Mathematical Modeling of Path Loss of Electromagnetic Signal at Very High Frequency Band Using Parabolic Equation

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***Abstract* - Path loss prediction plays a vital role in link budget analysis and in cell coverage prediction of radio system. In this work, the formalism of parabolic and Erickson models have been investigated as a method for calculating the radio propagation path loss and the results were compared with those obtained in measurement campaign. For comparative analysis, the field measurement data were taken in under G area, LAUTECH, Ogbomoso (8· 1227°N, 4.2436°E).A portable RF spectrum analyzer was used to capture the signal strength from the radio station operated at VHF band. The results shows that Erickson model overestimates the measured path loss with root mean square error (RMSE) 21.10 dBm while parabolic equation model was in good agreement with measured path loss having RMSE 10.90 dBm which is within the acceptable international standard value for urban area. It was also noted that parabolic equation has a lower spread correlation error with mean value 8.6654 dBm while Erickson model has higher correlation with average value 20.0085 dBm. For an accurate measurement in this environment, a new mathematical model for predicting path loss in this region is developed.**

***Key words:* Erickson model, parabolic equation, Path loss, Signal strength, VHF band.**

I. INTRODUCTION

Path loss models are used extensively in signal prediction, coverage optimization and interference analysis. Recently it is being used in estimating distances for safe operation of secondary users in TV white space. Peculiarities of these models give rise to high prediction errors when deployed in a different environment other than the one initially built for.

The method of parabolic wave equations was first proposed asa means of solving elliptic partial differential wave equations. The technique was used to solve the problem of electromagnetic wave propagation above a plane earth. The PE method was applied to solve the problem of trans-horizon radio wave propagation above a spherical earth, thereby making a breakthrough in electromagnetic wave propagation modeling. The electromagnetic parabolic equation model has broken a new path to study the path loss radio signal in the troposphere [1]. The parabolic equation gave an object description of electromagnetic propagation characteristics and path loss in the atmosphere [2]. The governing parabolic equation is [3], [4], [5] [6]

$$\frac{\partial^2 u(x,z)}{\partial z^2} + 2ik_o\frac{\partial u(x,z)}{\partial x} + {k_o}^2[m^2(Z-1)u](x,z) = 0 \quad (1)$$

where$u$ represent the electric field, x represent the range and z is the height. $k_o$is the free space wave number, m is the mid field index of refractive. PE has the advantage of high efficiency, stability and greater productivity in some situations but the algorithm is not effective for modeling arbitrary boundary conditions. Hence, in this study, simple mathematical model for predicting path loss and the validation of the parabolic equation model is presented.

II. THEORETICAL BACKGROUND

2.1 Propagation factor

In study of various signal propagation phenomena, Kerr define the one way generalized transmission equation which connects the power received (P_R) and transmitting antenna power (P_T) by [7]

$$\frac{P_R}{P_T} = G_T\left[\frac{F}{(2k_oR)}\right]^2 \quad (2)$$

where GT is the transmitting antennal power gain, R is the distance between the TX and RX, k_o is vacuum wavenumber equal to ${2\pi}/{\lambda}$ and F is the pattern propagation factor

The antenna radiation pattern function are represented in form of the pattern function $f(\theta,\emptyset)$ where $(\theta,\emptyset)$ are the zenith and azimuthal angles respectively pointing in the direction of the maximal transmission in a spherical coordinate system. This is represented as [8]

$$f(\theta,\emptyset) \equiv \frac{E(\theta,\varphi)}{E_o} \quad (3)$$

This radiation pattern is closely related to the average time Poynting vector of the radiated wave and the energy flow per unit area corresponding to the peak field E_o as shown in (4) while the antenna gain G is expressed as shown in (5)

$$S(\theta,\varphi) = |f(\theta,\varphi)|^2 S_o \quad (4)$$

$$G = \frac{4\pi}{\int_{(4\pi)} |f(\theta,\varphi)|^2 d\Omega} \quad (5)$$

The pattern propagation factor which is the ratio of the field magnitude at a point in space E (r) to the magnitude of the field at the same point under free-space conditions $E_o(r)$ is given as (6) [8, 9]. The quantity PF is the propagation factor which is the key part of equation (2) is the fundamental quantity to be computed by the parabolic equation and the path loss is expressed as shown in equation (8).

