On the New Trend of Modeling the Impact of Interacting Rate on the Vulnerability to Ebola Infection

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***Abstract*- By using an appropriate numerical method to model the impact of interacting rate of virus and uninfected cells, we have found that a bifurcation in the quantification of the impact of the disease condition has occurred between a decreased variation value of β at 99.8% and increased variation value of 105% and also the lower limit of the bifurcation interval to be 0.001188 and the upper limit of the bifurcation interval to be 0.00126 which is a vital intervention information in our bid to mitigate against the Ebola disease condition. The novel results that we have obtained which we have not seen elsewhere are fully presented and discussed quantitatively**.

***Key words*: Numerical Method, Population Depletion, Mathematical Modeling, Parallel Data, Bifurcation Interval**

I. INTRODUCTION

One of the process of understanding the dynamics of Ebola virus depends on the application a deterministic interacting system of a dynamical system that describes the growth of the density of uninfected cells at time *t*, the growth of the density of the infected cells at time *t*, the growth of the density of the virus at time *t*, the growth of the density of cytotoxic T-lymphocytes at time *t*,

This important mathematical construction has used the theory of population dynamics to clearly show the interaction between the four components of the spread of Ebola virus on the concept of a compartmental modeling [9]. Despite the interesting mathematical application of the uniqueness and existence of a solution, local stability, and stability analysis to mention but a few, there remain other open research questions yet to be fully explored. One of such research questions of concern bothers on the selection of the best fit parallel data which can be used to validate construction of the mathematical model on the spread of Ebola virus, as this idea is highly likely to provide a stronger mitigation measure. Other related mathematical contributions can be seen as read in works of [5] [1] [2] [7] [8] [3] [4]

II. MATHEMATICAL FORMULATION

Following Wester et al (2015), we consider the system of time dependent non-linear first ordinary differential equations of dynamical system of Ebola virus describing four distinct populations which are denoted by:

$X(t)$: density of uninfected cells at time t,

$I(t)$: density of infected cells at time t,

$V(t)$: density of virus at time t,

$T(t)$: density of cytotoxic T-lymphocytes at time t.

$$\frac{dX}{dt} = \lambda - \mu X(t) - \beta V(t)X(t) \quad (1)$$

$$\frac{dI}{dt} = \beta V(t)X(t) - \rho I(t)T(t) - \alpha I(t) \quad (2)$$

$$\frac{dV}{dt} = cI(t) - \gamma V(t) \quad (3)$$

$$\frac{dT}{dt} = \rho I(t)T(t) - \delta T(t) \quad (4)$$

With initial conditions $X(0) = X_0$, $I(0) = I_0$, $V(0) = V_0$, $T(0) = T_0$.

Other variables and parameters of the model includes

Parameter	Biological Description	Estimated Value Range	Units
λ	Growth Rate (Uninfected cells)	0.1 - 10	*ml/cells×days*
μ	Death Rate (Uninfected cells	0.02 - 0.03	1/*days*
β	Interaction Rate (Virus and Uninfected cells)	0.001 - 0.02	*ml/cells×days*
ρ	Interaction Rate (Infected cells and CTLs)	0.1	*ml/cells×days*
α	Death Rate (Infected Cells)	0.2 - 0.7	1/*days*
c	Growth Rate (Virus)	20 - 50	1/*days*
γ	Death Rate (Virus	2.4 - 3.8	1/*days*
δ	Death Rate (CTLs)	0.3 - 0.5	1/*days*

Source**:** Wester et al (2015),

III. METHOD OF ANALYSIS

For the purpose of clarity and focus, we have chosen to explore the deterministic implementation of the ODE 45 (which is computationally more efficient than ODE 23, ODE 23TB and ODE15s) in order to evaluate the effect of varying the value of the interacting rate (of virus and uninfected cell) of Ebola virus. The notation EPD represents the expected population deficit and EPI represents expected population increase.

