# Parameter Comparisons on Multivariate Time Series Analysis; a Case Study of GDP Growth Rate in Nigeria

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***Abstract*** **:In this work, multivariate Time Series "vector Autoregressive Model" was applied to analyze the Gross Domestics Product (GDP) growth rate of five (5) sectors namely: Agriculture, Industries, Building/Construction, Wholesales/Retails, and Services. The data was collected from the National Bureau of Statistics, range quarterly from 1985 to 2017, a total of 33years. Real (R) software was adopted to analyze the model. The data were grouped into 10 pairs of 2 parameter variables, 10 combinations of 3 parameters variables, 5 combinations of 4 parameters variables, and the complete 5 parameter variables. In each group, the best model was selected and Lag's using Akaike Information Criteria, in the group of 2 parameters variables the pairs of Building/Construction and Services was selected as the best model with the smallest of AIC (-11.8996) at lag (5). Building/ Construction, Wholesales/Retails, and Services pairs were selected, as the best model in the group of 3 parameters variables with an AIC value of (-17.0510). In the group of 4 parameters variables, the combination of Agriculture, Building/Construction, Wholesales/Retails, and Services was selected as the best model with an AIC value of (-22.0743) as the smallest at lag (5). In complete parameters variables, Agriculture, Industries, Building/Construction, Wholesales/Retails, and Services, the best lag was selected as lag (5) with the smallest AIC value of (-25.8996). In comparison, the best model of 2 parameters with an AIC value of (-11.973) at lag (5) was compared with that of the complete 5 parameters variables with an AIC value of (-25.8996), the results showed that the larger the parameters the better the model.**

***Keyword:*** **Agriculture, Industries, Building/Construction, Wholesale/Retail, Services, GDP, and Parameter**.

## I. INTRODUCTION

Over the years, Nigeria's economy has not been stable and, as a result, the country has faced some economic problems, threats, or shocks that have been internal and external for centuries. Internally, the outcome of expenditure and consumption trends, as well as insufficient public policy replacement and shifts in perceptions. The crises, as externally, may be the result of population growth, revolution or war, etc. Every country's economic growth reflects its capacity to increase the production of services, and goods. It specifically describes an increase in a country's Gross Domestic Product (GDP). Gross Domestic Product (GDP) is the market value of all services and goods produced in a country within one year. GDP growth rate is also referred to as an icon because it is a measure or an indicator of economic growth. A similar measure to Gross Domestic Product (GDP) is the Gross National Product (GNP) which account for the market value of total output in a country produced by its citizen within a year. The major difference is that Gross Domestic products include all the services and goods produced in a system irrespective of the country or nationality of the people producing it. The Gross Domestic Product (GDP) can be measured using three processes: Expenditure, Income, and Consumption. Nigeria as an emerging market with middle income and a mixed economy is rated as the 26th in the world in terms of Gross Domestic products and the largest economy in Africa (April 2014 bulletin). Nigeria is expected to have the highest average Gross Domestic Product in the system between 2010 and 2050. Nigeria's Economy is trying to maximize its crude oil and other resources to raise the standard of living of about 45% of its population who are living in poverty. Agriculture contributed 63% to Nigeria's growth rate in the year 1960 and decline to 34% in 1988. This fall is due to the discovery and exploration of oil which started in the early sixties which causes attention to leave Agriculture to the oil sector.