$$F = \left|\frac{E(r)}{E_o(r)}\right| \quad (6)$$

$$PF \equiv 20\log|F| \quad (7)$$

$$PL \equiv 20\log(2k_oR) - 20\log|F| \quad (8)$$

The pattern propagation factor for both horizontal and vertical polarization were computed using (9) and (10) respectively. The vertical electric dipole (VED) and vertical magnetic dipole (VMD) shown in equations (19) and (20) respectively are derived as [8]

$$F \equiv F_h(r) = \frac{|E_\varphi(r)|}{|E_\varphi^{fs}(r)|} \quad (9)$$

$$F \equiv F_v(r) = \frac{|H_\varphi(r)|}{|H_\varphi^{fs}(r)|} \quad (10)$$

The free space dipole fields are computed by using the dyadic Green's function $\Gamma(r, r_o)$ which is the solution of equation (11) where u is the unit dyadic Green's function and Γ is expressed in terms of a scalar Green's function G_o as shown in (12) where the scalar Green's function G_o satisfies (13) [8] [9] [10]

$$\nabla\times\nabla\times\Gamma(r,r_o) - k^2\Gamma(r,r_o) = u\delta(r-r_o) \quad (11)$$

$$\Gamma = \left(u + \frac{1}{k^2}\nabla\nabla\right)G_o \quad (12)$$

$$(\nabla^2 + k^2)G_o(r,r_o) = -\delta(r,r_o) \quad (13)$$

Hence, the appropriate outgoing wave solution of (13) is as shown in (14)

$$G_o(r,r_o) = \frac{1}{4\pi}\frac{e^{ik}|r-r_o|}{|r-r_o|} \quad (14)$$

$$E_{ed}(r) = -\frac{1}{e}\nabla\times(\mathrm{p}\times\nabla G_o) \quad (15)$$

$$H_{ed}(r) = iwp\times\nabla G_o \quad (16)$$

$$H_{md}(r) = -\nabla\times(m\times\nabla G_o) \quad (17)$$

$$E_{md}(r) = -iw\mu_o m\times\nabla G_o \quad (18)$$

$$H_{ed}(r) = \frac{-w}{4\pi R}e^{ik_oR}\sin\theta(\frac{r}{R})\left(k_o + \frac{i}{R}\right) \quad (19)$$

$$E_{md}(r) = \frac{w\mu_o}{4\pi R}e^{ik_oR}\sin\theta(\frac{r}{R})\left(k_o + \frac{i}{R}\right) \quad (20)$$

where$R = |r-r_o| = \sqrt{r^2 + r_o^2 - 2rr_o\cos\theta}$and θ is the polar angle. The propagation factor for horizontal polarization $F \equiv F_h$, then,

$$F_h(r) = \left|\frac{E_\varphi(r)}{E_{md}(r)}\right| \quad (22)$$

$$\therefore F_h(r) = \frac{4\pi}{k_ow\mu_o}\frac{|u|(r)|R^2|}{(r\,sin\theta)^{\frac{3}{2}}}[1 + (k_oR)^{-2}]^{-\frac{1}{2}} \quad (23)$$

While the vertical polarized radiation $F \equiv v$ is given as,

$$F_v(r) = \left|\frac{H_\varphi(r)}{H_{ed}(r)}\right| \quad (24)$$

$$\therefore F_v(r) = \frac{4\pi}{k_ow}\frac{|n(r)u(r)|R^2}{(r\sin\theta)^{\frac{3}{2}}}[1 + (k_oR)^{-2}]^{-\frac{1}{2}} \quad (25)$$

The propagation factor F in (x, z) coordinates is expressed by converting equations (19) and (20). The separation distance between TX and RX is given as

$$R = |r-r_o| = \sqrt{(r-r_o)^2 + 4rr_o sin^2\left(\frac{\theta}{2}\right)} \quad (26)$$

$$R = 2ae^{z+z_o/_{2a}}\sqrt{\sin h^2\left(\frac{z-z_o}{2a}\right) + sin^2\left(\frac{x}{2a}\right)} \quad (27)$$

$$\approx \left(1 + \frac{z+z_o}{2a}\right)\sqrt{(z-z_o)^2 + x^2} + \left(x^4/_{a^2}\right) \quad (28)$$

The angular dipole coefficient $^r/_R\sin\theta$ has the limiting forms

$$\frac{r}{R}\sin\theta = (^a/_R)e^{z/a}\sin(^x/_a) \quad (29)$$

$$With\ \frac{x}{R}, if\ ^x/_a \ll 1\ and\ ^{(z+z_o)}/_a \ll 1$$

$$With\ 1, if\ ^{(z-z_o)}/_x \ll 1.$$

From the altitudes and ranges of tropospheric propagations, the propagation factor, F, takes the following limits. The horizontal propagation factor is given as; [8], [11]

$$F_h(x) \approx \frac{4\pi R^2}{k_ow\mu_o}x^{-\frac{3}{2}}\frac{|w(x)|}{\sqrt{1 + (k_oR)^{-2}}} \quad (30)$$

$$\therefore F_h(x) = \frac{4\pi R^2}{k_ow\mu_o}\frac{|w(x)|}{\sqrt{1 + +(k_ox)^{-2}}}\ if\ ^{(z-z_\circ)}/_x \ll 1 \quad (31)$$

