Analytical Solution of First Kind Volterra Integro-Differential Equation Using Sadik Transform

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Abstract: **Volterra integro-differential equation generally appears when an initial value problem is to be converted into an integral equation. In this paper, authors determined the analytical solution of first kind Volterra integro-differential equation using Sadik transform. In this work, authors have considered that the kernel of first kind Volterra integro-differential equation is a convolution type kernel. Some numerical problems have been considered and solved with the help of Sadik transform for explaining the complete methodology. Results of numerical problems show that Sadik transform is very effective integral transform for determining the analytical solution of first kind Volterra integro-differential equation.**

Keywords: **Volterra integro-differential equation; Sadik transform; Convolution; Inverse Sadik transform.**

I. INTRODUCTION

Integral transforms (Laplace; Fourier; Hankel; Mellin; Hilbert; Stieltjes; Legendre; Jacobi; Gegenbauer; Laguerre; Hermite; Radon; Wavelet; Kamal; Laplace-Carson; Mohand, Elzaki, Aboodh; Sumudu; Shehu; Sawi; Sadik; Upadhyay transforms) are widely used mathematical techniques because these techniques provide the exact solution of the problem without large calculation work. Nowadays, these mathematical techniques are rapidly used for solving the problems of applied mathematics, theoretical mechanics, statistics, mathematical physics and pharmacokinetics. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of population growth and decay problems by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations.

Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms. Sadikali [64-65] gave Sadik transform and determined the solution of the problem of control theory using it.

The main aim of this paper is to determine the analytical solution of first kind Volterra integro-differential equation using Sadik transform.

II. DEFINITION OF SADIK TRANSFORM

The Sadik transform of the function $G(t)$ for all $t \geq 0$ is defined as [64]

$S\{G(t)\} = \frac{1}{v^\beta}\int_0^\infty G(t)e^{-tv^\alpha}dt = T(v^\alpha,\beta)$,

where v is complex variable and $\alpha \neq 0$ & β are any real numbers. Here S is called the Sadik transform operator.

Fundamental Properties of Sadik Transform

Linearity property [54-57]: $\begin{bmatrix} If\ S\{G_1(t)\} = T_1(v^\alpha,\beta) \\ and\ S\{G_2(t)\} = T_2(v^\alpha,\beta)\end{bmatrix}$ then $\begin{bmatrix} S\{aG_1(t)+bG_2(t)\} \\ = a\,T_1(v^\alpha,\beta)+bT_2(v^\alpha,\beta)\end{bmatrix}$, where a, b are arbitrary constants.

Proof: By the definition of Sadik transform, we have

$$S\{G(t)\} = \frac{1}{v^\beta}\int_0^\infty G(t)e^{-tv^\alpha}dt$$

$$\Rightarrow \begin{bmatrix} S\{aG_1(t)+bG_2(t)\} \\ = \frac{1}{v^\beta}\int_0^\infty [aG_1(t)+bG_2(t)]e^{-tv^\alpha}dt\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{aG_1(t)+bG_2(t)\} \\ = \begin{Bmatrix} a\left(\frac{1}{v^\beta}\int_0^\infty G_1(t)e^{-tv^\alpha}dt\right) \\ +b\left(\frac{1}{v^\beta}\int_0^\infty G_2(t)e^{-tv^\alpha}dt\right)\end{Bmatrix}\end{bmatrix}$$

$$\Rightarrow S\{aG_1(t)+bG_2(t)\} = aS\{G_1(t)\}+bS\{G_2(t)\}$$

$$\Rightarrow S\{aG_1(t)+bG_2(t)\} = a\,T_1(v^\alpha,\beta)+bT_2(v^\alpha,\beta),$$

where a, b are arbitrary constants.

*Change of scale property [55, 57]***:** If Sadik transform of function $G(t)$ is $T(v^\alpha,\beta)$ then Sadik transform of function $G(at)$ is given by $\frac{1}{a}T\left(\frac{v^\alpha}{a},\beta\right)$.

