Modelling Chicken Production in Nigeria Using the Autoregressive Integrated Moving Average Time Series Model

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***Abstract*- This research work modeled chicken production in Nigeria using the least square approach as well as the univariate Box-Jenkins Autoregressive Moving Average (ARMA) model method. The objective is to investigate the production trend and make logical forecast for poultry farmers to meet up with any future challenges that may arise due to the increase in demand for poultry meat. The maximum likelihood method of estimation was used to obtain the parameters of the fitted Autoregressive Integrated Moving Average (ARIMA) model. Yearly chicken production data for the period of 1961 to 2017 obtained from Food and Agricultural Organisation (FAO) was used to investigate the performance of the model. Augmented Dickey Fuller (ADF) test carried out to check for stationarity of the series shows that the original data was nonstationary and stationarity was attained after the first differencing. ARIMA (2, 1, 2) emerged as the best among other fitted models with Akaike Information Criterion (AIC) value of 1168.42 and log likelihood −579.21. The optimum model is given by:**

$$\hat{X}_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \theta_1 e_{t-1} + \theta_2 X_{t-2}$$

with estimated parameters φ_1 = 0.3169, φ_2 = − 0.5005, θ_1 = − 0.3948, θ_2 = 0.9377. Diagnostic checks via standardised residuals, ACF of residuals and p-values revealed that the model captures the data well enough. Forecast values for the year 2018 to 2023 were then obtained which show a considerable variation over the years. The Autoregressive Integrated Moving Average (ARIMA) model clearly performed excellently in studying the behavior of chicken production data and forecasting its future values.

***Keywords*- Chicken products, Autoregressive, Moving average, Least square, Akaike Information Criterion (AIC)**

I. INTRODUCTION

Over a long decade, meat consumption has been an integral part of human diet. Moreover, red meat consumption has formed a significant component of the balanced diet for many adults. (Cosgrove *et. al*., 2004; Henderson & Gregory, 2002). However, despite the fact that red meat is contains essential nutritional mineral like protein, nutrients and vitamins in human diet, many researchers have testified to the fact that its consumption has numerous negative effects on human health especially as one grows older; Azadbakht and Esmaillzadeh (2008), Alaejos *et. al.* (2008) and Alison *et. al.* (2010). As a result, many Nigerians seek other alternatives such as pig meat, poultry meat and products and so on which they consider safer. Coupled with the geometric increase in the population of Nigeria, this has led to tremendous increase in the rate of demand for poultry meat and the products. Therefore, there is the need to investigate the pattern of production of poultry products and examine which statistical model best describes the behavior of the series in order to make adequate planning.

II. METHOD

Least Square Method: Given a set of independent and dependent variables X and Y, of the form;

$$y = \beta_o + \beta_1 x \quad (1)$$

the least square method of parameter estimation gives the estimates of the parameters of the model. To estimate the β's, the total squared difference between the observed and predicted values which are often times called the residuals and which in matrix notation, $(\mathrm{Y} - \mathrm{X}\beta)$ is minimized. The square residual is $(\mathrm{Y} - \mathrm{X}\beta)'(\mathrm{Y} - \mathrm{X}\beta)$.

The Least Squares (LS) estimates for β_0 and β_1 are those for which the predicted values of the curve minimize the sum of the squared deviations from the observations. So, we shall obtain the estimates of β_0 and β_1 that minimize sum of squares of the residual given by;

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 \quad (2)$$

The procedure involves minimizing the vertical deviations from the line which leads to non-symmetric relationship between *x* and *y*. that is, if *x* is considered to be the dependent variable rather than *y,* different result would be expected. To obtain the minimizing values of β_i in (2) we shall differentiate (2) with respect to β_0 and β_1 as follows;

$$\frac{\partial S}{\partial \beta_o} = 0 \text{ and } \frac{\partial S}{\partial \beta_1} = 0 \quad (3)$$

which are;

$$\sum_i y_i = n\hat{\beta}_o + \hat{\beta}_1 \sum_i x_i \quad (4)$$

$$\sum_i x_i y_i = \hat{\beta}_0 \sum_i x_i + \hat{\beta}_1 \sum_i x_i^2 \quad (5)$$

Solving (4) and (5) simultaneously, we have;

$$\hat{\beta}_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} \quad (6)$$

$$\hat{\beta}_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} \quad (7)$$

which are the respective least squares estimates of β_0 and β_1.