## II. REVIEWS OF RELATED WORK

Stock investors have lost their savings several times as a result of weak market research. Over the years, researchers have studied potential methods that could help stock investors profitably control their investments. So this part provides an overview of the area of multivariate time series, univariate time series, and Gross Domestic Product of Economics. In describing the current market or growth rate of Gross domestics Product (GDP), Robert Engle, a finance professor at New York University stated, "We have no idea where things are going". This simply means high volatility [8]. [5] Used multivariate time series analysis to look at the relationship between inflation and job rates and Gross domestics Product (GDP). The multivariate time series analysis was performed using STATA software, and the results showed that the inflation rate does not affect Gross domestics Product (GDP), while the job rate has a negative relationship with Gross domestics Product (GDP). Granger causality was also used to assess the causality between the variables in the sample. In the short term, all independent variables have a unidirectional relationship with GDP, according to their results. [1] Studied the effects of interest rates, inflation rates, and GDP on Jordan's real economic growth rate. To verify the integration order of the variables, the unit root test was employed. The findings showed that, on the other hand, inflation induces interest rates; all the other variables are independent of each other. Regression analysis was also used to assess the relationship between growth rate and interest rate, showing that the current interest rate affects growth rate. Finally, the result also showed that the current GDP is changing. [9] Used a multivariate vector autoregressive model with impulse response function and other experiments to figure out how oil price shocks affect investment. The results showed that shocks in oil prices have a marginal effect on real GDP. [2] Investigated the effect of unemployment and inflation on Nigerian economic development. In checking the unit root property of the sequence, the Augmented Dickey-fuller technique was used. The Granger Casualty Trigger Test between GDP, unemployment, and inflation also suggested that all variables are stationary in the model. The outcome also showed that unemployment and inflation had a beneficial effect on economic development. The investigator also suggested that economic growth is not impaired by unemployment. [3] Performed a study using Vector autoregressive regression for 131 countries as well. The result indicates that, relative to developing countries, higher crude oil prices were more extreme for the poorer oil-importing countries. His research also revealed that an S10 rise in the price of crude oil could reduce economic growth by up to 4%. Decisions in financial investment most times depend seriously on the relationship over time among several similar time series. For instance, international currencies exchange rate series. Their contemporaneous variations play an important role in a decision relating to the spread of risk in investments [5]. Multivariate time series models are very important in modeling and identifying the joint structure on which these decisions depend. The aspect of principal component analyses provides good ideas insight into the multivariate structure and also a simple guide to model choice and assessment. Practically, an elaboration of this basic model is to incorporate time-variation in covariance matrices, [10]. Multivariate analysis is suitably applied in making such a forecast. These techniques have benefited from big improvements concerning technique of use, [7]. Checking for nonlinearity in multivariate time series was investigated by [13]. The researcher considered a multivariate extension of the test suggested by [11] to neglect nonlinearity, which used main components to address the dimensionality issue that is prevalent with the test. The results have shown, however, that the adjusted multivariate test makes a substantial difference by reducing the size without some kind of systemic level or loss of knowledge.

## III. METHODOLOGY

*Multivariate Time Series*

Let a K-dimensional vector of time series $U_t = (U_{1t}, U_{2t}, U_{3t}, \ldots, U_{kt})$.

A multivariate model is the (kx1) vector ($U_t$) where the $i^{th}$ row of ($U_t$) is ($U_{it}$). This implies that for any time t, $U_t = (U_{1t}, U_{2t}, U_{3t}, \ldots U_{kt})$.

From this definition above, when we want to model and provide a description of the interaction and counter interaction between groups of time series variables, a multivariate model is employed. These interactions are expected in the variable of interest in this work. So it is quite clear, that multivariate modeling deals with various considerations of different series.

*Linearity of multivariate Time Series $U_t$*

Truly speaking multivariate model is nonlinear; moreover linear series can often give an accurate approximation for making a decision.

$$U_t = \mu + \sum_{i>0}^{\infty} \psi a_{t-i} \qquad (3.1)$$

*Inevitability of multivariate model*

A multivariate series $\{U_t\}$ is a linear encounter of its lagged values, hence, multivariate time series is always a value of the model $U_t$ as a tool of its lagged values $U_{t-i}$ for i is greater than 0 plus new information at time t. This can be presented as

$$U_t = c + a_t + \sum_{j=1}^{\infty} \pi_j U_{t-j} \ , \qquad (3.2)$$

This equation must be a convergent series and the invertible condition of the model is that all $\pi$ must be less than 1 in modulus (circle).

*3.3.3 Stationarity*

This is another major condition, that multivariate data or time-series data must be subjected to before carrying out analysis. So an N-dimensional series $U_t$ is a weakly stationary series if the following condition holds; (a) $E(U_t)=\mu$, with N-dimensional standard vectors and (b) $Cov(U_t)=E[(U_t-\mu)(U_t-\mu)^1)] = E_a$, a constant NxN positive definite matrix.