Proof: By the definition of Sadik transform, we have

$$S\{G(at)\} = \frac{1}{v^\beta}\int_0^\infty G(t)e^{-tv^\alpha}dt$$

Put $at = p \Rightarrow adt = dp$ in above equation, we have

$$S\{G(at)\} = \frac{1}{a}.\frac{1}{v^\beta}\int_0^\infty G(p)e^{-\frac{pv^\alpha}{a}}dp$$

$$\Rightarrow S\{G(at)\} = \frac{1}{a}\left[\frac{1}{v^\beta}\int_0^\infty G(p)e^{-p\left(\frac{v^\alpha}{a}\right)}dp\right]$$

$$\Rightarrow S\{G(at)\} = \frac{1}{a}T\left(\frac{v^\alpha}{a},\beta\right).$$

Shifting property [55, 57]: If Sadik transform of function $G(t)$ is $T(v^\alpha,\beta)$ then Sadik transform of function $e^{at}G(t)$is given by $T(v^\alpha - a,\beta)$.

*Proof***:** By the definition of Sadik transform, we have

$$S\{e^{at}G(t)\} = \frac{1}{v^\beta}\int_0^\infty e^{at}G(at)e^{-tv^\alpha}dt$$

$$\Rightarrow S\{e^{at}G(t)\} = \frac{1}{v^\beta}\int_0^\infty G(t)e^{-(v^\alpha-a)t}dt$$

$$\Rightarrow S\{e^{at}G(t)\} = T(v^\alpha - a,\beta)$$

Sadik transform of the derivatives of the function $G(t)$ [54-57, 64]:

If $S\{G(t)\} = T(v^\alpha,\beta)$ then

a) $S\{G'(t)\} = v^\alpha T(v^\alpha,\beta) - \frac{G(0)}{v^\beta}$

b) $S\{G''(t)\} = v^{2\alpha}T(v^\alpha,\beta) - \frac{G'(0)}{v^\beta} - v^\alpha\frac{G(0)}{v^\beta}$

c) $\begin{bmatrix} S\{G^{(n)}(t)\} \\ = \begin{Bmatrix} v^{n\alpha}T(v^\alpha,\beta) - v^{(n-1)\alpha}\frac{G(0)}{v^\beta} \\ -v^{(n-2)\alpha}\frac{G'(0)}{v^\beta} - \cdots\ldots - \frac{G^{(n-1)}(0)}{v^\beta}\end{Bmatrix}\end{bmatrix}$

Sadik transform of integral of a function $G(t)$[55]:

If $S\{G(t)\} = T(v^\alpha,\beta)$ then $\begin{bmatrix} S\left\{\int_0^t G(t)dt\right\} \\ = \frac{1}{v^\alpha}T(v^\alpha,\beta)\end{bmatrix}$.

Proof: Let $H(t) = \int_0^t G(t)dt$.

Then $H'(t) = G(t)$and $H(0) = 0$.

Now by the property of Sadik transform of the derivative of function, we have

$$S\{H'(t)\} = v^\alpha S\{H(t)\} - \frac{H(0)}{v^\beta} = v^\alpha S\{H(t)\}$$

$$\Rightarrow S\{H(t)\} = \frac{1}{v^\alpha}S\{H'(t)\} = \frac{1}{v^\alpha}\ S\{G(t)\}$$

$$\Rightarrow S\{H(t)\} = \frac{1}{v^\alpha}T(v^\alpha,\beta)$$

$$\Rightarrow S\left\{\int_0^t G(t)dt\right\} = \frac{1}{v^\alpha}T(v^\alpha,\beta)$$

Convolution theorem for Sadik transforms [55-57]: If Sadik transform of functions $G_1(t)$ and $G_2(t)$ are $T_1(v^\alpha,\beta)$ and $T_2(v^\alpha,\beta)$ respectively then Sadik transform of their convolution $G_1(t) * G_2(t)$ is given by $S\{G_1(t) * G_2(t)\} = v^\beta S\{G_1(t)\}S\{G_2(t)\}$

$$\Rightarrow S\{G_1(t) * G_2(t)\} = v^\beta\ T_1(v^\alpha,\beta)T_2(v^\alpha,\beta),$$

where $G_1(t) * G_2(t)$ is defined by

$$\begin{bmatrix} G_1(t) * G_2(t) = \int_0^t G_1(t-x)\,G_2(x)dx \\ = \int_0^t G_1(x)\,G_2(t-x)dx\end{bmatrix}.$$