A. Stationarity Test

One of the critical tools used to investigate the presence of unit root in underlying time series models such as ARMA or ARIMA model is the Augmented Dickey Fuller (ADF) test. The test equation is given by;

$$\Delta y_t = \alpha_0 + \theta y_{t-1} + \gamma t + \alpha_1 \Delta y_{t-1} + \ldots + \alpha_p \Delta y_{t-p} + \varepsilon_t \quad (8)$$

Hypotheses:

$H_0 : \theta = 0$ (data is not stationary)

$H_0 : \theta < 0$ (data is stationary)

α = 0.05

Test statistic: $t_{\phi - 1} = \frac{\hat{\phi} - 1}{SE(\hat{\phi})}$

Akaike information criterion or the Schwartz Bayesian information criterion or dropping the lags to a statistically significant lag (p) approach can be used in determining the value of p. The decision is such that if the p-value obtained is greater than the critical value (α) then, we accept H_0.

B. Autoregressive Moving Average (ARMA)

Often times, the Autoregressive (AR) and the Moving Average (MA) processes are somewhat unrealistic by themselves. So, both of them can be mixed to form the ARMA (p, q) models which is extremely more useful. That is, the ARMA (p, q) series $\{X_t\}$is generated by;

$$X_t = [\varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \ldots + \varphi_p X_p]$$
$$+[e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \ldots + \theta_q e_{t-q}] \ \forall \ t \in \mathbb{Z} \quad (9)$$

where e_t is a White Noise process with variance; $V(e_t) = \sigma_e^2$

Thus, X_t is essentially the sum of an autoregression (on past values of X_t) and a moving average. The series will be stationary provided that the AR part is stationary. Thus, the sum of independent ARMA processes is again ARMA. Equation (9) can also be expressed as;

$$\Phi(B) X_t = \Theta(B) e_t \quad (10)$$

where; $\Phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \ldots \phi_p B^p)$ and B is the backward shift operator.

Non-seasonal ARIMA models are usually denoted by ARIMA (p, d, q), where the integrated part (denoted by "I") shows that the original values are substituted for by the difference between present and previous values, p is the order of the AR model, d is the number of times the series is differenced before stationarity is achieved and q is the MA process order. By definition we have;

$$\psi(B) X_t = \phi(B) \nabla^d X_t = \theta(B) e_t \quad (11)$$

where:

$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ is the autoregressive operator.

$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$ is the moving average operator.

$\psi(B) = \nabla^d \phi(B)$ is the genearalised autoregressive non-stationary operator.

Since $\psi(B) = \nabla^d \phi(B)$

$$\Rightarrow \psi(B) = \phi(B)(1-B)^d = 1 - \psi_1 B - \psi_2 B^2 - \ldots - \psi_{p+d} B^{p+d}$$

Then; (11) may be written as:

$$X_t = \psi_1 X_{t-1} + \ldots + \psi_{p+d} X_{t-p-d} + \theta_1 e_{t-1} + \ldots + \theta_q e_{t-q} + e_t \quad (12)$$

C. The method of Maximum Likelihood (ML)

This is a conventional method of parameter estimation which has been embraced by many researchers to provide solution to different problems. One major advantage of the method is that it gives the fulfillment of the underlying assumptions, thereby generating estimates with optimal properties. Moreover, in a set of large samples, the estimate of the variance using the ML is the least that can be achieved by any other methods of estimation. Given a regression model where the error terms are independent, identically and normally distributed, the probability density function (pdf) of each e_t is;

$$N(e_t; 0, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{e_t^2}{2\sigma^2}\right)$$

(13)

Their joint pdf is given by;

$$\prod_{t=1}^{T} N(e_t;0,\sigma^2) = (2\pi\sigma^2)^{\frac{T}{2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{t=1}^{T} e_t^2\right) \quad (14)$$

