Modelling Typhoid Fever with General Knowledge, Vaccination and Treatment for Susceptible Individual

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Keywords: Treatment, Basic Reproduction Number, Vaccination, Lyapunov Function.

I. INTRODUCTION


II. THE BASIC MATHEMATICAL MODEL

In This Paper, The Model In Equation (1) Was Adopted And Modified By Incorporating An Incidence Rate Which Include General Knowledge Parameter $m$

$$\frac{dS}{dt} = \lambda + \omega V + \omega_2 R - (\theta + \mu + (1-\psi_e)\frac{\beta S I}{1+mS} + \psi_e \gamma C) S$$

$$\frac{dV}{dt} = \delta S - (\omega_1 + \mu) V$$

$$\frac{dI}{dt} = (1-\rho)(1-\psi_e)\frac{\beta S I}{1+mS} - (\eta + d_2 + \mu) I - \alpha(1-\psi_e)\gamma C$$

$$\frac{dC}{dt} = \rho(\beta S C)S - (\mu + d_1)I_C - dIC$$

$$\frac{dR}{dt} = \eta l - (\omega_2 + \mu) R$$

2.1 Modified Model of Equation

We Obtain 5-Dimensional Non-Linear System Of Ordinary Differential Equations Describing The Transmission Of Typhoid Fever.

$$\frac{dS}{dt} = \Lambda + \omega V + \omega_2 R - (\theta + \mu + (1-\psi_e)\frac{\beta S I}{1+mS} + \psi_e \gamma C) S$$

At Equilibrium, $\Lambda + \omega V + \omega_2 R - \delta S - \mu S - (\beta S I + \gamma C) S = 0$ (3.1)

At Disease-Free Equilibrium $I = 0$

From Equation (3.5)

$$\eta l - (\omega_2 + \mu) R = 0$$
\[ R^0 = 0 \]

From Equation (3.2)
\[ \theta S - (\omega + \mu)V = 0 \]
\[ \frac{\theta S}{\omega + \mu} = (\omega + \mu)V \]
\[ V = \frac{\theta S}{(\omega + \mu)} \] (3.6)

Substitute \( V = \frac{\theta S}{(\omega + \mu)} \) and \( R = 0 \) into (3.1)

Also From (3.1) With Various Substitution
\[ S = \frac{\lambda (\omega + \mu) \theta}{\mu (\omega + \mu) + \theta} \]
\[ \Rightarrow S^0 = \frac{\lambda (\omega + \mu) \theta}{\mu (\omega + \mu) + \theta} \] (3.7)

Putting Equation (3.6) Into (3.7)
\[ V = \theta \left( \frac{\lambda (\omega + \mu) \theta}{\mu (\omega + \mu) + \theta} \right) \]
\[ \Rightarrow V = \frac{\theta \lambda (\omega + \mu) \theta}{\mu (\omega + \mu) + \theta} \] (3.8)

Model System (2) Has A Disease Free Equilibrium
\[ E_0 = (S^0, V^0, I^0, C^0, R^0) \left( \frac{\lambda (\omega + \mu) \theta}{\mu (\omega + \mu) + \theta}, 0,0,0 \right) \] (3.9)

2.3 Endemic Equilibrium

At Endemic Equilibrium, If We Let
\[ E = (S^*, V^*, I^*, C^*, R^*) \] Satisfies (S, V, I, C, R) > 0

From Equation (3.5)
\[ \eta I - (\omega + \mu)R = 0 \]
\[ \frac{\eta I}{\omega + \mu} = \frac{(\omega + \mu)R}{\omega + \mu} \]
\[ R^* = \frac{\eta I^*}{\omega + \mu} \]

From Equation (3.2)
\[ \theta S - (\omega + \mu)V = 0 \]
\[ \theta S = (\omega + \mu)V \]
\[ V^* = \frac{\theta S^*}{\omega + \mu} \]

From Equation (3.3)
Hence

\[ I = \frac{1}{(a+\mu)(a+\beta)(a+\gamma)(a+\delta)} \left[ -\eta_0 P \right] \]

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Dividing through by \( I_c^* \) gives

\[ \frac{(a+\mu)(a+\beta)(a+\gamma)(a+\delta)}{I_c^*} \left[ \frac{(-\eta_0 P)}{dT_d^*} \right] \]

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Now putting \( I^* \) in \( S^* \) gives new \( S^* \), \( I^* \), \( I_c^* \) in

Finally, the characteristic equation

\[ (P-1)BS + (1-P)S \]