The vertical propagation factor is given as;

$$F_v(x) = \frac{4\pi R^2}{k_ow}x^{-\frac{3}{2}}\frac{|m(x)w(x)|}{\sqrt{1 + (k_oR)^{-2}}} \quad (32)$$

$$\therefore F_v(x) = \frac{4\pi}{k_o w}\frac{\sqrt{x}|m(x)w(x)|}{\sqrt{1}+(k_o x)^{-2}} \quad if \ {(z-z_o)}/{x} \ll 1 \quad (33)$$

2.2 Parabolic wave equation

The parabolic equation method, an approximation of the Helmholtz wave equation, has proved to be a computationally efficient and accurate approach to study radio-wave propagation characteristics in straight and curved tunnels with arbitrary cross-section geometries [12], [13], [14], [15]. It is based on the paraxial approximation, which assumes that the wave propagation occurs mainly along or close to the axis of the tunnel. This enables the reduction of Helmholtz equation, from an elliptical equation into a parabolic equation with respect to the transverse components of the fields.

$$w(x) = e^{+i\bar{k}x}\varphi(x) \quad (34)$$

$$2i\bar{k}\frac{\partial\psi(x)}{\partial x} + \frac{\partial^2\psi(x)}{\partial_z{}^2} + (k^2(x)-k^2)\psi(x) = -\frac{\partial^2\psi(x)}{\partial x^2} \quad (35)$$

$$2i\bar{k}\frac{\partial\psi(x)}{\partial x} + \frac{\partial^2\psi(x)}{\partial_z{}^2} + (k^2(x)-k^2)\psi(x) = 0 \quad (36)$$

The parabolic equation in three dimensions can then be obtained as given by Zelley (1999) [16]

$$\frac{\partial u}{\partial x} + \frac{i}{2k}\times\frac{\partial^2 u}{\partial y^2} + \frac{i}{2k}\times\frac{\partial^2 u}{\partial x^2} = 0 \quad (37)$$

Expressing the three dimensions function u(x, y, z) as the sum of the two 2D functions

$$u(x,y,z) \approx u_1(x,y) + u_2(x,z) \quad (38)$$

The function $u_2(x,z)$ in equation (38) is propagation waves in (x, z) plane and is solved using 2D parabolic equation. The function $u_1(x,y)$ in equation (38) is propagation waves in the (x,y) plane. By applying equation (38), equation (37) can be written as

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{i}{2k}\times\frac{\partial^2 u_1}{\partial y^2} + \frac{i}{2k}\times\frac{\partial^2 u_2}{\partial x^2} = 0 \quad (39)$$

Equation (39) can further reduced to two 2D parabolic equations

$$\frac{\partial u_1}{\partial x} + \frac{i}{2k}\times\frac{\partial^2 u_1}{\partial y^2} = 0 \quad (40)$$

$$\frac{\partial u_2}{\partial x} + \frac{i}{2k}\times\frac{\partial^2 u_2}{\partial x^2} = 0 \quad (41)$$

Equations (40) and (41) are solved by split-step Fourier transformation (SSFT) method to obtain 3D space problem as

$$u_1(x+\delta x.y) = F^{-1}\left[F\left[\exp\left(-\frac{ip_1{}^2\delta x}{2k}\right).u_1(x,y)\right]\right] \quad (42)$$

$$u_2(x+\delta x.z) = F^{-1}\left[F\left[\exp\left(-\frac{ip_2{}^2\delta x}{2k}\right).u_2(x,z)\right]\right] \quad (43)$$

After computing $u_1(x,y)$and $u_2(x,z)$, respectively, using equation (38) the receiving position signal is obtained and this is transformed into propagation loss as [16] [17]

$$PL(dB) = -20\times log_{10}(|u|) + 20\times log_{10}(4\pi) + 20log_{10}(x) - 20log_{10}(\lambda) \quad (44)$$

2.3 Erickson model

Model 9999 is the Ericsson's implementation of Hata model. In this model parameter is possible according to propagation environment. The path loss PL is given as [18], [19]

$$PL(dB) = a_o + a_1\log(d) + a_2\log(h_b) + a_3\log(h_b)\log d - 3.2\left(log(11.75h_m)\right)^2 + g(f) \quad (45)$$

where$g(f)$is defined by :

$$g(f) = 44.49\,log(f) - 4.78\,(log(f))^2 \quad (46)$$

The parameters ao , a1 , a2 and a3 are constants, and can be change for better fitting specific propagation conditions. Default values are ao = 36.2, a1=30.2, a2=-12, and a3=0.4.

