academic performance was assessed through first quarter examination scores to establish baseline comparability between groups. The implementation spanned one academic quarter, following a structured timeline from October to December 2024.

Table 1 Distribution of Participants By Group

Participants	Control Group	Experimental Group	Total
Male	19	34	53
Female	32	24	56
Total	51 (47%)	58 (53%)	109 (100%)

Research Instruments

A comprehensive set of instruments was employed to collect both quantitative and qualitative data. Quantitative measures included pre-tests and post-tests adapted from established mathematical assessment instruments appropriate for grade 7 learners, problem-solving skills evaluations, and engagement questionnaires. Qualitative tools included structured classroom observation checklists, learner learning reflections, detailed teacher observation notes, semi-structured interview guides, and focus group discussion protocols. Weekly lesson plans served as structured guides for implementing the tangible mathematics intervention, ensuring consistent implementation while maintaining instructional fidelity.

Data Collection and Analysis

Data collection followed a systematic approach through multiple instruments. Quantitative data included pre-tests and post-tests to measure mathematical understanding, along with regular assessments of problem-solving skills and engagement levels. Qualitative data gathering involved systematic classroom observations, collection of learners' learning reflections, and maintenance of detailed teacher observation notes.

The quantitative analysis focused on comparing experimental and control groups through statistical procedures. Pre-test and post-test comparisons were conducted using the Mann-Whitney U Test, with effect size calculations to determine the practical significance of observed differences. The Wilcoxon Signed-Rank Test was employed to analyze engagement measures, tracking changes in learner participation and interest over time.

Qualitative analysis included systematic thematic analysis of classroom observations and content analysis of learner reflections, providing rich insights into how learners interact with and respond to tangible mathematics approaches. These analyses revealed patterns in learner engagement, understanding development, and problem-solving strategies. The integration of mixed methods brought together quantitative and qualitative findings through careful triangulation, enabling a comprehensive assessment of the intervention's effectiveness.

Ethical Considerations

This study adhered to strict ethical standards throughout all phases of research implementation. Prior to beginning the intervention, comprehensive informed consent was obtained from all participants and their guardians, with clear explanations of the nature of the experimental and control group conditions. The research design ensured equal educational opportunity by planning to provide control group learners access to tangible mathematics materials after the study conclusion, ensuring no learners would be disadvantaged in their educational experience.

Data privacy and protection measures were implemented with rigorous attention to confidentiality. All test results, observations, and participant feedback were securely stored with restricted access limited to the research team. Personal identifiers were removed from data during analysis, and pseudonyms were used in reporting to protect participant identities. Regular monitoring of learner well-being occurred throughout the intervention period to ensure that any potential negative effects could be promptly addressed. Additionally, the research received formal approval from the relevant institutional ethics committee prior to commencement.

The study design was developed with careful consideration of minimizing disruption to regular educational activities. Implementation was integrated into the standard curriculum schedule to avoid interference with learners' regular academic progression. Care was taken to avoid creating conditions that might place any participant at an educational disadvantage. This ethical framework prioritized participant welfare while maintaining research integrity, ensuring that the study contributed to educational knowledge without compromising learner learning opportunities or well-being.

RESULTS

The implementation of tangible mathematics yielded comprehensive positive outcomes that transformed multiple dimensions of mathematical learning. Both quantitative metrics and learners' qualitative responses converge to demonstrate the intervention's effectiveness across different aspects of mathematical engagement and understanding.

Quantitative Results

Analysis of pre-test and post-test performance, as shown in Table 2, revealed distinct patterns of improvement between the groups, with the experimental group showing notably larger gains despite starting from a lower baseline. The control group started with a slightly higher mean score (M = 7.61, SD = 4.10) compared to the experimental group (M = 4.45, SD = 3.07). After the intervention, the control group improved to a mean score of 33.55 (SD = 12.85), while the experimental group achieved a significantly higher mean score of 64.17 (SD = 27.22).