3.1 MATLAB Algorithm

```
function dN=nwa10(t,N)

dN=zeros(4,1);

lambda=0.2;

meu=0.024;

beta=0.0012;

p=0.1;

alpha=0.25;

c=22.5;

gamma=2.42;

delta=0.36;

 X=N(1)

% I=N(2)

% V=N(3)

% T=N(4)

dN(1)=lambda-meu*N(1)-beta*N(3)*N(1)

dN(2)=beta*N(3)*N(1)-p*N(2)*N(4)-alpha*N(2)

dN(3)=c*N(2)-gamma*N(3)

dN(4)=p*N(2)*N(4)-delta*N(4)
```

IV. RESULTS

The results are presented and displayed as shown in the figures and tables below. For purpose of clarity, the term V(t) represents unmodified virus at time t while Vm (t) represents the modified virus at time

Table 1: Evaluating the effect of decreasing the interacting rate of virus and uninfected cells by 99.8% using ODE45 numerical method.

t (days)	V(*t*)	Vm(*t*)	EPD(%)
1	0.400000000000000	0.400000000000000	0.00000000000000
3	1.387939901079178	1.387914626869217	0.001820987345436
5	0.566316591173700	0.566279859098428	0.006486137938588
7	0.285419618469304	0.285383127462285	0.012785038118479
9	0.161934522783045	0.161901244804860	0.020550267856922
11	0.097700462308006	0.097671410951599	0.029735126857344
13	0.060985980368290	0.060961410722751	0.040287366688951
15	0.038831423325099	0.038811141737940	0.052229831984063
17	0.025019597641007	0.025003238808633	0.065384074552088
19	0.016293051274757	0.016280062143526	0.079721907281760
21	0.010677999509070	0.010667813820569	0.095389482755426
23	0.007046870091454	0.007038985762615	0.111884123545114
25	0.004678791224672	0.004672720377622	0.129752467224797
27	0.003120201098962	0.003115573883926	0.148298615677489
29	0.002094104986479	0.002090589119470	0.167893540755915
31	0.001410482182410	0.001407824281262	0.188439186366263

Table 2: Evaluating the effect of increasing the interacting rate of virus and uninfected cells by105% using ODE45 numerical method.

t (days)	V(*t*)	Vm(*t*)	EPI(%)
1	0.400000000000000	0.400000000000000	0.000000000000000
3	1.387939901079178	1.388571789731296	0.045527090303232
5	0.566316591173700	0.567235328872876	0.162230404952735
7	0.285419618469304	0.286332987664931	0.320009255329201
9	0.161934522783045	0.162768257765390	0.514859319690286
11	0.097700462308006	0.098429139105539	0.745827379235098
13	0.060985980368290	0.061603065986060	1.011848319963549
15	0.038831423325099	0.039341574877371	1.313759601341702
17	0.025019597641007	0.025431775175211	1.647418715994431
19	0.016293051274757	0.016620921331659	2.012330602616075
21	0.010677999509070	0.010935622818829	2.412655193888424
23	0.007046870091454	0.007246711064274	2.835882742640083
25	0.004678791224672	0.004833014448696	3.296219399803357
27	0.003120201098962	0.003238037916546	3.776577657879265
29	0.002094104986479	0.002183857559772	4.285963400722626
31	0.001410482182410	0.001478519078254	4.823662198107703

Table 3: Evaluating the effect of increasing the interacting rate of virus and uninfected cells by 110% using ODE45 numerical method.

t (days)	V(*t*)	Vm(*t*)	EPI(%)
1	0.400000000000000	0.400000000000000	0.000000000000000
3	1.387939901079178	1.389203742570961	0.091058805269606
5	0.566316591173700	0.568154904671636	0.324608801258330
7	0.285419618469304	0.287248462481620	0.640756238875473
9	0.161934522783045	0.163605433370441	1.031843339319893
11	0.097700462308006	0.099162433566794	1.496381106343447
13	0.060985980368290	0.062225650112055	2.032712659988456
15	0.038831423325099	0.039857753337178	2.643039899634614
17	0.025019597641007	0.025850176621390	3.319713579331118
19	0.016293051274757	0.016954904036686	4.062178107512748
21	0.010677999509070	0.011199060455015	4.879761845863784
23	0.007046870091454	0.007451897144892	5.747616291793722
25	0.004678791224672	0.004992047048800	6.695229795154178
27	0.003120201098962	0.003360118082268	7.689151298153019
29	0.002094104986479	0.002277278074745	8.747082378825176
31	0.001410482182410	0.001549704411579	9.870541500306750

Table 4: Evaluating the effect of increasing the interacting rate of infected and uninfected cells by 120% using ODE45 numerical method.