If the elements $x_1,\ldots,x_T$ are a set of fixed variables then, the conditional pdf of the sample $y_1,\ldots,y_T$ is;

$$f(y_1,\ldots,y_T,\ x_1,\ldots,x_T) =$$

$$(2\pi\sigma_2)^{-\frac{T}{2}} \exp - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \alpha - Bx_t) \quad (15)$$

The ML principle assumes that the estimates of the parameters, as well as σ^2 should be obtained by selecting suitable values which maximize the probability measure attributed to $y_{1,}\ldots\ldots y_T$. Suppose α, β and σ^2 are arguments of the function f and not its parameters and $y_1,\ldots,y_T$ are data values rather than random variables, then the relation becomes a likelihood function (L) usually denoted by L (α, β, σ^2). Its log, which has the same maximizing values as f, is given by;

$$\log L = -\frac{T}{2}\log 2\pi - \frac{T}{2}\log(\sigma^2)$$

$$-\frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \alpha - \beta x_t)^2 \quad (16)$$

Therefore, given the value of σ^2, L is maximized by $\hat{\alpha}$ and $\hat{\beta}$ that minimize the sum of squares. Then, the maximum-likelihood estimator for σ^2 is obtained by solving the first-order relation;

$$\frac{\partial \log L}{\partial \sigma^2} = -\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{t=1}^{T}(y_t - \alpha - \beta x_t)^2 = 0 \quad (17)$$

D. Order determination

In order to select the appropriate form of an ARMA model from a set of possible models, there is the need to examine the autocorrelation (ACF) and partial autocorrelation (PACF) plots of the stationary series. Makridakis *et al.* (1993) clearly stated that Box and Jenkins (1976) provided a theoretical framework and guiding principles to determine appropriate values of p and q. A major important criterion usually considered when choosing an appropriate model is the Akaike Information Criterion (AIC) developed by Akaike (1970) and proposed in Akaike (1974). It makes comparison between the quality of a set of possible models and ranks them in order, starting from the best. The "best" or "optimum" model usually, neither under-fits nor over-fits the model. So, it best represents the model. In general;

$$AIC = 2k - 2\ln(L) \quad (18)$$

Where; k = number of parameters.

If n is the number of observations, then the AIC is given by:

$$AIC = 2k + n\ln\left(\frac{RSS}{n}\right) \quad (19)$$

where RSS = residual sum of squares.

E. Forecasting in ARMA process

For ARMA models, the one-step forecast model is given by;

$$X_{n+1} = \phi_1 X_n + \phi_2 X_{n-1} + \ldots + \phi_p X_{n-p+1} + e_{n+1}$$
$$+\theta_1 e_n + \theta_2 e_{n-1} + \ldots + \theta_q e_{n-q+1} \quad (20)$$

Hence, the optimal forecast is then given by:

$$f_{n,1} = \phi_1 X_n + \phi_2 X_{n-1} + \ldots + \phi_p X_{n-p+1}$$
$$+e_{n+1} + \theta_1 e_n + \theta_2 e_{n-1} + \ldots + \theta_q e_{n-q+1} \quad (21)$$

where, a step forecast error given by:

$$\varepsilon_{n,1} = X_{n+1} - f_{n,1} = e_{n+1} \quad (22)$$

And the optimal forecast error, $\{e_t\}_{t=n-q+1}^{n}$ can be estimated by starting with $f_{0,1} = 0$ and then forming the e_t recursively by;

$$e_t = X_t - f_{t-1,1} \quad (23)$$

III. RESULTS AND DISCUSSION

A. Descriptive Analysis of Variables

Descriptive statistics of the variable used in the analysis are as shown in table I. It shows that the mean value of Chicken Production in Nigeria between 1961 and 2017 is 104800 (in '000) and is between 37360 (Minimum) and 192300 (Maximum) on monthly bases.