Proof: By the definition of Sadik transform, we have

$$S\{G_1(t) * G_2(t)\} = \frac{1}{v^\beta}\int_0^\infty [G_1(t) * G_2(t)]e^{-tv^\alpha}dt$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left\{\int_0^t G_1(t-x)\,G_2(x)dx\right\}dt\end{bmatrix}$$

By changing the order of integration, we have

$$\begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) \left\{ \frac{1}{v^\beta} \int_x^\infty e^{-tv^\alpha} G_1(t-x) \right\} dtdx \end{bmatrix}$$

Put $t - x = p$ so that $dt = dp$ in above equation, we have

$$\begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) \left\{ \frac{1}{v^\beta} \int_0^\infty e^{-(p+x)v^\alpha} G_1(p)\, dp \right\} dx \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) e^{-xv^\alpha} \left\{ \frac{1}{v^\beta} \int_0^\infty e^{-pv^\alpha} G_1(p)\, dp \right\} dx \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) e^{-xv^\alpha} \left[S\{G_1(t)\} \right] dx \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = [S\{G_1(t)\}] \int_0^\infty G_2(x) e^{-xv^\alpha}\, dx \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = [\, T_1(v^\alpha, \beta)] v^\beta \left[\frac{1}{v^\beta} \int_0^\infty G_2(x) e^{-xv^\alpha}\, dx \right] \end{bmatrix}$$

$$\Rightarrow \left[S\{G_1(t) * G_2(t)\} = v^\beta S\{G_1(t)\} S\{G_2(t)\} \right]$$

$$\Rightarrow S\{G_1(t) * G_2(t)\} = v^\beta\, T_1(v^\alpha, \beta) T_2(v^\alpha, \beta).$$

Table 1 Sadik Transform of Frequently Encountered Functions [54-57, 64]

S.N.	$G(t)$	$S\{G(t)\} = T(v^\alpha, \beta)$
1.	1	$\frac{1}{v^{\alpha+\beta}}$
2.	t	$\frac{1}{v^{2\alpha+\beta}}$
3.	t^2	$\frac{2!}{v^{3\alpha+\beta}}$
4.	$t^n, n \in N$	$\frac{n!}{v^{(n+1)\alpha+\beta}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{(n+1)\alpha+\beta}}$
6.	e^{at}	$\frac{1}{v^\beta (v^\alpha - a)}$
7.	$sinat$	$\frac{a}{v^\beta (v^{2\alpha} + a^2)}$
8.	$cosat$	$\frac{v^\alpha}{v^\beta (v^{2\alpha} + a^2)}$
9.	$sinhat$	$\frac{a}{v^\beta (v^{2\alpha} - a^2)}$
10.	$coshat$	$\frac{v^\alpha}{v^\beta (v^{2\alpha} - a^2)}$

III. INVERSE SADIK TRANSFORM

If $S\{G(t)\} = T(v^\alpha, \beta)$ then $G(t)$ is called the inverse Sadik transform of $T(v^\alpha, \beta)$ and mathematically it is defined as $G(t) = S^{-1}\{T(v^\alpha, \beta)\}$, where S^{-1} is the inverse Sadik transform operator.

Linearity property of inverse Sadik transforms: $\begin{bmatrix} If\ S^{-1}\{\, T_1(v^\alpha, \beta)\} = G_1(t) \\ and\ S^{-1}\{T_2(v^\alpha, \beta)\} = G_2(t) \end{bmatrix}$ *then*

$$\begin{bmatrix} S^{-1}\{a\, T_1(v^\alpha, \beta) + bT_2(v^\alpha, \beta)\} \\ = aS^{-1}\{\, T_1(v^\alpha, \beta)\} + bS^{-1}\{T_2(v^\alpha, \beta)\} \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} S^{-1}\{a\, T_1(v^\alpha, \beta) + bT_2(v^\alpha, \beta)\} \\ = a\, G_1(t) + b\, G_2(t) \end{bmatrix}$, where a, b are arbitrary constants.