Table 2 Summary of Pretest-Posttest Performance among the Groups

Measure	Control Group	Experimental Group
Pretest Mean (SD)	7.61 (4.10)	4.45 (3.07)
Posttest Mean (SD)	33.55 (12.85)	64.17 (27.22)
Mean Gain (SD)	25.94 (10.26)	59.72 (26.51)
Range Pretest	0-20	0-12
Range Posttest	22-71	20-94
N	51	58

The between-group analysis using the Mann-Whitney U test, as shown in Table 3, revealed significant differences at both testing points. At pretest, there was a significant difference (U = 901, z = -3.51, $p < .001$) with a medium effect size (r = 0.336). The posttest comparison yielded even more pronounced differences (U = 721, z = -4.60, $p < .001$) with a medium to large effect size (r = 0.441). This increase in effect size from pretest to posttest indicates that the gap between groups widened during the intervention period, favoring the experimental group.

Table 3 Results of Between-Group Comparisons Using Mann-Whitney U Test

Test Stage	U-statistic	z-score	p-value	Effect Size (r)	Interpretation
Pretest	901	-3.51	< .001	0.336	Medium Effect
Posttest	721	-4.60	< .001	0.441	Medium to Large Effect

Effect size interpretations: small (0.1-0.3), medium (0.3-0.5), large (>0.5)

The within-group analysis using the Wilcoxon Signed-Rank Test, as shown in Table 4, revealed significant improvements across both groups. The control group demonstrated significant improvement (W = 1277, z = 5.76, $p < .001$) with a large effect size (r = 0.806). Similarly, the experimental group showed significant improvement (W = 1680, z = 6.38, $p < .001$) with an even larger effect size (r = 0.838).

Table 4 Results of Within-Group Analysis Using Wilcoxon Signed-Rank Test

Group	W-statistic	z-score	p-value	Effect Size (r)	Interpretation
Control Group	1277	5.76	< .001	0.806	Large Effect
Experimental Group	1680	6.38	< .001	0.838	Large Effect

Effect size interpretations: small (0.1-0.3), medium (0.3-0.5), large (>0.5)

Following the significant results of the Mann-Whitney U test (U = 29, z = -8.77, $p < .001$), post-hoc power analysis was conducted, as shown in Table 5. With the observed effect size (r = 0.844), $\alpha = .05$, and N = 109, the achieved power ($1 - \beta$) was .999, indicating extremely high statistical power. The 95% confidence interval for the effect size was calculated as [0.781, 0.907], demonstrating the robustness of the intervention's impact.

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

Table 5 Effect Size Analysis

Measure	Value	95% CI Lower	95% CI Upper
Effect Size (r)	0.844	0.781	0.907
Achieved Power	0.999	-	-
Number Needed to Treat	2.37	2.20	2.54

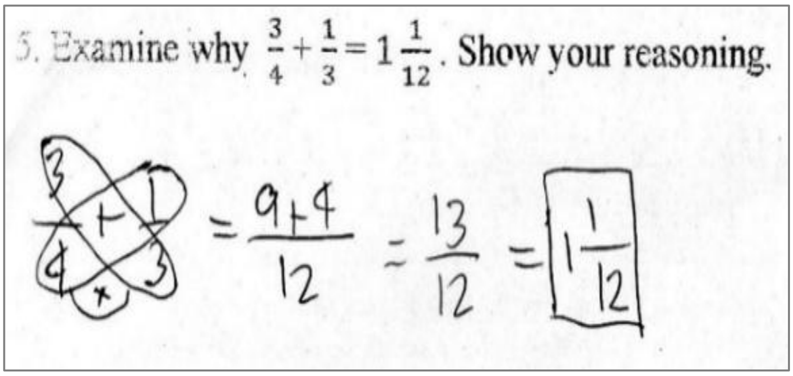
The Number Needed to Treat (NNT) of 2.37 [CI: 2.20, 2.54] provides a particularly practical interpretation of the intervention's effectiveness. This means that for approximately every two to three learners who receive the tangible mathematics intervention, one additional learner achieves mastery compared to conventional instruction.