TABLE I SUMMARY OF THE DATA

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
37360	64380	114700	104000	134800	192300

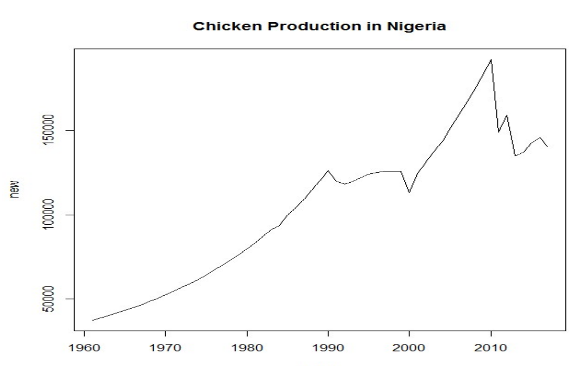

Fig.1: Time Plot of the Data

Fig.1 shows the chicken production values (in '000). It suggests the presence of trend effect in the series. However, it does not provide sufficient evidence to establish the stationarity property of the series. So, we shall consider a standard test of stationarity.

B. Stationarity Test

1) ADF test of the original series

The result of the ADF stationarity test is as shown in table II.

H_0: Data is not Stationary

H_1: Data is Stationary.

$\alpha = 0.05$

TABLE II ADF TEST

	ADF-value	**Prob. (p-value)**
Value	-2.6917	0.2956

Since the p-value (0.2956) exceeds the critical value (0.05), we do not reject the null hypothesis. This implies non-stationarity of the data. Although the result of ADF test shows weak stationarity of the series but a critical examination of the time plot suggests a trend of non-stationarity. To further examine its behaviour, we shall difference the series and then re-run the unit root test.

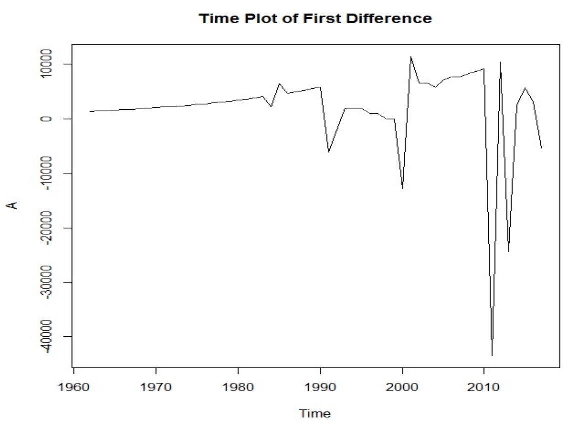

Fig. 2: Time Plot of the First Difference

Fig. 2 shows that the series is now stationarity (i.e. constant mean and variance).

To justify this claim, ADF test of the first difference carried out is as shown in Table III.

2) ADF Test of the First Difference of the series

H_0: data is not stationary.

H_1: data is stationary.

$\alpha = 0.05$

TABLE III ADF TEST OF THE FIRST DIFFERENCE

	ADF-value	**Prob. (p-value)**
1st diff ADF value	-3.7157	0.03148

From the ADF test, it was found that the p-value (0.03148) is less than the critical value (0.05). Hence, we reject the null hypothesis and conclude that the data is stationary

C. Model Fitting, Selection and Diagnostics

In this section, two different approaches were considered; the Deterministic as well as the stochastic approach. Under the deterministic, least square trend was fitted for the series and the estimated trend equation was obtained. The stochastic part includes inspection of the ACF and PACF plots, fitting of the appropriate model as suggested by the plots, selection of adequate model as well as model diagnostic check.

1) Least Square Trend

(20) and (21) show the trend equations. These would help us to examine the movement of the series along the trend line, the growth rate and its direction over time.

Least square model: $Y_t = a + bX_t$ (20)

Trend model by least square: $T = a + bt$ (21)

TABLE IV LEAST SQUARE ESTIMATION OF THE TREND VALUES

Method: Least Square. Included Observations: 57				
Variables	Estimate	Std. Error	t-value	Pr(>|t|)
Intercept	33880.0	3490.6	9.706	161e-13
time	2418.7	104.7	23.103	<2e-16
---Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 13000 on 55 degrees of freedom; Multiple R-squared: 0.9066, Adjusted R-squared: 0.9049; F-statistic: 533.7 on 1 and 55 DF, p-value: <2.2e-16				

Source: R-Language

Intercept value of 33880.0 in table IV shows that the chicken production in Nigeria will experience 33880.0 increases with constant time. Also, there will be an increase of 2418.7 per unit increase in time (i.e. yearly increase of 2419).