*Vector Autoregressive (VAR) Model*

In modeling dynamics between a set of variables, the VAR model provides us with an approach. This method is specifically concerned with the dynamics of multiple variables. The VAR model is also a reduced version of the dynamic model that involves constructing an equation that makes each endogenous variable a function of its previous values as well as the previous values of all other endogenous variables. The autoregressive VAR (P) model of the basic P-lag vector is of the form:

$$U_t = \theta_0 + \theta_1 U_{t-1} + \theta_2 U_{t-2} + \cdots + \theta_p U_{t-p} + a_t \qquad (3.3)$$

The VAR(P) model can be written in the matrix form as

$$\begin{pmatrix} U_{1t} \\ U_{2t} \\ \vdots \\ U_{kt} \end{pmatrix} = \begin{pmatrix} \theta_{10} \\ \theta_{20} \\ \vdots \\ \theta_{k0} \end{pmatrix} + \begin{pmatrix} \theta_{11}^1 & \theta_{12}^1 & \dots & \theta_{1k}^1 \\ \theta_{21}^1 & \theta_{22}^1 & \dots & \theta_{2k}^1 \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^1 & \theta_{k2}^1 & \dots & \theta_{kk}^1 \end{pmatrix} \begin{pmatrix} U_{1t-1} \\ U_{2t-1} \\ \vdots \\ U_{kt-1} \end{pmatrix} +$$

$$\begin{pmatrix} \theta_{11}^2 & \theta_{12}^2 & \dots & \theta_{1k}^2 \\ \theta_{21}^2 & \theta_{22}^2 & \dots & \theta_{2k}^2 \\ \vdots & & & \vdots \\ \theta_{k1}^2 & \theta_{k2}^2 & \dots & \theta_{kk}^2 \end{pmatrix} \begin{pmatrix} U_{1t-2} \\ U_{2t-2} \\ \vdots \\ U_{kt-2} \end{pmatrix} + \cdots +$$

$$\begin{pmatrix} \theta_{11}^p & \theta_{12}^p & \dots & \theta_{1k}^p \\ \theta_{21}^p & \theta_{22}^p & \dots & \theta_{2k}^p \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^p & \theta_{k2}^p & \dots & \theta_{kk}^p \end{pmatrix} \begin{pmatrix} U_{1t-p} \\ U_{2t-p} \\ \vdots \\ U_{kt-p} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \\ \vdots \\ a_{kt} \end{pmatrix}$$

*Order of Selection*

In this work, we will emulate the proposed tool of [4], and that consisting of model specification, estimation, and diagnostic checking on multivariate analysis. We will use the recent procedure of [4]. But this approach of selecting the model order of multivariate time series was first proposed by [12]. Behind the approach is to compare different sets of the multivariate model that amount to examining the hypothesis of testing;

$H_0;\ \emptyset_\rho = 0$ Versus $H_\propto;\ \emptyset_\rho \neq 0$

*Information Criteria*

Models are successful in concluding any mathematical model based on the knowledge criteria. So, we understand that all parameters are based on chance, consist of two properties. Firstly, components are concerned with the model data's goodness of fit test, while the second component penalizes more complex models.

These methods are;

$$AIC(\ell) = In|\tilde{\text{E}}_{a,\ell}| + \frac{2}{T}\ell K^2 \qquad (3.4)$$

$$BIC(\ell) = In|\tilde{\text{E}}_{a,\ell}| + \frac{In(T)}{T}\ell K^2 \qquad (3.5)$$

$$HQ(\ell) = In|\tilde{\text{E}}_{a,\ell}| + \frac{2In[In(T)]}{T}\ell K^2$$

*Model Checking*

Model-checking is a major aspect of the examination of the model; it is also known as a diagnostic test. In model design, this plays a significant role, such as multivariate normality, a typical model is said to be adequate.

## IV. RESULT

*Data*

The data used in this work was collected from the National Bureau of Statistics (NBS), the data contain quarterly government records for sectors GDP of Agric, Industries, B/ construction, W/Retail, and Services growth rate from 1985-2017, a total of 33 years.