Qualitative Results

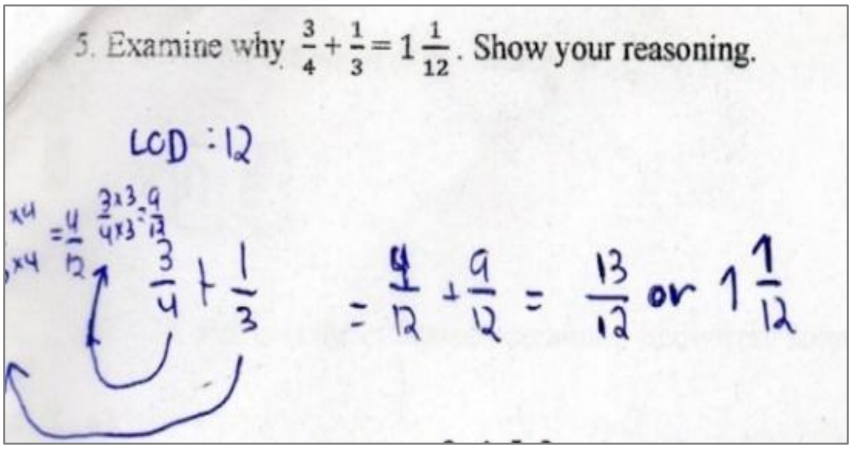
The qualitative analysis of learner work samples revealed significant differences in problem-solving approaches between conventional and tangible mathematics approaches. While control group learners predominantly relied on computational approaches, experimental group learners developed sophisticated visual representation strategies that demonstrated deeper conceptual understanding.

Exhibit 1 shows the responses of learners, CPoL2 and CPoL7, in the control group.

CPoL2



CPoL7

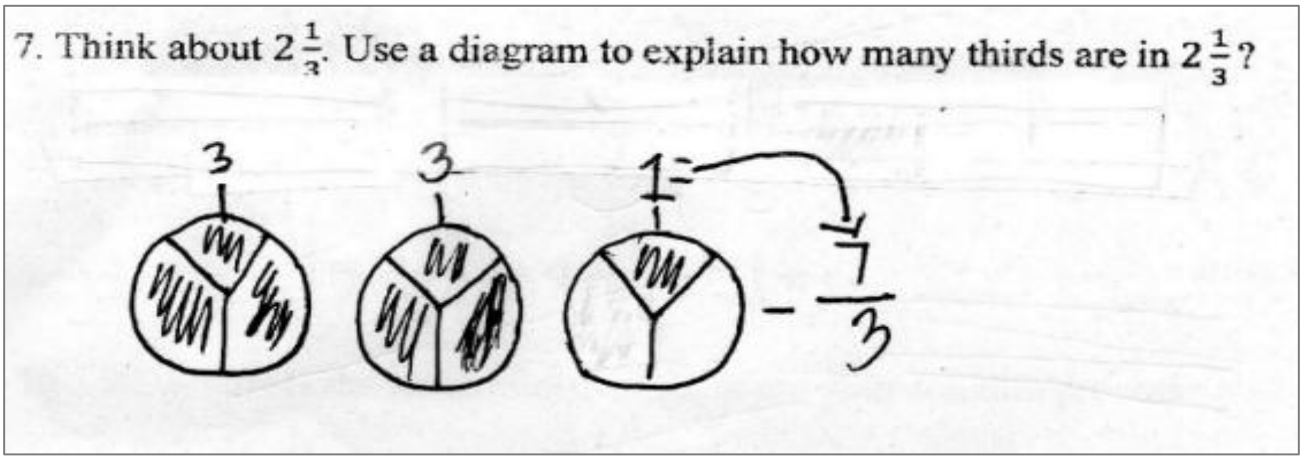


Learners in the experimental group consistently demonstrated stronger conceptual understanding through their use of visual representations, systematic approaches to problem-solving, and clear reasoning about mathematical relationships. Their solutions showed strong connections between visual and symbolic representations, suggesting that the tangible mathematics approach supported the development of more robust mathematical understanding.

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

Exhibit 2 shows the responses of learners, EPoL3 and EPoL42, in the experimental group.

