On the Semi-Implicit Iterative Scheme for a General Class of Map

Alfred Olufemi Bosede[1], Adegoke Stephen Olaniyan[2], Omolara Fatimah Bakre[3]

[1,2]Department of Mathematics, Lagos State University, Ojo, Lagos, Nigeria

[3]Department of Mathematics& Statistics, Federal College of Education (Technical), Akoka, Lagos

Abstract **- Our aim in this paper is to consider a different type of implicit iteration scheme called Semi-Implicit iteration (SII) scheme and study its strong convergence as well as the stability for a general class of maps in a normed linear space. We establish the rate of convergence and present numerical example which validate that the SII iterative scheme converges better than implicit Mann, implicit Ishikawa iteration and implicit S-iteration schemes.**

Keywords: **Semi-Implicit Iterative Scheme, Generalized Contractive-Like Operators, Common Fixed Point, T-Stability, Rate of Convergence**

I. INTRODUCTION AND PRELIMINARY DEFINITIONS

It has been established over the years that implicit iterations have an advantage over explicit iteration schemes because they provide better approximations when compared to explicit iteration schemes. In recent years, implicit iteration schemes for approximating fixed points for non-linear mappings has been proposed and studied by different authors. Several authors have explored implicit iterations in terms of its qualitative features such as convergence, stability and rate of convergence in various spaces (see [4], [6], [7], [8], [9], [14], [15], [16], [20], [21]). However, authors such as ([14], [20], [21]) went ahead to define new implicit iterations. Chugh et al. [14, 2015] defined a new 3-step implicit iteration and studied its strong convergence, stability, rate of convergence and data dependence in convex metric spaces. They concluded that their proposed scheme has better rate of convergence than implicit & explicit Mann and implicit & explicit Ishikawa iteration schemes. Isa and Mujahid [21, 2017] introduced an implicit S-iteration process and studied its convergence in the framework of W-hyperbolic spaces. They concluded that S-iteration process has better rate of convergence than implicit Mann iteration and implicit Ishikawa iteration processes. Recently, Isa [20, 2018] introduced a new 3-step implicit process and proved that it is faster than the other implicit iteration process. Our contribution in this paper is inspired by the above described scheme, therefore we propose a new 3-step iteration called Semi-Implicit iteration scheme for a general class of map T. We show that the scheme converges to a fixed point of T; the scheme is T-stable and has a better convergence rate in comparison with existing schemes.

*Algorithm 1.1. Let (*E, ||.||*) be a normed linear space and* D *a non empty convex closed subset of* E *and* T: D → D *a self map of* D. *Then the Semi Implicit iteration (SII) scheme we propose is as follows: Pick* $x_0 \in$ E *and let sequence* $\{x_n\}_{n=0}^{\infty}$ *be generated by*

$$x_{n+1} = (1-\alpha_n)y_n + \alpha_n T x_n;$$
$$y_n = (1-\beta_n)z_n + \beta_n T y_n; \quad (1)$$
$$z_n = (1-\gamma_n)x_n + \gamma_n T z_n,$$

where sequences $\{\alpha_n\}$, $\{\beta_n\}$ *and* $\{\gamma_n\} \subset [0, 1]$

If $y_n = 0$ in (SII), we get Ishikawa-type iteration:

$$x_{n+1} = (1-\alpha_n)y_n + \alpha_n T x_n;$$
$$y_n = (1-\beta_n)x_n + \beta_n T y_n. \quad (2)$$

Moreover, if $\beta_n = \gamma_n = 0$ in (SII), we get Mann-type iteration:

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n. \quad (3)$$

Remark 1.2. Note that (3) is equivalent to explicit Mann iteration scheme hence the title of our work

Zamfirescu [18, 1972] obtained some interested operators which satisfy, for each pair of points x, y in X with $0 < a < 1$, $0 \le b < ½$ and $0 \le c < ½$ such that at least one of the following is true:

(i) $d(T_x, T_y) \le a d(x, y)$;

(ii) $d(T_x, T_y) \le b[d(x, T_x) + d(y, T_y)]$; (4)

(iii) $d(T_x, T_y) \le c[d(x, T_y) + d(y, T_x)]$.

These class of operators have been adjudged to be the most general contractive-like operators and have been investigated by various authors (see [1], [2], [6], [8]).

We can observe that (4) is equivalent to the following:

$$d(T_x, T_y) \le h\max\{d(x.y), \tfrac{1}{2}[(d(x, T_x) + (y, T_y)], \tfrac{1}{2}[d(x, T_y) + d(y, T_x)]\} \quad (5)$$

where $0 \le h < 1$.