Table 2 Inverse Sadik Transform Of Frequently Encountered Functions [54, 56-57]

S.N.	$T(v^\alpha, \beta)$	$G(t) = S^{-1}\{T(v^\alpha, \beta)\}$
1.	$\frac{1}{v^{\alpha+\beta}}$	1
2.	$\frac{1}{v^{2\alpha+\beta}}$	t
3.	$\frac{1}{v^{3\alpha+\beta}}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^{(n+1)\alpha+\beta}}, n \in N$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^{(n+1)\alpha+\beta}}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v^\beta (v^\alpha - a)}$	e^{at}
7.	$\frac{1}{v^\beta (v^{2\alpha} + a^2)}$	$\frac{sinat}{a}$
8.	$\frac{v^\alpha}{v^\beta (v^{2\alpha} + a^2)}$	$cosat$
9.	$\frac{1}{v^\beta (v^{2\alpha} - a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{v^\alpha}{v^\beta (v^{2\alpha} - a^2)}$	$coshat$

IV. ANALYTICAL SOLUTION OF FIRST KIND VOLTERRA INTEGRO-DIFFERENTIAL EQUATION USING SADIK TRANSFORM

In this part of the paper, authors determine the analytical solution of first kind Volterra integro-differential equation using Sadik transform. In this work, we have considered that the kernel of first kind Volterra integro-differential equation is a convolution type kernel.

Convolution type first kind Volterra integro-differential equation is given by

$$\left. \begin{matrix} \int_0^t K_1(t-u)\, \omega(u) du + \\ \int_0^t K_2(t-u)\, \omega^{(n)}(u) du \\ = F(t), K_2(t-u) \neq 0 \end{matrix} \right\} \quad (1)$$

$$\text{with} \left.\begin{matrix} \omega(0)=\delta_0, \omega'(0)=\delta_1, \\ \omega''(0)=\delta_2, \dots\dots, \\ \omega^{(n-1)}(0)=\delta_{n-1} \end{matrix}\right\} \quad (2)$$

$$\text{where} \left.\begin{matrix} \begin{bmatrix} K_1(t-u), K_2(t-u) \\ = convolution\,type\,kernels \\ of\ integral\ equation \end{bmatrix} \\ \begin{bmatrix} \omega(t) = unknown \\ function \end{bmatrix} \\ \begin{bmatrix} \omega^{(n)}(t) = nth\ derivative \\ of unknown\, function \end{bmatrix} \\ \begin{bmatrix} F(t) = known \\ function \end{bmatrix} \\ \begin{bmatrix} \delta_0, \delta_1, \delta_2, \dots, \delta_{n-1} \\ = real\ numbers \end{bmatrix} \end{matrix}\right\}$$

Taking Sadik transform of both sides of (1), we have

$$\begin{bmatrix} S\left\{\int_0^t K_1(t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t K_2(t-u)\,\omega^{(n)}(u)du\right\} \\ = S\{F(t)\} \end{bmatrix} \quad (3)$$

Applying convolution theorem of Sadik transform on (3), we have

$$\begin{bmatrix} v^{\beta} S\{K_1(t)\}S\{\omega(t)\} \\ +v^{\beta} S\{K_2(t)\}S\{\omega^{(n)}(t)\} = S\{F(t)\} \end{bmatrix} \quad (4)$$

Applying the property "Sadik transform of derivative of functions" on (4), we get

$$\begin{bmatrix} v^{\beta} S\{K_1(t)\}S\{\omega(t)\} \\ +v^{\beta} S\{K_2(t)\} \begin{bmatrix} v^{n\alpha} S\{\omega(t)\} \\ -v^{(n-1)\alpha}\frac{\omega(0)}{v^{\beta}} \\ -v^{(n-2)\alpha}\frac{\omega'(0)}{v^{\beta}} \\ -v^{(n-3)\alpha}\frac{\omega''(0)}{v^{\beta}} \\ -\cdots\dots \\ -\frac{\omega^{(n-1)}(0)}{v^{\beta}} \end{bmatrix} = S\{F(t)\} \end{bmatrix} \quad (5)$$