EPoL3



EPoL42

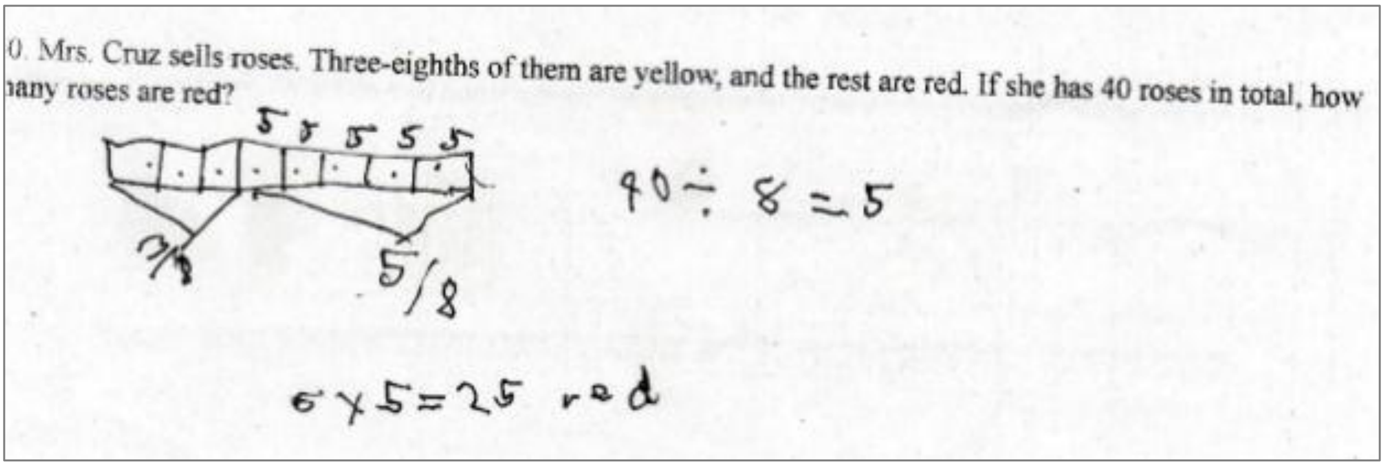
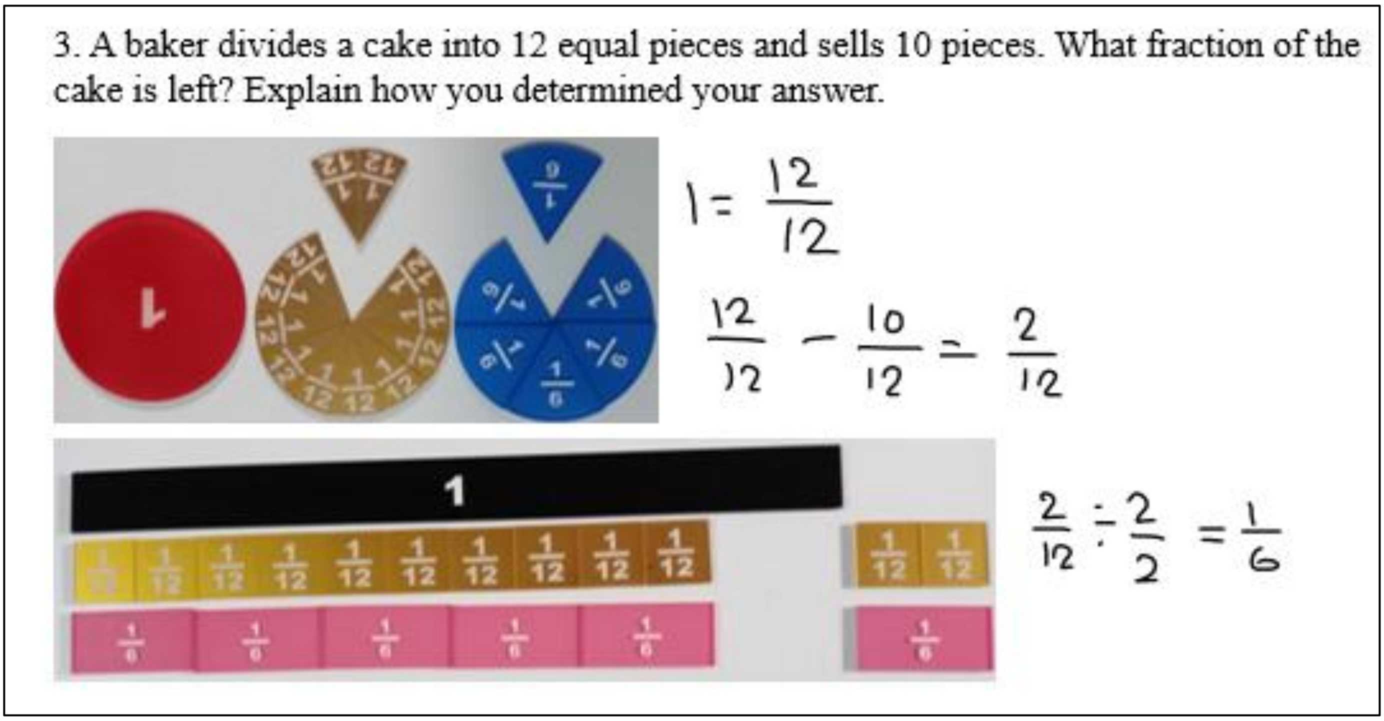