The Fitted Model:

$$T = 33880.0 + 2418.7t, \quad (R^2 = 0.9049)$$

This shows that chicken production increases by 2418.7 $\cong 2419$ ('000) every year.

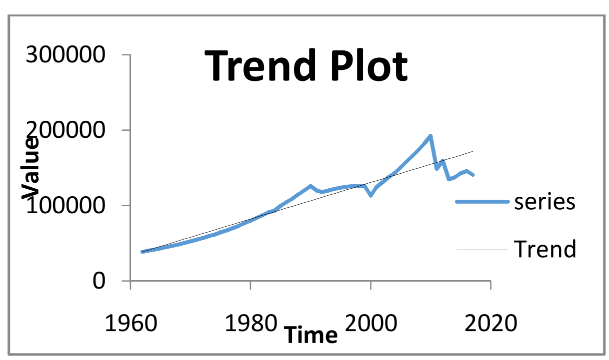

Fig. 3: Trend Plot

It can also be noticed from Fig. 3 that the positive upward trend of the least square line (Trend) is in accordance with the trend direction of the data.

2) Stochastic Approach

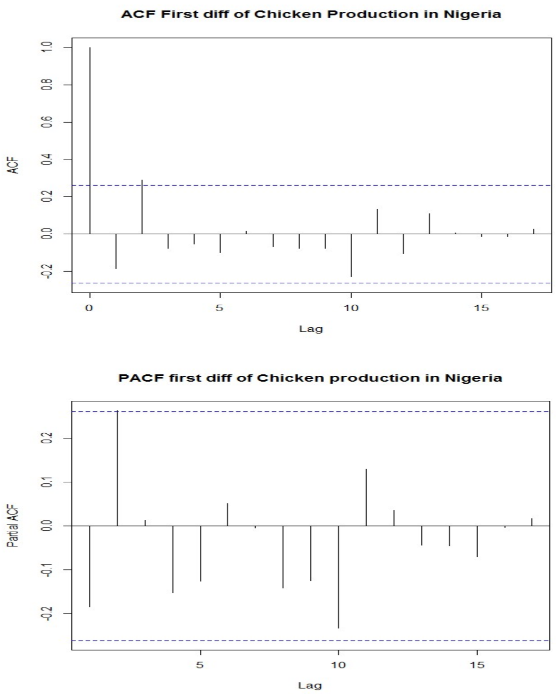

Fig. 4: ACF and PACF of the First Difference of chicken production

Fig. 4 suggests that ARMA model would be appropriate for the series**,** similar to what was observed Fig. 1. Hence, ARIMA model was considered for the series. The following are the feasible tentative models with their corresponding AIC.

TABLE V LIST OF FITTED ARIMA MODELS

ARIMA model	AIC	Log-likelihood
(1, 1, 1)	1172.76	-583.38
(1, 1, 2)	1169.96	-580.59
(2, 1, 1)	1171.82	-581.52
(2, 1, 2)	**1168.42**	**-579.21**
(2, 1, 3)	1169.79	-578.9
(3, 1, 2)	1172.51	-945.32
(3, 1, 3)	1171.51	-964.43

Source: R-Language

Table V shows that the best model is ARIMA (2, 1, 2) since it has the minimum AIC value of **1168.42** and log likelihood = -**579.21**

TABLE VI PARAMETER ESTIMATION VALUES

	AR (1)	AR (2)	MA (1)	MA (2)
Coefficient	0.3169	-0.5005	-0.3948	0.9377
S.E.	0.1530	0.1812	0.0731	0.1431
log likelihood = -579.21, AIC = 1168.42				

Table VI shows the parameter estimation of autoregressive of order two with moving average of order two with their corresponding standard errors. This is the parameter estimates of the model with the smallest AIC after the first difference.

D. Diagnostics Checking

In order to check whether the data is well captured by the model, we carried out the following diagnostic tests for residual.