III. METHODOLOGY AND MEASUREMENT CAMPAIGN

The measurement campaign was conducted in LAUTECH area around under G, Ogbomoso, South western, Nigeria (8·1227°N, 4.2436°E).A portable RF spectrum analyzer was used to capture the signal strength from the radio station operated at VHF band. A digital measuring wheel was used to determine the distance from the transmitter and the path loss at each point from the transmitter was obtained using (47)

$$PL(dBm) = P_T + G_T - R_X \quad (47)$$

where P_T is the transmitting power, G_T is the antenna gain and R_X is the receiving power.

Simulation of path loss with distance was obtained using parabolic equation model (8) while MATLAB R2017b software was used for data analysis. In this work, two statistical analyzed tools namely; the root mean square error, RMSE (μ), and spread-correlation root mean square error (SC-RMSE). Statistics were applied to test the performance of fit for the proposed model with respect to the parabolic equation and Erickson models. The root mean square error between the measured path loss (Pm) and computed path loss (Pc) is given as;

$$RMSE, \mu = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(P_m - P_p\right)^2} \qquad (48)$$

The SC-RMSE which is used to extract the impact of dispersion from the overall error which has the effect of reducing the error associated with a noisy link with standard deviation, σ is compute using,

$$SC - RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{n}\left(P_m - P_p\right)^2 - \sigma} \qquad (49)$$

IV. RESULTS AND DISCUSSION

Table 1 depicts the variation of measured path loss with distance between transmitter and receiver. It also shows the simulation of path loss with distance using parabolic equation as in (8) and Erickson model (45). A linear relationship between the measured path loss and distance was noticed up to 200 m away from the transmitter with a loss of 127 dBm. At the distance above 200 m, the relationship is non-linear. As shown in figure 1, the results obtained from the simulation of parabolic and Erickson models were compared with measured data. It was observed that Erickson model overestimates the measured path loss with root mean square error 21.10 dBm while parabolic model, though slightly higher than the measured values but it was in good agreement with measured path loss having root mean square error 10.90 dBm which is within the acceptable international standard for urban area [20][21].

Table 1: Variation of path loss with distance for measured and computed values

d (m)	Measured (dBm)	Parabolic (dBm)	Erickson (dBm)	SC-RMSE Parabolic	SC-RMSE Erickson
50	108.30	125.514	136.860	16.980	28.428
100	125.40	131.534	142.881	5.675	17.263
150	138.00	135.056	146.403	0.814	7.939
200	127.20	137.555	148.902	9.219	21.527
250	125.60	139.493	150.840	13.602	25.010
300	138.10	141.077	152.423	0.924	14.057
350	138.20	142.416	153.762	3.124	15.319
400	140.58	143.576	154.922	0.981	14.076
450	147.70	144.599	155.945	1.267	7.772
500	132.90	145.514	156.860	12.293	23.802
550	134.00	146.342	157.680	12.013	23.528
600	136.00	147.097	158.445	10.731	22.275
650	128.00	147.793	159.139	19.589	31.018
700	131.20	148.436	159.783	17.003	28.450
750	131.30	149.036	160.382	17.509	29.515
800	132.80	149.596	160.943	16.556	28.008
850	132.10	150.123	161.469	17.799	29.240
900	134.60	150.619	161.966	15.768	27.227
950	135.20	151.089	162.435	15.635	27.096
1000	133.50	151.534	162.881	17.811	29.252
1050	143.40	151.958	163.305	8.007	19.714
1100	144.90	152.362	163.709	6.905	18.607
1150	145.60	152.748	164.095	6.565	18.289
1200	140.10	153.118	164.465	12.707	24.209
1250	142.80	153.473	165.819	10.291	22.854
1300	143.50	153.813	165.960	9.918	21.484
1350	143.90	154.141	165.488	9.843	21.412
1400	144.60	154.457	165.804	9.436	21.024
1450	145.80	154.762	166.108	8.503	20.121
1500	147.90	155.056	166.403	6.573	18.297
1550	146.20	155.341	166.688	8.692	20.295
1600	148.40	155.617	166.963	6.639	18.358
1650	149.90	155.884	167.231	5.273	17.111
1700	151.20	156.143	167.490	4.053	16.056
1750	153.60	156.395	167.742	0.995	13.871
1800	153.68	156.640	167.986	0.865	14.039
1850	153.45	156.878	168.224	1.923	14.516
1900	154.02	157.109	168.456	1.240	14.171
1950	160.20	157.335	168.682	0.442	8.022
2000	161.30	157.555	168.902	2.452	7.086