Now using (2) in (5), we have

$$\begin{bmatrix} v^{\beta} S\{K_1(t)\}S\{\omega(t)\} \\ +v^{\beta} S\{K_2(t)\} \begin{bmatrix} v^{n\alpha} S\{\omega(t)\} \\ -v^{(n-1)\alpha}\frac{\delta_0}{v^{\beta}} \\ -v^{(n-2)\alpha}\frac{\delta_1}{v^{\beta}} \\ -v^{(n-3)\alpha}\frac{\delta_2}{v^{\beta}} \\ -\cdots\dots \\ -\frac{\delta_{n-1}}{v^{\beta}} \end{bmatrix} = S\{F(t)\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \begin{bmatrix} v^{\beta} S\{K_1(t)\} \\ +v^{n\alpha+\beta} S\{K_2(t)\} \end{bmatrix} S\{\omega(t)\} \\ = \begin{bmatrix} S\{F(t)\} \\ +S\{K_2(t)\} \begin{pmatrix} v^{(n-1)\alpha}\delta_0 \\ +v^{(n-2)\alpha}\delta_1 \\ +v^{(n-3)\alpha}\delta_2 \\ +\cdots\dots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{\omega(t)\} = \dfrac{\begin{bmatrix} S\{F(t)\} \\ +S\{K_2(t)\} \begin{pmatrix} v^{(n-1)\alpha}\delta_0 \\ +v^{(n-2)\alpha}\delta_1 \\ +v^{(n-3)\alpha}\delta_2 \\ +\cdots\dots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix}}{\begin{bmatrix} v^{\beta} S\{K_1(t)\} \\ +v^{n\alpha+\beta} S\{K_2(t)\} \end{bmatrix}}, \\ \begin{bmatrix} v^{\beta} S\{K_1(t)\} \\ +v^{n\alpha+\beta} S\{K_2(t)\} \end{bmatrix} \neq 0 \end{bmatrix} \quad (6)$$

The inverse Sadik transform of both sides of (6) gives the required analytical solution of given convolution type first kind Volterra integro-differential equation.

V. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following first kind Volterra integro-differential equation

$$\begin{bmatrix} \int_0^t (t-u)\,\omega(u)du \\ +\int_0^t (t-u)^2\,\omega'(u)du \\ = 3t - 3sint \end{bmatrix} \quad (7)$$

with $\omega(0) = 0$ (8)

Taking Sadik transform of both sides of (7), we have

$$\begin{bmatrix} S\left\{\int_0^t (t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t (t-u)^2\,\omega'(u)du\right\} \\ = S\{3t - 3sint\} \end{bmatrix} \quad (9)$$

Applying convolution theorem of Sadik transform on (9), we have

$$\begin{bmatrix} v^{\beta} S\{t\}S\{\omega(t)\} + v^{\beta} S\{t^2\}S\{\omega'(t)\} \\ = S\{3t - 3sint\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v^{\beta} S\{t\}S\{\omega(t)\} + v^{\beta} S\{t^2\}S\{\omega'(t)\} \\ = 3S\{t\} - 3S\{sint\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v^{\beta}\left(\frac{1}{v^{2\alpha+\beta}}\right)S\{\omega(t)\} + v^{\beta}\left(\frac{2}{v^{3\alpha+\beta}}\right)S\{\omega'(t)\} \\ = \left(\frac{3}{v^{2\alpha+\beta}}\right) - \left\{\frac{3}{v^{\beta}(v^{2\alpha}+1)}\right\} \end{bmatrix} \quad (10)$$

Applying the property "Sadik transform of derivative of functions" on (10), we get

$$\left[\begin{Bmatrix} v^{\beta}\left(\frac{1}{v^{2\alpha+\beta}}\right)S\{\omega(t)\} \\ +v^{\beta}\left(\frac{2}{v^{3\alpha+\beta}}\right)\left\{v^{\alpha}S\{\omega(t)\}-\frac{\omega(0)}{v^{\beta}}\right\} \end{Bmatrix} \\ =\left(\frac{3}{v^{2\alpha+\beta}}\right)-\left\{\frac{3}{v^{\beta}(v^{2\alpha}+1)}\right\}\right] \quad (11)$$