Now, let us consider some of the contractive mappings that will be used in the sequel.

Let E be a normed linear space and D a non-empty convex closed subset of E and T: $D \to D$ be a self map of D for which there exist the real number a $\in$ [0, 1) and x, y $\in D$ such that:

$$\| x - T_y \| \leq a \| x - y \| \quad (6)$$

The contractive condition (5) according to Zamfirescu [18] implies

$$\|T_x - T_y\| \leq \delta\|x-y\| + 2\delta \|x-T_x\| \quad (7)$$

where $\delta \in [0,1)$. Observe that (7) will become (6) only if x is a fixed point of T.

In Osilike [12, 1995] the following contractive definition was used: for each $x, y \in E$, there exist $L \geq 0$ and $a \in [0, 1)$ such that

$$\|T_x - T_y\| \leq a\|x-y\| + L\|x-T_x\| \quad (8)$$

to prove several stability results which are generalizations and extensions of most of the results of Rhoade [5, 2010] as hinted in Bosede et al. [4,2018].

Imoru and Olatinwo [11, 2008] gave the following general contractive definition which is more general than (8):

for each $x, y \in E$, there exists $a \in [0,1)$ and a monotone-increasing function $\varphi : R^+ \to R^-$ with $\varphi(0) = 0$ such that

$$\|T_x - T_y\| \leq \delta \|x - y\| + \varphi(\|x - T_x\|) \quad (9)$$

The suggested assumption of (6) by Bosede and Rhoades [2] put an end to (9) by putting $x=p$ in (9) and in addition, Chidume and Olaleru [6] validated with several examples that (6) is more general than (7), (8) and (9).

We will need the following lemmas and definition to prove our main results.

Lemma 1.3. [19]*Let δ be a real number satisfying $0 \leq \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ a sequence of positive numbers such that $lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying*

$$u_{n+1} \leq \delta u_n + \epsilon_n, \forall \, n \geq 0 \quad (10)$$

then $\lim_{n\to\infty} u_n = 0$.

Lemma 1.4. [17]Let $\{a_n\}_{n=0}^{\infty}$ and $\{\epsilon_n\}_{n=0}^{\infty}$ be non negative real sequences satisfying the following inequality $a_{n+1} \leq (1 - \lambda_n)a_n + \epsilon_n$, such that $\lambda_n \in (0, 1)$, for all $n \leq n_0$, $\sum_{n=0}^{\infty} \lambda_n = \infty$ and $\epsilon_n = o(\lambda_n)$. Then $\lim_{n\to\infty} a_n = 0$.

Definition 1.5.[19]Let $\{a_n\}$ and $\{b_n\}$ be two real convergent sequences with limits a and b respectively. Then $\{a_n\}$ is faster than $\{b_n\}$ if

$$\lim_{n\to\infty} \left|\frac{a_n - a}{b_n - b}\right| = 0$$

Definition 1.6.[19] Let u_n and v_n be two fixed point iterations that converge to the same fixed point p on a normed space X such that the error estimates $||u_n - p|| \leq a_n$ and $||v_n - p|| \leq b_n$ are available such that $\{a_n\}$ and $\{b_n\}$ are two sequences of positive numbers that converge to zero. If $\{a_n\}$ converges faster than $\{b_n\}$ then we say that $\{u_n\}$ converges faster to p than $\{v_n\}$.

Definition 1.7. [9]Let (X, ||.||) be a normed linear space and $T: X \to X$ a self map, $x_0 \in X$ and the iteration procedure defined by

$$x_{n+1} = f(T, x_n), \quad (11)$$

such that the generated sequence $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. Let $\{u_n\}_{n=0}^{\infty}$ be an arbitrary sequence in X, and $\epsilon_n = ||u_{n+1} - f(T, u_n)||$, for $n \geq 0$, then the iteration (11) is T-Stable if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies that $\lim_{n\to\infty} u_n = p$.

II. MAIN RESULT

Theorem 2.1.Let (X, ||.||) be a normed linear space, E a nonempty, convex and closed subset of X. Let T: E →E be a self map satisfying $F(T) \neq \varnothing$ and the inequality:

$$\|Tx-Ty\| \leq a \|x-y\|, \quad (12)$$

where $0 \leq a < 1$. Then, for $x_0 \in E$, the sequence $\{x_n\}$ defined by (1) converges to the fixed point p of T, provided $\sum\alpha_n = \infty$.