*Variability of 2 Variables*

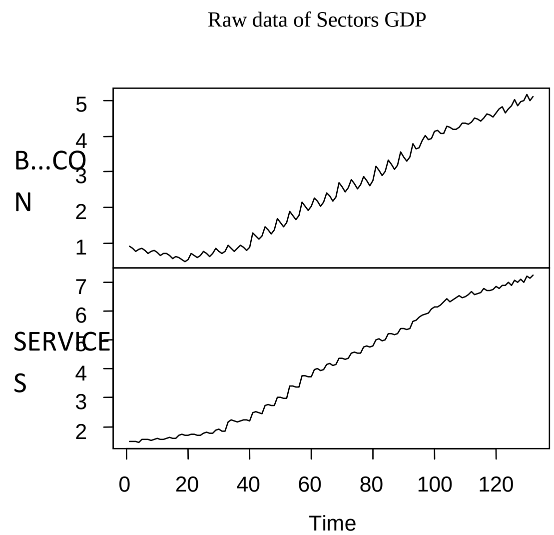


Figure 1a; Plot of the Raw Data of B.Con and Services

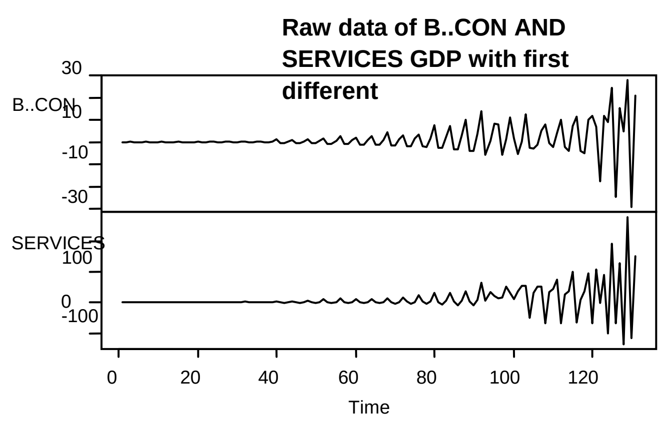


Figure 1b; First Difference Plot of the Raw Data of B.Con and Services

Table 1; Model Selection on 2 Parameter Variables

| S/N | Variables | VAR – ORDER | AIC | BIC | HQ |
|---|---|---|---|---|---|
| 1 | Agric and Industries | VAR(6) | -8.4799 | -7.9558 | -8.2669 |
| 2 | Indust. and B/construct | VAR(5) | -8.9133 | -8.4765 | -8.358 |
| 3 | B/const. and W/Retails | VAR(7) | -10.6739 | -10.2022 | -10.7819 |
| 4 | W/Retails & Services | VAR(5) | -10.9594 | -10.5226 | -10.7819 |
| 5 | Agric and Services | VAR(6) | -10.9454 | -10.4334 | -10.7324 |
| 6 | Agric and B/construct | VAR(9) | -10.8197 | -10.2599 | -10.5618 |
| 7 | Agric and W/Retails | VAR(8) | -10.2026 | -9.7056 | -.9.9649 |
| 8 | Indust. and W/Retails | VAR(5) | -8.1399 | -7.5708 | -7.9624 |
| 9 | Indust .and Services | VAR(5) | -9.1725 | -8.7357 | -8.9950 |
| 10 | B/const and Services | VAR(5) | -11.8843 | -11.4475 | -11.7068 |

It was observed in table4.1 that, the data set of industries and wholesale/ retails have the highest AIC, BIC, and HQ (-8.1399, -7.5708,-7.9624) respectively. This indicates that data set 8 with VAR (5) is the worst pair of 2 parameter variables. While data set of Building/ Construction and Services VAR (5) with AIC, BIC, and HQ (-11.8843,-11.4475, -11.7068) respectively, have the smallest AIC, BIC and HQ, was selected to be the best model of 2 parameter variables.