Now using (8) in (11), we have

$$\left[\begin{Bmatrix} v^{\beta}\left(\frac{1}{v^{2\alpha+\beta}}\right)S\{\omega(t)\} \\ +v^{\beta}\left(\frac{2}{v^{3\alpha+\beta}}\right)\{v^{\alpha}S\{\omega(t)\}\} \end{Bmatrix} \\ =\left(\frac{3}{v^{2\alpha+\beta}}\right)-\left\{\frac{3}{v^{\beta}(v^{2\alpha}+1)}\right\}\right]$$

$$\Rightarrow\left[\begin{matrix} S\{\omega(t)\} \\ =\frac{1}{v^{\beta}}-\frac{v^{2\alpha}}{v^{\beta}(v^{2\alpha}+1)}=\frac{1}{v^{\beta}(v^{2\alpha}+1)} \end{matrix}\right] \quad (12)$$

Taking inverse Sadik transform of both sides of (12), we get the required solution of (7) with (8) as

$$\left[\omega(t)=S^{-1}\left\{\frac{1}{v^{\beta}(v^{2\alpha}+1)}\right\}=sint\right].$$

Problem: 2 Consider the following first kind Volterra integro-differential equation

$$\left[\begin{matrix} \int_0^t(t-u)\,\omega(u)du \\ +\frac{1}{4}\int_0^t(t-u-1)\,\omega''(u)du \\ =\frac{sin2t}{2} \end{matrix}\right] \quad (13)$$

with $[\omega(0)=1,\omega'(0)=0]$ (14)

Taking Sadik transform of both sides of (13), we have

$$\left[\begin{matrix} S\left\{\int_0^t(t-u)\,\omega(u)du\right\} \\ +\frac{1}{4}S\left\{\int_0^t(t-u-1)\,\omega''(u)du\right\} \\ =S\left\{\frac{sin2t}{2}\right\} \end{matrix}\right] \quad (15)$$

Applying convolution theorem of Sadik transform on (15), we have

$$\left[\begin{matrix} v^{\beta}S\{t\}S\{\omega(t)\}+\frac{1}{4}v^{\beta}S\{t-1\}S\{\omega''(t)\} \\ =\frac{1}{2}S\{sin2t\} \end{matrix}\right]$$

$$\Rightarrow\left[\begin{Bmatrix} v^{\beta}\left(\frac{1}{v^{2\alpha+\beta}}\right)S\{\omega(t)\} \\ +\frac{1}{4}v^{\beta}\left(\frac{1}{v^{2\alpha+\beta}}-\frac{1}{v^{\alpha+\beta}}\right)S\{\omega''(t)\} \end{Bmatrix} \\ =\frac{1}{2}\left\{\frac{2}{v^{\beta}(v^{2\alpha}+4)}\right\}\right] \quad (16)$$

Applying the property "Sadik transform of derivative of functions" on (16), we get

$$\left[\begin{Bmatrix} \left(\frac{1}{v^{2\alpha}}\right)S\{\omega(t)\} \\ +\frac{1}{4}\left(\frac{1}{v^{2\alpha}}-\frac{1}{v^{\alpha}}\right)\left[\begin{matrix} v^{2\alpha}S\{\omega(t)\}-\frac{\omega'(0)}{v^{\beta}} \\ -v^{\alpha}\frac{\omega(0)}{v^{\beta}} \end{matrix}\right] \end{Bmatrix} \\ =\left\{\frac{1}{v^{\beta}(v^{2\alpha}+4)}\right\}\right] \quad (17)$$

Now using (14) in (17), we have

$$\left[\begin{Bmatrix} \left(\frac{1}{v^{2\alpha}}\right)S\{\omega(t)\} \\ +\frac{1}{4}\left(\frac{1}{v^{2\alpha}}-\frac{1}{v^{\alpha}}\right)\left(v^{2\alpha}S\{\omega(t)\}-\frac{v^{\alpha}}{v^{\beta}}\right) \end{Bmatrix} \\ =\left\{\frac{1}{v^{\beta}(v^{2\alpha}+4)}\right\}\right]$$

$$\Rightarrow S\{\omega(t)\}=\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+4)} \quad (18)$$

Taking inverse Sadik transform of both sides of (18), we get the required solution of (13) with (14) as

$$\left[\omega(t)=S^{-1}\left\{\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+4)}\right\}=cos2t\right].$$

Problem: 3 Consider the following first kind Volterra integro-differential equation

$$\left[\begin{matrix} \int_0^t cos(t-u)\,\omega(u)du \\ +\int_0^t sin(t-u)\,\omega'''(u)du \\ =1+sint-cost \end{matrix}\right] \quad (19)$$

with $\left[\begin{matrix}\omega(0)=1, \\ \omega'(0)=1,\omega''(0)=-1\end{matrix}\right]$ (20)