Proof: Let p $\in$ F (T). From (1) and (12), we have

$$\|x_{n+1} - p\| \leq (1-\alpha_n)\| y_n - p\| + \alpha_n \| Tx_n - p\|$$

$$\leq (1-\alpha_n)\| y_n - p\| + a\,\alpha_n \|x_n - p\|, \quad (13)$$

Again from (1) and (12), we have

$$\|y_n - p\| \leq (1-\beta_n)\|z_n - p\| + \beta_n \| Ty_n - p\|$$

$$\leq (1-\beta_n)\|z_n - p\| + a\beta_n \| y_n - p\|$$

Therefore,

$$\|y_n - p\| \leq \frac{1-\beta_n}{1-a\beta_n}\|z_n - p\| \quad (14)$$

Furthermore,

$$\| z_n - p \| \leq (1-\gamma_n) \| x_n - p\| + \gamma_n \| Tz_n - p\|$$

$$\leq (1-\gamma_n) \| x_n - p\| + a\,\gamma_n \|z_n - p\|$$

Thus,

$$\|z_n - p \| \leq \frac{1-\gamma_n}{1-a\gamma_n}\| x_n - p\| \quad (15)$$

Assuming $\frac{An}{Bn} = \frac{1-\beta_n}{1-a\beta_n}$, then

$$1 - \frac{An}{Bn} = 1 - \frac{1-\beta_n}{1-a\beta_n} = \frac{\beta_n(1-a)}{1-a\beta_n} \geq \beta_n(1-a) \quad (16)$$

Therefore, $\frac{An}{Bn} \leq 1 - \beta_n(1-a) \leq 1$. (17)

In the same vein,

$\frac{1-\gamma_n}{1-a\gamma_n}$= 1-$\gamma_n(1-a) \leq$ 1. (18)

Making use of (17) in (14), (18) in (15) and all in (13), we have

$||x_{n+1}$- p|| ≤ [aα_{n}+ (1- α_n)]||x_n-p||

≤ [1- α_n(1 – a)]||x_n-p||. (19)

Hence,

$$\left||x_n-p\right|| \leq \prod_{t=1}^{n}[1-\alpha_t(1-a)]\left||x_0-p\right||. \quad (20)$$

Using the fact that 0 ≤a < 1 and $\alpha_n \epsilon$ [0, 1), it follows that

$$\left|\left|x_n-p\right|\right| \leq e^{-\sum_{t=1}^{n}\alpha_t(1-a)}\left||x_0-p\right|| \quad (21)$$

Since$\sum \alpha_n = \infty$, then (21) and application of Lemma (1.4) yields $lim_{n\to\infty} ||x_n-p||$ = 0.We conclude that $\{x_n\}$ converges to p.

Corollary 2.2. Let (X, ||.||) be a normed linear space, E a non empty, convex and closed subset of X and T: E → E a self map such that, F(T) ≠ ∅ and

|| Tx - y || ≤ a|| x – y ||, (22)

where 0 ≤ a < 1. Let the sequences $\{x_n\}$ and $\{y_n\}$ be defined by (2) and (3) with$\sum \alpha_n = \infty$, then for $x_0 \epsilon$ E, we have:

(i) (2) converges strongly to the fixed point p of T;

(ii) (3) converges strongly to the fixed point p of T.

Theorem 2.3.Let E be a Banach space, T: E →E be a self map of E with a fixed point satisfying (6) for each y ϵ E and 0 ≤ a < 1. Then the sequence $\{x_n\}_{n=0}^{\infty}$ defined in (1) with 0 < α <α_n , 0 <β<β_n and 0 <γ<γ_n is T-stable.

Proof: Let$\{t_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ be an arbitrary sequences where

$u_n=\beta_n Tu_n + (1-\beta_n)v_n$,

$v_n=\gamma_n Tv_n + (1-\gamma_n)t_n$.

Let $\epsilon_n = ||t_{n+1}-(1-\alpha_n)t_n - \alpha_n Tu_n||$ and suppose $\lim_{n\to\infty}\epsilon_n = 0$.