Presentation of 3 Parameter Variables

Variability of 3 Variables

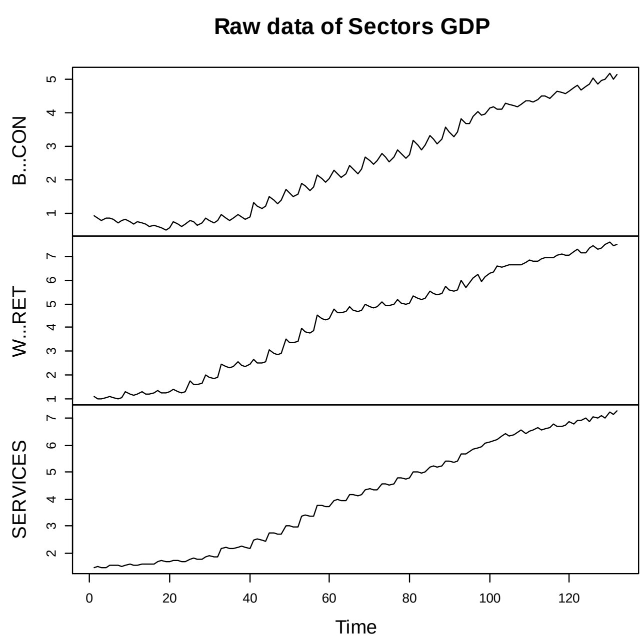


Figure 2a; Plot of the Raw Data of B.Con, Wholesale/Retail and Services

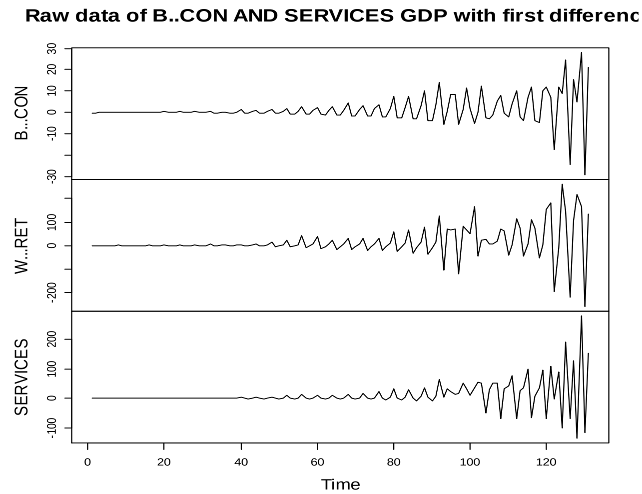


Figure 2b; First Difference Plot of the Raw Data of B.Con, Wholesale/Retail and Services

Table 2; Model Selection of 3 Parameter Variables

| S/N | VARIABLES | VAR-ORDER | AIC | BIC | HQ |
|---|---|---|---|---|---|
| 1 | Agric, Industries, B/construction. | VAR(6) | -14.2395 | -13.2402 | -13.8236 |
| 2 | Industries, B/construction W/Retails | VAR(5) | -14.0276 | -13.0449 | -13.6283 |
| 3 | B/construction, W/Retails, Services | VAR(5) | -17.0510 | -16.0682 | -16.6516 |
| 4 | Agric, Industries. W /Retails | VAR(5) | -13.6047 | -12.6214 | -13.2049 |
| 5 | Agric, Industries. and Services | VAR(6) | -14.4149 | -13.4028 | -13.9862 |
| 6 | Agric, B/construction, and W/Retails | VAR(5) | -10.8197 | -10.2599 | -10.5618 |
| 7 | Agric, B/construction, and Services | VAR(6) | -10.2026 | -9.7056 | -.9.9649 |
| 8 | Agric, W/Retails, and Services | VAR(6) | -8.1399 | -7.5708 | -7.9624 |
| 9 | Industries, B/construction, and Services | VAR(5) | -9.1725 | -8.7357 | -8.9950 |
| 10 | Industries, W/Retails, and Services | VAR(5) | -11.8843 | -11.4475 | -11.7068 |

From table2; we observed that the VAR (6) order of the model of Agric, W/Retails, and Services has the highest value of AIC, BIC, and HQ (-8.1399, -7.9624, -7.9624) respectively. This indicates that the model is the poorest in the 3 parameter variables combinations. We also observed that the VAR (5) model of B/construction, W/Retails, Services have the smallest AIC, BIC, and HQ value of (-17.0510, -16.0682, -16.6516) respectively is to be selected as the best model in the combinations of 3 parameter variables.