Taking Sadik transform of both sides of (19), we have

$$\left[\begin{matrix} S\left\{\int_0^t cos(t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t sin(t-u)\,\omega'''(u)du\right\} \\ =S\{1+sint-cost\} \end{matrix}\right] \quad (21)$$

Applying convolution theorem of Sadik transform on (21), we have

$$\left[\begin{matrix} v^{\beta}S\{cost\}S\{\omega(t)\}+v^{\beta}S\{sint\}S\{\omega'''(t)\} \\ =S\{1\}+S\{sint\}-S\{cost\} \end{matrix}\right]$$

$$\Rightarrow\left[\begin{matrix} v^{\beta}\left(\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right)S\{\omega(t)\} \\ +v^{\beta}\left(\frac{1}{v^{\beta}(v^{2\alpha}+1)}\right)S\{\omega'''(t)\} \\ =\left(\frac{1}{v^{\alpha+\beta}}\right)+\left(\frac{1}{v^{\beta}(v^{2\alpha}+1)}\right)-\left(\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right) \end{matrix}\right] \quad (22)$$

Applying the property "Sadik transform of derivative of functions" on (22), we get

$$\begin{bmatrix} \left(\frac{v^{\alpha}}{(v^{2\alpha}+1)}\right) S\{\omega(t)\} \\ +\left(\frac{1}{(v^{2\alpha}+1)}\right) \begin{bmatrix} v^{3\alpha} S\{\omega(t)\} \\ -\left(\frac{v^{2\alpha}}{v^{\beta}}\right)\omega(0) \\ -\left(\frac{v^{\alpha}}{v^{\beta}}\right)\omega'(0) \\ -\left(\frac{1}{v^{\beta}}\right)\omega''(0) \end{bmatrix} \\ = \left(\frac{1}{v^{\alpha+\beta}}\right) + \left(\frac{1}{v^{\beta}(v^{2\alpha}+1)}\right) - \left(\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right) \end{bmatrix} \quad (23)$$

Now using (20) in (23), we have

$$\begin{bmatrix} \left(\frac{v^{\alpha}}{(v^{2\alpha}+1)}\right) S\{\omega(t)\} \\ +\left(\frac{1}{(v^{2\alpha}+1)}\right) \begin{bmatrix} v^{3\alpha} S\{\omega(t)\} \\ -\left(\frac{v^{2\alpha}}{v^{\beta}}\right) \\ -\left(\frac{v^{\alpha}}{v^{\beta}}\right) \\ +\left(\frac{1}{v^{\beta}}\right) \end{bmatrix} \\ = \left(\frac{1}{v^{\alpha+\beta}}\right) + \left(\frac{1}{v^{\beta}(v^{2\alpha}+1)}\right) - \left(\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right) \end{bmatrix}$$

$$\Rightarrow \left[S\{\omega(t)\} = \frac{1}{v^{2\alpha+\beta}} + \left(\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right)\right] \quad (24)$$

Taking inverse Sadik transform of both sides of (24), we get the required solution of (19) with (20) as

$$\begin{bmatrix} \omega(t) = S^{-1}\left\{\frac{1}{v^{2\alpha+\beta}} + \frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right\} \\ = S^{-1}\left\{\frac{1}{v^{2\alpha+\beta}}\right\} + S^{-1}\left\{\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+1)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + cost.$$

Problem: 4 Consider the following first kind Volterra integro-differential equation

$$\begin{bmatrix} \int_0^t (t-u)^2\,\omega(u)du \\ -\frac{1}{12}\int_0^t (t-u)^3\,\omega'''(u)du = \frac{t^4}{12} \end{bmatrix} \quad (25)$$

with $\begin{bmatrix} \omega(0) = 0, \\ \omega'(0) = 3, \omega''(0) = 0 \end{bmatrix}$ (26)

Taking Sadik transform of both sides of (25), we have

$$\begin{bmatrix} S\left\{\int_0^t (t-u)^2\,\omega(u)du\right\} \\ -\frac{1}{12} S\left\{\int_0^t (t-u)^3\,\omega'''(u)du\right\} = \frac{1}{12} S\{t^4\} \end{bmatrix} \quad (27)$$