Exhibit 3 shows the sample Tangible Mathematics activities conducted in the classroom.



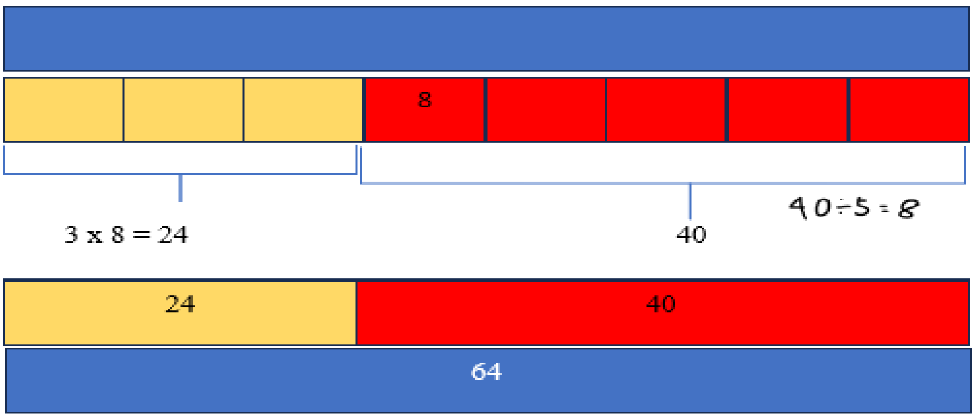
2. John Paul has a strip of paper that is $\frac{8}{10}$ meter long. If he wants to cut it into pieces that are $\frac{1}{5}$ meter each. How many complete 1/5 pieces can he make? Will there be any paper left over? Explain your answer.

$$\frac{8}{10} \div \frac{1}{5} = \frac{8}{10} \times \frac{5}{1}$$

$$= \frac{40}{10}$$

$$= 4$$

6. Mrs. Rosa sells roses. Three-eighths of them are yellow, and the rest are red. If she has 40 red roses, how many roses are there in total?



Algebraically,

Method 1.

Let x be the total number of roses.

$\frac{3}{8}x$ – yellow roses

40 – red roses

$$\frac{3}{8}x + 40 = x$$

$$40 = x - \frac{3}{8}x$$

$$40 = \frac{5}{8}x$$

$$320 = 5x$$

$$64 = x$$

Method 2.

Let x be the total number of roses.

$\frac{3}{8}x$ – yellow roses

$$x = \frac{8x}{8}$$

$$\frac{8x}{8} - \frac{3x}{8} = \frac{5x}{8}$$

$\frac{5x}{8}$ = 40 red roses

$$\frac{5x}{8} = 40$$

$$5x = 320$$

$$x = 64$$

These activities collectively illustrate the concrete-pictorial-abstract learning progression, moving learners from hands-on manipulation through visual representation to symbolic mathematical thinking. The implementation demonstrates how tangible materials can effectively scaffold complex mathematical concepts,

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

supporting learners' transition from physical understanding to abstract mathematical reasoning while maintaining engagement through multi-sensory learning experiences.