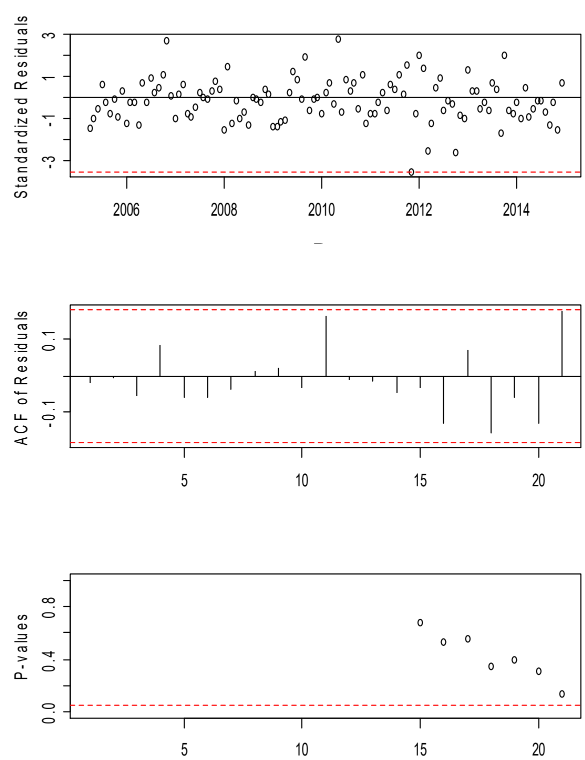

Fig. 5: Standardize Residual, Residual ACF and P-values.

Fig. 5 shows that the residual plots are rectangular, scattered and irregular round a zero, horizontal level without any form of trend and the points are within the tolerance line in the p-value plot. These show that the model captures the data well enough. Therefore, it can be used to fit the model and predict future values.

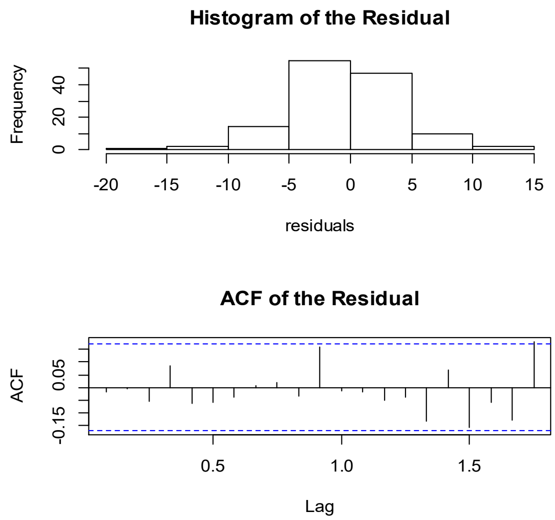

Fig. 6: Histogram and ACF of Residuals

The histogram and the ACF of the residual plots in Fig. 6 show a considerable conformity property as the histogram follows a normal distribution and the ACF do not exceed the significant bound from lag 1 to the end.

3. Forecast of the data using the ARIMA model

TABLE VII FORECAST OF THE DATA

Year	Forecast
2018	139558.8
2019	136196.7
2020	135696.4
2021	137220.5
2022	137953.8
2023	138102.1

Source: R-Language

The Time Plot of the Data with the Forecasted Values is as shown in fig. 7.

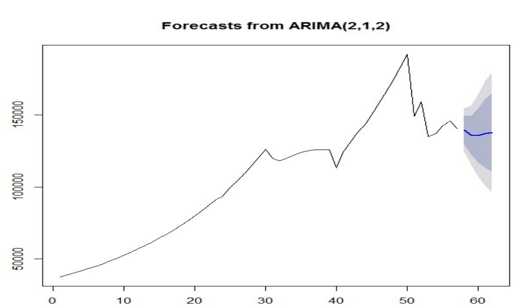

Fig. 7: Time Plot of the Data with the Forecasted Values

The plot shows considerable variation over those years, the forecast starts from the year 2018 to 2023.

IV. CONCLUSION

In conclusion, chicken production values in Nigeria have positive increase over the years. Hence, little more effort would be required by poultry farmers in order to meet up with increasing demand for the products due to increase in population. Moreover, the Autoregressive Integrated Moving Average (ARIMA) time series model is an effective tool in examining chicken production figures and should be employed at regular intervals to monitor the production rate.

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