*Presentation of 4 Parameter Variables*

*Variability of 4 Variables*

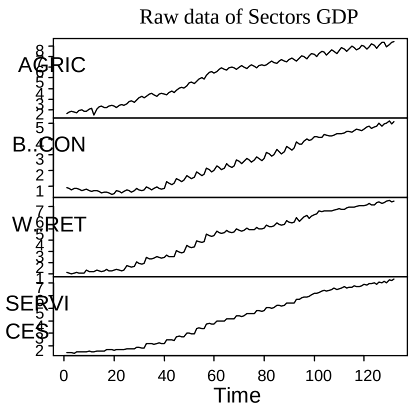


Figure 3a; Plot of the Raw Data of Agric, B.Con, Wholesale/Retail and Services

Raw data of AGRIC, B..CON, W..RET AND SERVICES GDP with the first difference

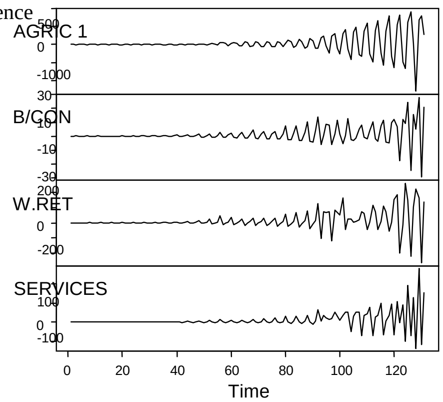


Figure 3b; First Difference Plot of the Raw Data of Agric, B.Con, Wholesale/Retail and Services

Table 3 Model Selection of 4 Parameter Variables

| S/N | Variables | VAR - ORDER | AIC | BIC | HQ |
|---|---|---|---|---|---|
| 1 | Agric, Indust., B/const. W/Retails | VAR(5) | -19.5445 | -17.7974 | -18.8346 |
| 2 | Indust., B/const. W/Retails, Serv. | VAR(5) | -20.3677 | -18.6205 | -19.6577 |
| 3 | Agric, indust., B/const. Services | VAR(5) | -20.7235 | -18.9764 | -20.0135 |
| 4 | Agric, B/const., W /Retails, and Serv. | VAR(5) | -22.0743 | -20.9967 | -22.0339 |
| 5 | Agric, Indust. W /Retails and Services | VAR(5) | -19.7718 | -18.0246 | -19.0618 |

From the order of selecting the model of table 3 in chapter four, we have; K=4, the parameter of the variables (number of variables used in the model), C= VAR (p); stand for the vector order of the model. T=132 which is the sample size. On applying the sequential likelihood ratio test, using three information criteria on the data, we then select the best model by choosing the model with the smallest AIC value. From the table of the order of selection, we see that the AIC BIC and HQ of combination set of Agric, B/construction, W/Retails, and Services is the best model with AIC=-22.7438, BIC=-20.9967, and HQ=-22.0339 respectively.

*Presentation of 5 Parameter Variables*

*Variability of the Complete Variables*

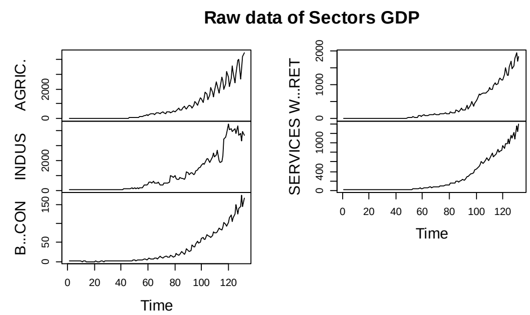


Figure 4a; Plot of the Raw Data

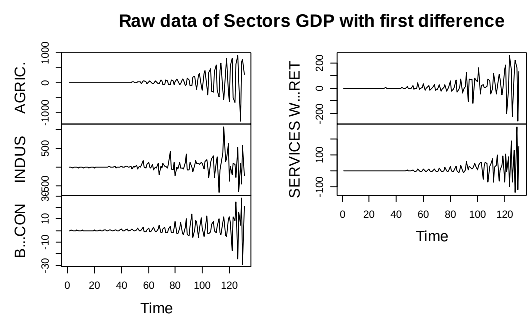


Figure 4.4b; Plot of First Difference of the Raw Data Order of Selecting Model on the Complete Parameter Variables

Summary table: 4

| | p | AIC | BIC | HQ | M(p) | p-value |
|---|---|---|---|---|---|---|
| | | | | | | |
| [1,] | 0 | -12.5550 | -12.555 | -12.555 | 0 | 0 |
| [2,] | 1 | -22.4248 | -21.8788 | -22.2029 | 1152.9597 | 0.0000 |
| [3,] | 2 | -23.0928 | -22.0009 | -22.6491 | 112.5352 | 0.0000 |
| [4,] | 3 | -23.9793 | -22.3413 | -23.3137 | 129.6844 | 0.0000 |
| [5,] | 4 | -25.0614 | -22.8775 | -24.1740 | 142.4419 | 0.0000 |
| [6,] | 5 | -25.8996 | -23.4015 | -25.0222 | 134.0173 | 0.0000 |
| [7,] | 6 | -25.7988 | -22.7229 | -25.0222 | 21.5344 | 0.6625 |
| [8,] | 7 | -25.7692 | -22.0473 | -24.3162 | 20.5621 | 0.7168 |
| [9,] | 8 | -25.7295 | -21.4616 | -24.0546 | 26.2778 | 0.3929 |

In the order of selecting the model in table 4, K=5, the parameters of the model (no. of variables used in the work).