Making use of (6), we will now prove that $\lim_{n\to\infty} t_n$ = p

We have

$||t_{n+1}-$ p|| ≤$||t_{n+1}$-(1 - α_n)t_n- α_nTu_n ||

+ || t_n (1 - α_n) - (1 - α_n+α_n) p + α_n Tu_n ||

≤ ϵ_n + (1 - α_n) || t_n– p ||+ α_n || Tu_n - p||. (23)

Applying (6) and substituting in (23), we get

|| t_{n+1}- p|| ≤ ϵ_n + (1 - α_n) || t_n- p||+a α_n||u_n- p||, (24)

||u_n - p|| = ||β_n T u_n + (1-β_n) v_n- (1-β_n + β_n) p||

≤ β_n ||Tu_n -p||+ (1-β_n) ||v_n - p||

≤aβ_n || u_n - p||+(1- β_n) || v_n - p||,

thus

||u_n - p|| = $\frac{1-\beta_n}{1-a\beta_n}$ ||v_n-p||. (25)

Also,

||v_n-p|| = ||γ_nTv_n+ (1-γ_n) t_n- (1- γ_n+ γ_n) p||

≤ γ_n ||Tv_n - p||+ (1-γ_n) ||t_n- p||

≤ aγ_n||v_n- p||+(1- γ_n) || t_n- p||,

thus

||v_n - p|| = $\frac{1-\gamma_n}{1-a\gamma_n}$ || t_n- p ||. (26)

Substituting (26) in (25) and keeping in mind (17) and (18), then (24) becomes

||t_{n+1}- p|| ≤ ϵ_n+ [1- (1-a)α_n] ||t_n-p||. (27)

Application of Lemma 1.3 in (27) results to

$\lim_{n\to\infty} t_n$ = p.

Conversely, let $\lim_{n\to\infty} t_n = p$, we will now show that $\lim_{n\to\infty}\epsilon_n = 0$. Indeed,

ϵ_n = || t_{n+1}- (1 - α_n) t_n- α_n Tu_n||

= || t_{n+1}+ (1 - α_n+ α_n)p - p - (1 - α_n)t_n- α_nTu_n||

≤ ||t_{n+1}- p|| + || (1 - α_n + α_n)p - (1 - α_n)t_n- α_nTu_n||

≤ ||t_{n+1}- p||+ (1 - α_n) ||t_n- p|| + aα_n||u_n- p||. (28)

Substituting (25) and (26) in (28) and making use of (17) results in

ϵ_n ≤||t_{n+1}- p|| + [1 - (1 - a) α_n]||t_n- p||. (29)

Since $\lim_{n\to\infty}$ || t_n - p|| = 0, we obtain $\lim_{n\to\infty}\epsilon_n$ = 0.

Therefore the Semi Implicit Iterative Scheme [SII] (1) is T-Stable.

Corollary 2.4. Let E be a Banach space, T: E →E be a self map of E with a fixed point satisfying (6) for each y ϵ E and 0 ≤ a < 1. Then, the sequence $\{x_n\}_{n=0}^{\infty}$ with 0 < α <α_n and 0 <β<β_n defined in (2) and (3) is T- stable.

III. RATE OF CONVERGENCE

Theorem 3.1.Let D be a nonempty, closed and convex subset of a normed linear space (E, ||.||) and T: D → D a contractive type mapping with F(T) ≠ Ø. Then, the sequence $\{x_n\}$ *defined by the (SII) Iteration with* $\sum (1 - \alpha_n) = \infty$ *converges to a fixed point p of T faster than implicit Mann and Ishikawa Iteration.*

Proof: Using implicit Mann, we have

$$||x_{n+1} - p|| \leq \alpha_n || x_n - p|| + (1- \alpha_n)||Tx_{n+1} - p||$$

$$\leq \alpha_n || x_n - p|| + a\,(1- \alpha_n)|| x_{n+1} - p||,$$

therefore,

$$||x_{n+1} - p|| \leq \frac{\alpha_n}{1 - a(1-\alpha_n)} ||x_n - p||$$

.

.

.

$$\leq \left(\frac{\alpha_n}{1 - a(1-\alpha_n)}\right)^n ||x_0 - p||.$$

So let $b_n = \left(\frac{\alpha_n}{1 - a(1-\alpha_n)}\right)^n || x_0 - p||.$ (30)

For Implicit Ishikawa, we have

$$||x_{n+1} - p|| \leq \alpha_n||x_n - p|| + (1- \alpha_n)||Ty_n - p||$$

$$\leq \alpha_n||x_n - p|| + a(1- \alpha_n)||y_n - p||, \quad (31)$$

thus

$$||y_n - p|| \leq \beta_n||x_{n+1} - p|| + (1- \beta_n)||Tx_{n+1} - p||$$

$$\leq \beta_n||x_{n+1} - p|| + a(1- \beta_n)||x_{n+1} - p||. \quad (32)$$

Therefore,

$$||x_{n+1} - p|| \leq \alpha_n||x_n - p|| + a(1- \alpha_n)[\beta_n + a(1- \beta_n)]||x_{n+1} - p||$$

$$\leq \frac{\alpha_n}{1 - a(1-\alpha_n)[\beta_n + a(1-\beta_n)]} ||x_n - p||$$

.