t (days)	V(t)	Vm(t)	EPI(%)
1	0.400000000000000	0.400000000000000	0.000000000000000
3	1.387939901079178	1.390467840523633	0.182136088348650
5	0.566316591173700	0.569996570495220	0.649809555092240
7	0.285419618469304	0.289085739177488	1.284466964059883
9	0.161934522783045	0.165290147175363	2.072210628498294
11	0.097700462308006	0.100642969676032	3.011764016785712
13	0.060985980368290	0.063487481697122	4.101764559207832
15	0.038831423325099	0.040908443479194	5.348812833115946
17	0.025019597641007	0.026705991416415	6.740291349224648
19	0.016293051274757	0.017641630176079	8.277018703128313
21	0.010677999509070	0.011743867167822	9.981903987227646
23	0.007046870091454	0.007878845398830	11.806309703147445
25	0.004678791224672	0.005325108239813	13.813760522853791
27	0.003120201098962	0.003617593718940	15.941043676413980
29	0.002094104986479	0.002475715398211	18.223079272380403
31	0.001410482182410	0.001702084690789	20.673959020220511

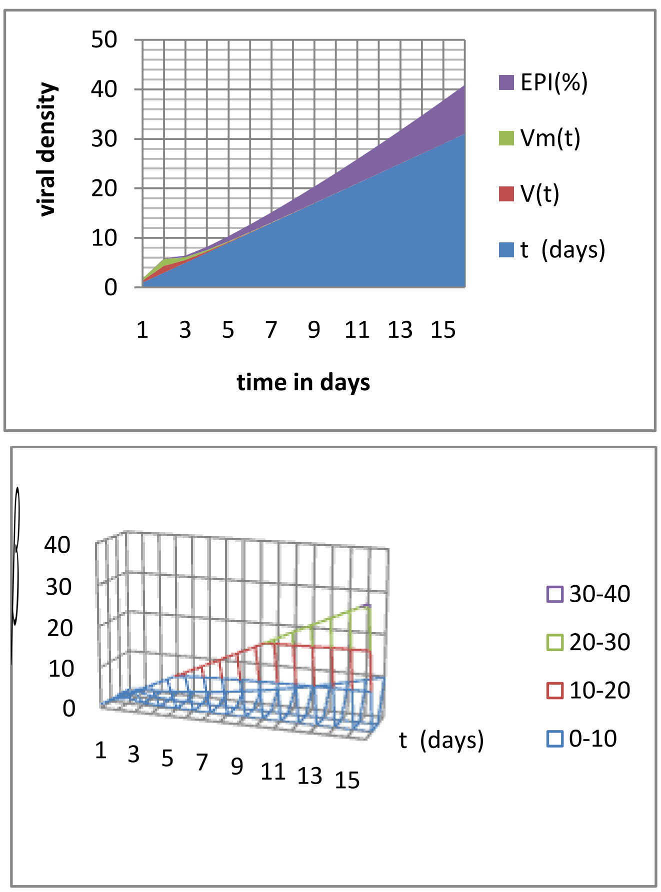

Figure 1: A comparison of modified and unmodified viral density for an increased value of β