Applying convolution theorem of Sadik transform on (27), we have

$$\begin{bmatrix} v^{\beta} S\{t^2\} S\{\omega(t)\} \\ -\frac{1}{12} v^{\beta} S\{t^3\} S\{\omega'''(t)\} = \frac{1}{12}\left(\frac{24}{v^{5\alpha+\beta}}\right) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v^{\beta}\left(\frac{2}{v^{3\alpha+\beta}}\right) S\{\omega(t)\} \\ -\frac{1}{12} v^{\beta}\left(\frac{6}{v^{4\alpha+\beta}}\right) S\{\omega'''(t)\} = 2\left(\frac{1}{v^{5\alpha+\beta}}\right) \end{bmatrix} \quad (28)$$

Applying the property "Sadik transform of derivative of functions" on (28), we get

$$\begin{bmatrix} \left(\frac{2}{v^{3\alpha}}\right) S\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{v^{4\alpha}}\right) \begin{bmatrix} v^{3\alpha} S\{\omega(t)\} \\ -\left(\frac{v^{2\alpha}}{v^{\beta}}\right)\omega(0) \\ -\left(\frac{v^{\alpha}}{v^{\beta}}\right)\omega'(0) \\ -\left(\frac{1}{v^{\beta}}\right)\omega''(0) \end{bmatrix} = 2\left(\frac{1}{v^{5\alpha+\beta}}\right) \end{bmatrix} \quad (29)$$

Now using (26) in (29), we have

$$\begin{bmatrix} \left(\frac{2}{v^{3\alpha}}\right) S\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{v^{4\alpha}}\right)\left[v^{3\alpha} S\{\omega(t)\} - 3\left(\frac{v^{\alpha}}{v^{\beta}}\right)\right] = 2\left(\frac{1}{v^{5\alpha+\beta}}\right) \end{bmatrix}$$

$$\Rightarrow \left[\left\{\left(\frac{2}{v^{3\alpha}}\right) - \frac{1}{2}\left(\frac{1}{v^{\alpha}}\right)\right\} S\{\omega(t)\} = \left(\frac{2}{v^{5\alpha+\beta}}\right) - \frac{3}{2}\left(\frac{1}{v^{3\alpha+\beta}}\right)\right]$$

$$\Rightarrow \begin{bmatrix} \left\{\frac{\left(\frac{4}{v^{3\alpha}}\right) - \left(\frac{1}{v^{\alpha}}\right)}{2}\right\} S\{\omega(t)\} \\ = \left\{\frac{\left(\frac{4}{v^{5\alpha+\beta}}\right) - 3\left(\frac{1}{v^{3\alpha+\beta}}\right)}{2}\right\} \end{bmatrix}$$

$$\Rightarrow \left[S\{\omega(t)\} = \left\{\frac{1}{v^{2\alpha+\beta}} - \frac{2}{v^{\beta}(4 - v^{2\alpha})}\right\}\right]$$

$$\Rightarrow \left[S\{\omega(t)\} = \left\{\frac{1}{v^{2\alpha+\beta}} + \frac{2}{v^{\beta}(v^{2\alpha}-4)}\right\}\right] \quad (30)$$

Taking inverse Sadik transform of both sides of (30), we get the required solution of (25) with (26) as

$$\begin{bmatrix} \omega(t) = S^{-1}\left\{\frac{1}{v^{2\alpha+\beta}} + \frac{2}{v^{\beta}(v^{2\alpha}-4)}\right\} \\ = S^{-1}\left\{\frac{1}{v^{2\alpha+\beta}}\right\} + 2S^{-1}\left\{\frac{1}{v^{\beta}(v^{2\alpha}-4)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + sinh2t.$$

VI. CONCLUSIONS

In this paper, authors successfully determined the analytical solution of first kind Volterra integro-differential equation using Sadik transform and four numerical problems solved in numerical problem session of the paper for explaining the complete methodology. The results of numerical problems show that the Sadik transform is very useful integral transform for handling the problem of determining the analytical solution of first kind Volterra integro-differential equation. In future, Sadik transform can be used for determining the analytical solution of system of first kind Volterra integro-differential equations.

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