.

.

$$\leq \left(\frac{\alpha_n}{1 - a(1-\alpha_n)[\beta_n + a(1-\beta_n)]}\right)^n ||x_0 - p||.$$

So let

$$c_n = \left(\frac{\alpha_n}{1 - a(1-\alpha_n)[\beta_n + a(1-\beta_n)]}\right)^n ||x_0 - p||. \quad (33)$$

For (SII) Iteration, making use of (14) and (15) in (13) we get,

$$||x_{n+1} - p|| \leq \frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}|| x_n - p|| + a\alpha_n|| x_n - p||.$$

Therefore

$$||x_{n+1} - p|| \leq \frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n) + a\alpha_n(1-a\beta_n)(1-a\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}||x_n - p||$$

.

.

.

$$\leq \left(\frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n) + a\alpha_n(1-a\beta_n)(1-a\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}\right)^n ||x_0 - p||.$$

Also, let

$$a_n = \left(\frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n) + a\alpha_n(1-a\beta_n)(1-a\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}\right)^n ||x_0 - p||$$

Making use of definitions (1.5) and (1.6), we get

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0 \text{ and } \lim_{n\to\infty} \frac{a_n}{c_n} = 0.$$

Evidently,

$$\lim_{n\to\infty} \frac{a_n}{c_n} = \frac{\left(\frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n) + a\alpha_n(1-a\beta_n)(1-a\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}\right)^n ||x_0 - p||}{\left(\frac{\alpha_n}{1 - a(1-\alpha_n)[\beta_n + a(1-\beta_n)]}\right)^n ||x_0 - p||} = 0.$$

which implies that the (SII) Iteration converges faster than Implicit Ishikawa Iteration.

Also,

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\left(\frac{(1-\alpha_n)(1-\beta_n)(1-\gamma_n) + a\alpha_n(1-a\beta_n)(1-a\gamma_n)}{(1-a\beta_n)(1-a\gamma_n)}\right)^n ||x_0 - p||}{\left(\frac{\alpha_n}{1 - a(1-\alpha_n)}\right)^n ||x_0 - p||} = 0.$$

This implies that the (SII) Iteration converges faster than Mann Iteration.

It implies that (SII) Iteration converges faster than both Implicit Mann and Implicit Ishikawa iteration processes.

Let us now buttress the above proof with numerical example.

Example 3.2.(See [21]) Let E = [0,1] be defined by $T_x = \frac{1}{2}$, $x \neq 0$, $a \in [0,1]$ and $\alpha_n = \beta_n = \gamma_n = 1- \frac{1}{n}$ for $n \geq 2$ .The comparison of the convergences of the implicit Mann (IMI), implicit Ishikawa (III), implicit S-iteration(ISI)[15] and semi implicit iteration (SII) to the fixed point p = 0 are given with initial value x_1= 1 in table below.

Table 1.Comparison of the Convergence Rate of IMI, III, ISI and SII

n	IMI	III	ISI	SII
2	0.666666666666667	0.615384615384615	0.307692307692308	0.388504155124654
5	0.406349206349209	0.352704628530670	0.022044039283167	0.023512284740250
10	0.28377319275152	0.240691952056443	0.000470101468860	0.000379697556274
16	0.22329413874241	0.187699995568689	0.000005728149279	0.000003734341016
20	0.19940865344744	0.167113839554526	0.000000318744353	0.000000187028417
25	0.17813377193108	0.148920204678483	0.000000008876336	0.000000004680741
30	0.16247771019742	0.135609685043003	0.000000000252593	0.000000000121966
35	0.15033562847356	0.125328510781087	0.000000000007295	0.000000000003268
40	0.14056334382810	0.117078595772533	0.000000000000213	0.000000000000089
46	0.13102258080520	0.109013978938918	0.000000000000003	0.000000000000001
47	0.12928481035291	0.107063257860022	0.000000000000001	0.000000000000000
50	0.12564512901850	0.104523598655989	0.000000000000000	0.000000000000000

Remark 3.3. From the above example, it is observed that semi implicit iteration (SII) iteration has faster convergence rate than implicit Mann iteration (IMI), im plicit Ishikawa iteration (III) and implicit S-iteration (ISI) proposed in [21].