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The Dominant Eigen Value Of 5.2 Is The Ro After Substituting The Value Of \( S^* \) into:

\[ R_0 = \frac{\alpha(P-1)\beta(\omega + \mu) + \alpha(P-1)\gamma(\omega + \mu)(\mu + d_1 + \alpha)}{\mu(\theta + \omega + \mu) + m(\omega + \mu)\eta + d_2 + \mu(\mu + d_1 + \alpha)} \]

\[ R_0 = \frac{\alpha(P-1)\beta(\omega + \mu) + \alpha(P-1)\gamma(\omega + \mu)(\mu + d_1 + \alpha)}{\mu(\theta + \omega + \mu) + m(\omega + \mu)\eta + d_2 + \mu(\mu + d_1 + \alpha)} \]

VI. LOCAL STABILITY OF DISEASE FREE EQUILIBRIUM

\[ J(F_0) = \left[ \begin{array}{cccc} \frac{dS}{dS} & \frac{dS}{dV} & \frac{dS}{dV} & \frac{dS}{dS} \\ \frac{dS}{dV} & \frac{dS}{dV} & \frac{dS}{dS} \\ \frac{dS}{dV} & \frac{dS}{dS} & \frac{dS}{dS} \\ \frac{dS}{dS} & \frac{dS}{dS} & \frac{dS}{dS} \end{array} \right] \]

(6.1) So That,
Let \( a = -(1-\rho)\beta_S^0 - (\eta + d_1 + \mu) \)
\( b = -(1-\rho)\beta_S^0 + \alpha \)
\( c = (1-\rho)\mu^0 \)
\( d = \rho\beta_S^0 - (\eta + d_2 + \mu) \)

Hence,
\[
J(E_0) = \begin{vmatrix}
-\theta & 0 & 0 & \omega_T & 0 & 0 \\
0 & a & c & 0 & -\beta_S^0 & 0 \\
0 & 0 & (1-\rho)\beta_S^0 - (\eta + d_2 + \mu) & (1-\rho)\gamma_S^0 + \alpha & \rho\beta_S^0 - (\eta + d_2 + \mu) \\
0 & 0 & 0 & \rho\beta_S^0 - (\eta + d_2 + \mu) \\
\end{vmatrix}
\]

Then \( \text{Tr}(J(E_0)) = -\theta + \omega_T(\theta + \omega_T + \mu) + \mu^2(\theta + \omega_T) \)

\( \text{Det}(J(E_0)) = \mu(\theta + \omega_T + \mu)(ad - bc) \)

Simplification gives
\[
(ad - bc) = \frac{bc}{ad} \begin{vmatrix} \frac{1}{ad} \end{vmatrix}
\]

Therefore \( \text{Det}(J(E_0)) > 0 \) if and only if \( R_0 < 1 \)

VII. LOCAL STABILITY OF ENDEMIC EQUILIBRIUM

If We Let...
\[ \text{det}(J(E^*)) = \mu(\theta + \alpha_1 + \mu)(ch - fg) \]

Simplification gives

\[ (ch - fg) = eh \left(1 - \frac{fg}{eh}\right) \]

So, \( \text{det}(J(E^*)) = \mu(\theta + \alpha_1 + \mu)(ch - fg) = \mu(\theta + \alpha_1 + \mu)eh\left(1 - \frac{fg}{eh}\right) \]

Where \( R_C = \frac{fg}{eh} \)

\[ \text{det}(J(E^*)) = \mu(\theta + \alpha_1 + \mu)eh(1 - R_C) \]

Therefore, \( \text{det}(J(E^*)) > 0 \) if and only if \( R_C < 1 \)

VIII. GLOBAL STABILITY

Note: \( \frac{dv}{dt} = V_x, x + V_y, y \)

Disease Free-Equilibrium At Global Stability

To Investigate the Global Stability of \( X_0 \), Consider the Lyapunov Function.

\[ L(I_e, I) = I + d \]

We Take Asymptotic Infectious And Symptomatic Infectious, \( L(I_e, I) \leq 0 \) for \( R_0 \leq 1 \)

We Take the Derivative of Equation (8.1) Gives

\[ L(I_e, I) = I + d \]

If we substitute \( I = \frac{dI}{dt} = \frac{(1 - p)(\beta \mu + \beta_2 \mu)}{1 + ms} - (\eta + d_1 + \mu)d + ad \]