The thematic analysis of learner responses in Table 6 revealed six distinct but interconnected themes that illuminate how the manipulatives enhanced mathematical learning: Enhanced Understanding and Learning, Enjoyment and Engagement, Physical and Visual Appeal, Challenge and Motivation, Mathematical Connections, and Collaborative Learning. These themes provide crucial insights into the mechanisms behind the quantitative improvements observed in learner engagement and understanding.

Table 6 Thematic Analysis of Learner Responses

Major Themes	Sub-themes	Supporting Quotes
Enhanced Understanding and Learning	Faster Comprehension	"I like fraction circles and bars because it helps me understand quickly" "Because I understand and directly think into my brain"
	Ease of Learning	"It helps me understand fraction easier" "I really like them because they are easy to understand"
	Learning Support	"It makes me understand the lesson easily" "I like it the most because it can help you get more knowledge"
Enjoyment and Engagement	Fun in Learning	"It's fun when we play using fraction circles and bars" "Understand more and makes learning fun"
	Excitement and Happiness	"Can make me happy and excited during the activities" "I really like with it comes on activities, for me it's very happy"
	Personal Preference	"I like it because it is my favorite subject and fraction is my favorite lesson" "I like it because it is my favorite subject and my favorite algebra"
Physical and Visual Appeal	Visual Features	"The shape and color" "I like the colors"
	Hands-on Experience	"I like it when we arrange it" "All the tasks and activities we do during math time"
	Ease of Use	"Because it's easy to use, and it's not confusing" "How it makes things easier"
Challenge and Motivation	Positive Challenge	"It is challenging to everyone; it is nice and joyful" "Having a time limit while answering (No pressure though)"
	Competitive Element	"I like it being competitive" "What I like best about using these is that it makes math more interesting and gives a bit me of pressure"
Mathematical Connections	Conceptual Understanding	"I really like it because hands-on activities help me see connections between numerals and algebra" "When it's time to answer least to greatest fractions"
	Practical Application	"Fun and help me how to count and make it into half" "Because I understand and think on it"
Collaborative Learning	Teamwork	"I like it because it is fun and teamwork"
	Social Interaction	"All the tasks and activities we do during math time, using fraction bars and circles are the best"

Learner feedback themes further validated these results, with 53.3% expressing strong appreciation for visual and hands-on learning approaches, and 73.3% reporting high levels of engagement and motivation, as shown in Table 7. Learners also consistently reported enhanced understanding, improved learning efficiency, and positive experiences with the collaborative learning environment.

ISSN No. 2454-6194 | DOI: 10.51584/IJRIAS |Volume X Issue V May 2025

Table 7 Thematic Analysis of Learner Feedback With Frequency Distribution

Theme	Frequency (%)	Key Manifestations in Problem-Solving
Enhanced Learning Experience	13.3%	Improved conceptual understanding Better problem-solving strategies Increased learning efficiency
Engagement and Motivation	73.3%	Higher participation in problem-solving Increased willingness to tackle challenges Sustained effort in complex problems
Physical and Visual Properties	53.3%	Enhanced visualization of problems Better representation of mathematical concepts Improved spatial reasoning
Positive Learning Environment	13.3%	Collaborative problem-solving Supportive learning atmosphere Active engagement in mathematical discourse

Learner Engagement

The implementation of tangible mathematics catalyzed significant improvements across multiple dimensions of learner engagement, as revealed in Table 8. Active participation showed a remarkable increase from 3.12 to 3.84, with the percentage of learners "Always" participating rising substantially from 50% to 86.2%. This dramatic improvement demonstrates a significant enhancement in overall classroom engagement.