REFERENCES

[1] O. Bosede, *Strong Convergence of Noor Iteration for a General Class of Functions*. Bulletin of Mathematical Analysis and Applications.2011; Vol. (2011):140− 145.

[2] O. Bosede and B. E. Rhoades, *Stability of Picard and Mann Iteration for a General Class of Functions*. Journal of Advanced Mathematical Studies. 2010; 3 (2): 1 − 3.

[3] O. Bosede,Some *Common Fixed Point Theorems in Normed Linear Spaces*. Acta Univ. Palacki. Olomuc. Fac. rer. nat., Mathematica. 2010; 49(1): 17 − 24.

[4] O. Bosede, H. Akewe, O. F. Bakre and A. S. Wusu, *On the Equivalence of Implicit Kirktype Fixed Point Iteration Schemes for a General Class of Maps*. Journal of Applied Mathematics and Physics, 2019, 7, 123-137.

[5] E. Rhoades, *Fixed Point Theorems and Stability Results for Fixed Point Iteration Procedures*. Indian Journal of Pure and Applied Mathematics, Vol. 24, no. 11, pp. 691703; 1993.

[6] E. Chidume and J. O. Olaleru, *Picard Iteration Process for a General Class of Contractive Mappings*. Journal of Nigerian MathematicalSociety, Vol.33, (2014).pages19–23.

[7] F. Gursoy, V. Karakaya and B. E. Rhoades, *Data Dependence Results of New Multi-Step and S-iterative Schemes for Contractive- like Operators.* Fixed Point Theory and Applications, 76 (2013), 112.

[8] H. Akewe, G.A. Okeke and A. Olayiwola, *Strong Convergence and Stability of Kirk-Multistep-Type Iterative Schemes for Contractive-type Operators.* Fixed Point Theory Appl. 45(2014), 24 pages.

[9] H. Akewe, and J. O. Olaleru, *On Convergence and Stability of Generalized Noor Iterations for a General Class of Operators*. British Journal of Mathematics and Computer Science 3(3): 437−447, 2013.

[10] M. O. Olatinwo, *Stability Results for Some Fixed Point Iterative Processes in Convex Metric Spaces*. International Journal of Engineering, Vol. 9, pp. 103106; 2011.

[11] M. O. Olatinwo and C. O. Imoru, Some *convergence results for the Jungck-Mann and the Jungck-Ishikawa iteration processes in the class of generalized Zamfirescu operators*. Acta MathematicaUniversitatisComenianae, LXXVII, 2,299304, 2008.

[12] M. O. Osilike, Stability *Results for Ishikawa Fixed Point Iteration Procedure*. Indian Journal of Pure and Applied Mathematics. 1995/96; 26(10): 937-941.

[13] P. Ky and T. Quoc, Stability *and convergence of implicit iteration processes*. Vietnam Journal of Mathematics, Vol. 32,No. 4, pp. 467473,2004.

[14] R. Chugh, P. Malik and V. Kumar, *On a new faster implicit fixed point iterative scheme in convex metric space*. J. Funct. Spaces 2015(2015), Article ID 905834, 11pages.

[15] R. Chugh, P. Malik andV. Kumar, *On analytical and numerical study of implicit fixed point iterations.* Cogent Mathematics (2015), 2:1021623.

[16] R. Chugh and V. Kumar, *Convergence of SP Iterative Scheme with*

Mixed Errors for Accretive Lipschitzian and Strongly Accretive Lipschitzian Operators in Banach Spaces. International Journal of Computer Mathematics, Vol. 90, no. 9, pp.18651880, 2013.

[17] S.M. Soltuz, *The Equivalence of Picard, Mann and Ishikawa Iterations Dealing with Quasi-Contractive Operators*. Math. Commun., 10(2005), 81-88.

[18] T. Zamfirescu, Fixed *Point Theorems in Metric Space*. Arch. Math. (Basel), 23, 292-298 (1972).

[19] V. Berinde, *Picard Iteration Converges Faster Than Mann Iteration for a Class of Quasi-Contractive Operators*. Fixed Point Theory and Applications, no. 2, pp.94105, 2004.

[20] Y. Isa, On *the Rate of Convergence of Different Implicit Iteration in Convex Metric Space*. Konuralp Journal of Mathematics.2018.

[21] Y. Isa and A. Mujahid, *Convergence Rate of Implicit Iteration Process and a Data Dependence Result*. European Journal of Pure and Applied Mathematics.2017.