And \( I_e = \frac{(\beta \mu + \beta_2 \mu)S}{1 + ms} - (\eta + d_1 + \mu)I + ad_e \) (8.2)

\[ L(I_e, I) = \frac{(1 - p)(\beta \mu + \beta_2 \mu)}{1 + ms} - (\eta + d_1 + \mu)I + ad \]

\[ \frac{dL}{dt} = \frac{(1 - p)(\beta \mu + \beta_2 \mu)}{1 + ms} - (\eta + d_1 + \mu)I + ad - \left(\frac{p}{1 + ms}\right)(\alpha \mu + a)\]

\[ L(I_e, I) = \frac{(1 - p)(\beta \mu + \beta_2 \mu)}{1 + ms} - (\eta + d_1 + \mu)I + ad_e \]

\[ \frac{dL}{dt} = \frac{(1 + ms)(\alpha \mu + a)!}{(1 + ms)!} \]

Note: If \( abcd = 0 \) Then \( A = 0, B = 0, C = 0, D = 0 \).

\[ \alpha_1 + \mu = 0 \quad \mu = -\alpha_1 \]

\[ \frac{dL}{dt} = -(\eta + d_2 + \alpha_1)I - ((-\eta_1 + d_1 + \alpha)E - \alpha)I_e \]

Since \( \kappa > 0 \) And Sufficiently Small And \( \delta > 0 \) Then

\[ \frac{dL}{dt} = -((\eta + d_2 + \alpha_1)I - ((-\eta_1 + d_1 + \alpha)E - \alpha)I_e \]

Then \( L < 0 \) if \( L < L_0 \).

Thus \( L \) Is Globally Stable For All Initial Conditions

\[ L(t) \rightarrow L \text{ as } t \rightarrow \infty \]

IX. ANALYSIS OF CONTROL STRATEGIES FOR DISEASE FREE EQUILIBRIUM

\[ R_C = \frac{[1 - (P - 1)(\beta_1 + \beta_2 + \mu)]}{1 + ms} - (\eta + d_1 + \mu)I + ad \]

\[ \frac{dL}{dt} = -(\eta + d_2 + \alpha_1)I - ((-\eta_1 + d_1 + \alpha)E - \alpha)I_e \]

It is really clear that \( R_C \) increases as \( y \) increases. This agrees with the belief that higher transmission increases the basic reproduction number.

To see the effect of \( R_0 \) on \( R_C \), we note that

\[ R_C = \frac{[1 - (P - 1)(\beta_1 + \beta_2 + \mu)]}{1 + ms} - (\eta + d_1 + \mu)I + ad \]

\[ \frac{dL}{dt} = -(\eta + d_2 + \alpha_1)I - ((-\eta_1 + d_1 + \alpha)E - \alpha)I_e \]

\[ \frac{dL}{dt} = \frac{(\alpha \mu + a)!}{(1 + ms)!} \]

Thus we have

\[ \frac{dL}{dt} = \frac{(\alpha \mu + a)!}{(1 + ms)!} \frac{L_a}{(\eta + d_2 + d_1 + \alpha)E - \alpha} \]

\[ \frac{dL}{dt} = \frac{(\alpha \mu + a)!}{(1 + ms)!} \frac{L_a}{(\eta + d_2 + d_1 + \alpha)E - \alpha} \]

X. SENSITIVITY ANALYSIS OF \( R_C \)

\[ R_C = \frac{1}{\mu_0 + \mu_1 + \mu_2 + \mu_3} \]
11.1 Results

\[
\frac{R}{\theta} = \frac{R_e}{\theta} = \frac{\theta}{(\theta+\alpha+\beta)^2} \left[ \frac{\theta (\alpha+\mu) \mu (\mu+d_2+\eta) (\mu+d_1+\alpha)}{(\mu+d_2+\eta)(\mu+d_1+\alpha)} \right]
\]

Hence

\[
\frac{R}{\theta} = \frac{\theta}{(\theta+\alpha+\beta)^2}
\]

XI. RESULTS AND DISCUSSION

11.2 Discussion

Figure 1-3, Reveals The Effect Of General Knowledge In Eradicating Typhoid Fever. It Implies That At \( M=0.01 \), The Susceptible Class Increases Slightly. But At \( M=0.5 \) And 1.0, The Susceptible Class Increases Drastically While Infected Reduces To The Minimum.

XII. CONCLUSIONS

The Simulation Results Reveals The Effect Of The General Knowledge, That Is The Lower The General Knowledge \( m \) The Little Higher The Susceptible Compartment, Because Of The Presence Of Other Parameters. Also, The Higher The General Knowledge The Highest Level Of Susceptible Class Is Observed. This Means, When There’s Presence Of Vaccination, Treatment And General Knowledge, There Is High Tendency For Disease Eradication. It Is Also Clearly
Show That General Knowledge Play Vital Role Than Education In Disease Eradication. Hence Education Is More Than Schooling.

REFERENCES