P= VAR (lag); which stands for the vector order of the model. T=132, stand for the sample size. On applying the sequential likelihood ratio test, using the information criteria on the data. We subjected the data into VAR (p) models, in the order; we selected the best model in table 4.5 we see that the order selected by AIC, BIC, and HQ of the VAR (5) model, have the least value of AIC, BIC, and HQ. So statistically speaking, the VAR (5) model of the sector's GDP order of selection is the best in modeling the data of Sectors GDP in Nigeria.

Table 5; Model Summary

| Variables | | Model/ AIC | |
|---|---|---|---|
| 1 | Building &Services | VAR (5) | -11.5116 |
| 2 | B/ Con, W/Ret, and Service | VAR (5) | -17.1111 |
| 3 | Agric, B/con, W/Ret, and Service | VAR (5) | -22.1823 |
| 4 | Agric, Indus, B/con, W/Ret, and Service | VAR (5) | -25.8996 |

## V. DISCUSSION

In the summary table, we observed that, the greater the parameters (K), the better the model, as we see in the summary table by comparing model 1 of Building/ Construction and Services of two (2) parameters, K=2 with AIC values of -12.5116 with that of model 4 of Agriculture, Industries, Building/ Construction, Wholesales/ Retails and Services GDP sectors of five (5) parameters with AIC of -25.8996. However, we group the data set into various groups by combining in 2 variables, 3 variables, 4 variables, and the complete 5 variables in each group, we model the best pair, the reasons behind this is to compare the least parameter group with that of the large parameter group. The information criteria of Akaike, Bayesian, and Hannan-Quinn were adopted as the means of selecting the best model with their lag (p). From the results, the group of Building/Construction and Services with lag (5) was selected as the best model of 2 parameter variables for 2x2 vectors. Also, the best model of 3 parameter variables from table2 was selected to be a group of Building/Construction, Wholesales/Retails, and Services. In a group of 4 parameter variables of table3 model, we selected the combination of Agriculture, Building/Construction, Wholesales/retails, and Services as the best model with the least value of AIC with lag(5). Finally, we also model the complete (5) variables as 5x5 vectors, from the results in table 4, the model was selected to be the best at lag (5). From the summarized table in table5, we confirm that, the larger the parameters the better the model becomes.

## VI. SUMMARY

This paper discusses the Gross Domestic Product (GDP) growth rate of Nigeria's agricultural, manufacturing, building/construction, wholesale/retail, and service sectors from 1985 to 2017. [11] developed the autoregressive model VAR (p) used, and [12] developed the order of selection in multivariate time series (2008). The data used was collected from the National Bureau of Statistics and analyzes with Real (R) software. However, we group the data set into groups by pairing 10 set of two variables, 10 set of three variables, 5 set of four variables, and all five variables in each group, and then we model the best pair, with the goal of comparing the least parameter group to the largest parameter group. The Akaike, Bayesian, and Hannan-Quinn knowledge parameters were used to decide the best model with their lag (p). The category of Building/Construction and Services with lag (5) was chosen as the best model of two parameter variables for 2x2 vectors based on the results. A category of Building/Construction, Wholesales/Retails, and Services was also chosen as the best model of three parameter variables from table 2. We chose the combination of Agriculture, Building/Construction, Wholesales/Retails, and Services as the best model with the least value of AIC with lag (5), from a group of four parameter variables in the table 3 model. Finally, we model all five variables as 5x5 vectors, and the model with the best lag was chosen from the results in table 4. When the best model of two parameters with an AIC value of (-11.973) at lag (5) was compared to the best model of all five parameters with an AIC value of (-25.8996), the results revealed that the larger the parameters, the better the model.

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