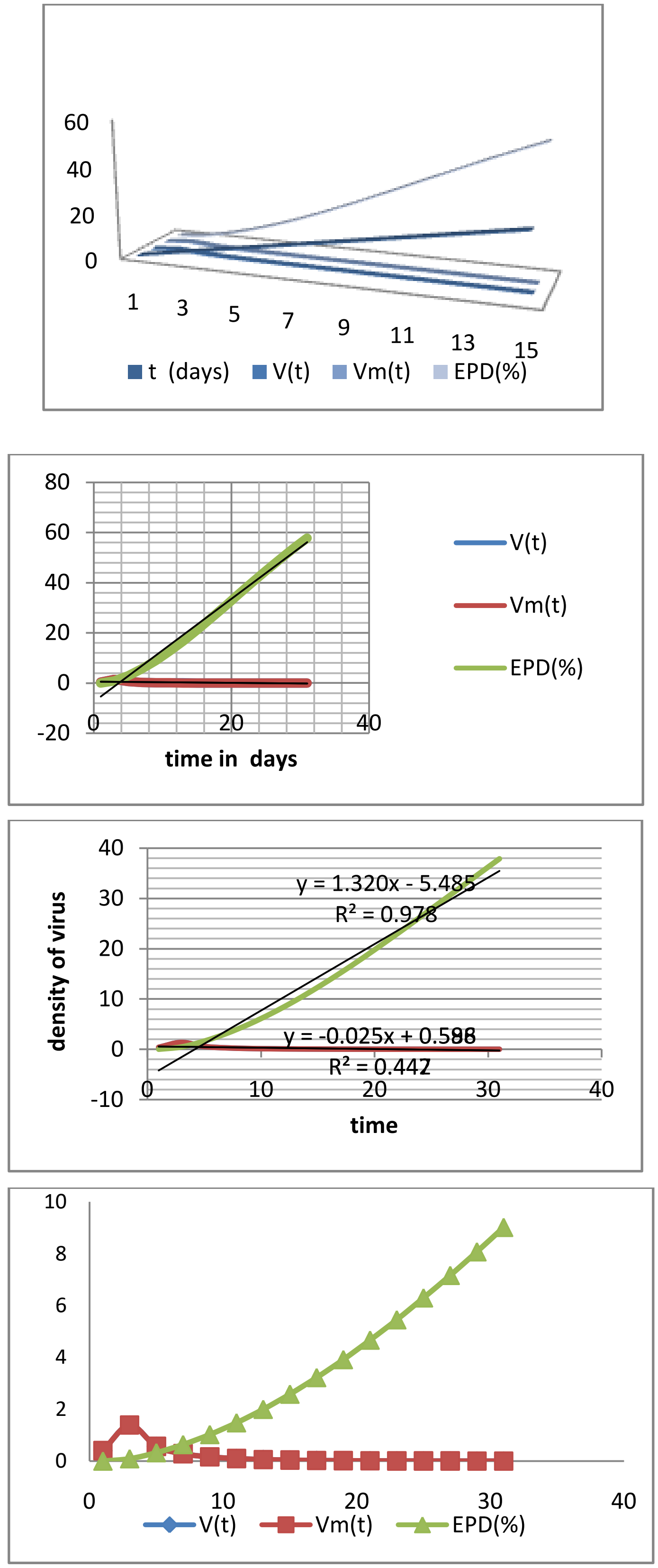

Figure 2: A comparison of modified and unmodified viral density for varying reduced values of β

V. DISCUSSIONS OF RESULTS

What do we learn from these tables and figures of results? When the value of β is decreased by 5%, we have observed that the V(t) population is generally vulnerable to a depletion indicating a better likelihood that the disease will be under control at day 31. For other values of β varied by 10%, 50%, 95%, 99%, we have found a consistent general depletion of the disease impact , that is, at a relatively lower reduced interacting rate (of virus and uninfected cells) , the estimated proportion that depleted is 0.019 approximately by day 31. Thereafter, at other variations of the same model parameter values such as 105%, 110%, 120%, we have found the general increase in the same disease condition.

On the basis of this simulation analysis, we have observed that when the interacting rate (of the virus and uninfected cell) thereby denoted by β has occurred at fure 1 with EPI of 59.82% approximately. Therefore, the disease in question is highly likely to go out of control if an appropriate health mitigation measure is not taken early enough.

Without lost of generality we have found that a bifurcation in the quantification of the impact of the disease condition has occurred between a decreased variation value of β 99.8% and increased variation value of 105%. We have found the lower limit if the bifurcation interval to be 0.001188 and the upper limit of the bifurcation interval to be 0.00126. It is interesting to observe that the metric of the selected bifurcation interval is 0.000072.

Our present contribution to this body of knowledge to the discipline of mathematical modeling and numerical simulation of health information is a sound cutting edge novel result over the earlier modelers.

VI. CONCLUSION AND FURTHER RESEARCH

A small varying value of β (interaction rates of the virus and uninfected cells) shows a bigger value of depletion at five percent whereas a relatively decreased value of 99.8% is associated with a small value of depletion. In contrast, an increased variation of β is an indication that a diseased condition is getting out of control.

In this present study, we did not look at the pattern of disease control if $\beta(0) = 0.4$ is slightly varied. This will be the subject of our next investigation. After a second thought, on the philosophy of algebra there can be another improved lower limit of bifurcation and another improved upper limit of bifurcation. This idea can be cautiously and mathematically handled in our next investigation.

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