Table 8 Pre-Post Comparison of Learner Engagement

Parameters	Pre-Implementation Mean	Post-Implementation Mean	Mean Difference
A. Participation			
Active participation	3.12	3.84	+0.72
Following directions	3.86	3.86	0.00
Asking questions	3.28	3.64	+0.36
Working with classmates	3.41	3.78	+0.37
Completing tasks	3.29	3.66	+0.37
B. Feelings			
Enjoying Lessons	3.34	3.71	+0.37
Excitement	3.38	3.74	+0.36
Confidence	2.86	3.59	+0.73
Interest	2.84	3.57	+0.73
Pride	3.36	3.72	+0.36
C. Thinking			
Understanding math	3.26	3.74	+0.48
Connecting to past lessons	3.31	3.76	+0.45
Explaining answers	3.17	3.59	+0.42
Finding multiple solutions	3.24	3.71	+0.47
Applying to new situations	3.19	3.62	+0.43
D. Reflections			
Understanding operations	3.43	3.83	+0.40
Grasping concepts	3.21	3.64	+0.43
Visualizing expressions	3.29	3.67	+0.38
Explaining mathematical ideas	3.24	3.66	+0.42
Seeing connections	3.28	3.67	+0.39

Learner confidence, previously identified as an area of concern, showed one of the study's most significant transformations. The confidence metric rose markedly from 2.86 to 3.59, with the proportion of learners reporting they are "Always" confident more than doubling from 27.6% to 63.8%. Perhaps most tellingly, the percentage of learners who were only "Sometimes confident" plummeted from 37.9% to a mere 1.7%, suggesting a fundamental shift in mathematical self-efficacy.

Mathematical interest demonstrated parallel growth, with a 0.73-point increase from 2.84 to 3.57. The percentage of learners "Always" interested more than doubled, rising from 32.8% to 69.0%. Concurrent with this increase, the proportion reporting only occasional interest fell dramatically from 41.4% to 5.2%, indicating deeper and more consistent engagement with mathematical content.

The analysis of classroom observation feedback revealed six significant themes that characterized the implementation of tangible mathematics instruction: Strategic Preparation and Organization, Structured Instructional Delivery, Pedagogical Effectiveness, Collaborative Learning Environment, Learner Engagement and Confidence, and Inquiry-Based Learning. These themes demonstrated a comprehensive and well-structured implementation of tangible mathematics instruction, with strong attention to both preparation and execution, while maintaining consistent focus on learner engagement and understanding.

DISCUSSION

The findings of this study provide significant insights into the effectiveness of tangible mathematics instruction for grade 7 learners, with implications that extend beyond immediate academic performance to broader educational practice. This section examines the key findings in relation to existing literature, theoretical frameworks, and practical applications.

Bridging Concrete and Abstract Understanding

One of the most significant findings is how tangible mathematics facilitated the transition from concrete to abstract mathematical thinking. The experimental group's superior performance in transferring understanding to algebraic contexts aligns with Siegler et al.'s (2011) integrated theory of whole number and fractions development, which emphasizes the importance of extending rather than replacing existing knowledge. The qualitative analysis revealed that learners using manipulatives demonstrated a deeper conceptual understanding of fraction relationships, supporting Skemp's (1976) distinction between relational and instrumental understanding. This finding is particularly important given that Grade 7 represents a critical transition point where learners must begin to engage with more abstract mathematical concepts.

The experimental group's use of visual representations to solve problems demonstrates how tangible approaches can scaffold the development of mathematical thinking, consistent with Kelly's (2006) research showing that manipulatives enhance problem-solving by providing external representations that support working memory. This successful bridging of concrete and abstract understanding addresses one of mathematics education's most persistent challenges, offering a practical pathway for supporting learners through this critical transition.

Enhanced Engagement

The remarkable improvements in learner engagement metrics align with research by Moyer (2001) and Carbonneau et al. (2013) on the motivational impact of hands-on approaches. The substantial increase in confidence levels (from 2.86 to 3.59) is particularly noteworthy as it suggests a transformation in learners' mathematical identity. As Singh et al. (2002) noted, confidence in mathematics is strongly associated with achievement and persistence. The study's findings suggest that tangible mathematics may help disrupt negative cycles of low confidence and disengagement that often characterize mathematics learning.

The thematic analysis of learner feedback revealed that the approach created an environment where mathematical challenge became engaging rather than intimidating. This unexpected finding contradicts common assumptions about learners avoiding mathematical difficulty and suggests that appropriate concrete supports can transform learners' relationship with challenging mathematical content. This aligns with Schoenfeld's (2016) research on problem-solving development, which emphasizes the importance of creating supportive environments for mathematical risk-taking.