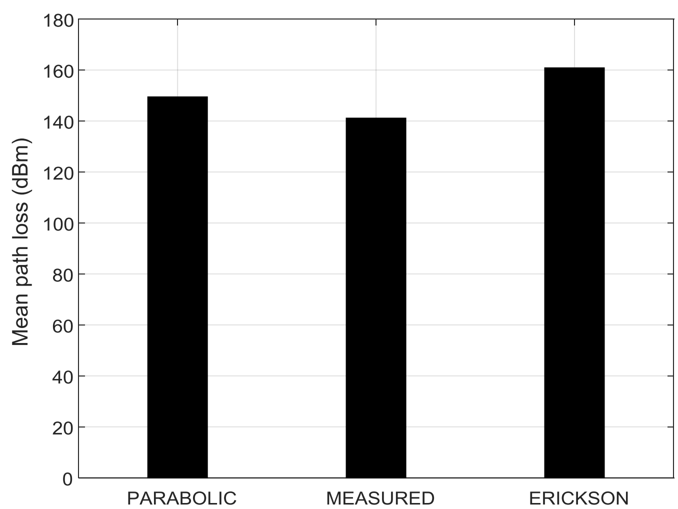

Fig 1: mean path loss for measured and the computed data.

Figure 2 shows the graphical representation of spread correlation error against the separation distance between the transmitter and receiver. It was noted that parabolic model has a lower spread correlation error over the distance with mean value 8.6654 dBm while Erickson model has the higher average spread correlation error value 20.0085 dBm. This implies that, for wider range path loss estimation and prediction, parabolic equation model would perform better than Erickson model. Furthermore, a new mathematical model as a function of distance has been developed for path loss prediction in this environment. The developed model is proposed using general model power 1 as in (50).

$$\boldsymbol{PL(dBm) = 90.71 \times d^{0.06603}} \quad (50)$$

The statistical analysis of the goodness of fit for the new model are; SSE = 1899, R^2 = 0.5552, adjusted R^2 = 0.5435 and RMSE = 7.0691

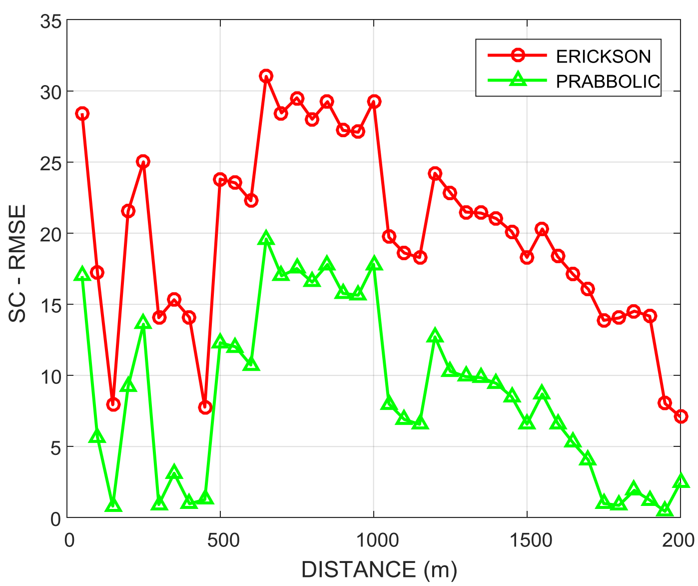

Fig 2: The graphical representation of SC – RMSE against distance for parabolic and Erickson model

V. CONCLUSION

In this work, estimation of path loss of radio signal at VHF band in urban environment has been investigated using the formalism of parabolic and Erickson models. The computation results of the two models were compared with measured data. The obtained results revealed that Erickson model overestimates the measured path loss with root mean square error (RMSE) 21.10 Bm while parabolic equation model was in good agreement with measured path loss having RMSE 10.90 dBm which is within the acceptable international standard value for urban area. It was also noted that parabolic equation has a lower spread correlation error with mean value 8.6654 dBm while Erickson model has higher correlation with average value 20.0085 dBm. For an accurate measurement in this environment, a new mathematical model for predicting path loss in this region is developed.

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