Pedagogical Implications

The implementation analysis provides valuable insights into effective pedagogical practices. The structured progression from concrete to abstract understanding, with systematic support at each stage, proved critical to the intervention's success. This aligns with Fyfe et al.'s (2014) research on knowledge transfer principles, emphasizing the importance of carefully designed instructional sequences that support the transition from concrete to abstract understanding.

The classroom observation data revealed that teacher implementation practices significantly influenced outcomes, with Strategic Preparation and Organization emerging as a key theme. This finding resonates with research by Puchner et al. (2008), highlighting how teacher preparation and systematic implementation approaches are crucial for maximizing the effectiveness of manipulative-based instruction.

The study also demonstrated the importance of collaborative learning environments, with peer interaction enhancing the effectiveness of tangible approaches. This aligns with Vygotsky's (1978) social constructivist perspective, suggesting that mathematical understanding develops through social interaction as well as individual cognitive processes. The improvement in learners' ability to explain their mathematical thinking (from 3.17 to 3.59) indicates enhanced mathematical communication, an essential skill for deeper conceptual understanding.

Future Directions

The study focused specifically on rational number operations; the effectiveness of tangible approaches for other mathematical domains requires further investigation. Future research might explore how tangible mathematics instruction can support learning across different mathematical topics and how it might be integrated into comprehensive mathematics curricula.

The dramatic effect size observed ($r = 0.844$) exceeds those typically reported in educational interventions, suggesting the potential for tangible mathematics to significantly impact mathematics education practice. Further research is needed to identify the specific mechanisms through which tangible approaches enhance understanding and how these might be optimized for different learner populations and contexts.

Integration with Digital Technologies

An area deserving further exploration is the potential integration of physical manipulatives with digital technologies. As noted by Fokides and Alatzas (2023), digitally enhanced tangible materials show promise for teaching fractions. The current study focused primarily on physical manipulatives, but future research might investigate how hybrid approaches combining physical and digital representations could further enhance mathematical understanding and engagement. This integration could potentially address scalability challenges while maintaining the benefits of concrete, tactile learning experiences.

CONCLUSION

This study provides compelling evidence for the effectiveness of tangible mathematics instruction in enhancing grade 7 learners' understanding, problem-solving skills, and engagement in rational number operations. The dramatic reversal of the initial performance gap between groups challenges conventional assumptions about persistent disadvantages and demonstrates how effectively designed tangible mathematics instruction can transform learning outcomes. The exceptionally large effect size ($r = 0.844$) documented in this study surpasses typical mathematics interventions, providing compelling evidence for the potency of this approach in addressing challenging mathematical concepts.

Tangible mathematics instruction significantly enhances learner understanding of rational numbers, as demonstrated by the substantial difference in performance between experimental and control groups. The integration of visual and tactile learning experiences effectively addresses fundamental gaps in mathematical understanding, helping learners build stronger connections between concrete and abstract mathematical concepts.

Learner engagement and confidence in mathematics significantly improve through tangible mathematics instruction, as evidenced by the marked increases in participation, confidence, and interest levels. This indicates that hands-on approaches create a more positive and engaging learning environment, which is crucial for sustained mathematical development.

Proper development and implementation of manipulative materials are crucial for their effectiveness, as shown by the systematic development process, expert validation, and teacher feedback in this study. This highlights that carefully designed and validated materials are essential for successful mathematics instruction.

The findings suggest that integrating tangible mathematics approaches in grade 7 classrooms can transform both cognitive and affective dimensions of mathematics learning, particularly in the challenging domain of rational numbers. Future research might explore the long-term impact of tangible mathematics instruction on learners' mathematical development and investigate the effectiveness of this approach across different mathematical domains and educational levels.

ACKNOWLEDGEMENT

This research was supported by the Department of Science and Technology – Science Education Institute, the Philippine Science High School -- Central Mindanao Campus and the Mindanao State University -- Iligan Institute of Technology. The researchers thank the school administrators, teachers, and learners who participated in this study for their cooperation and valuable contributions.

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