

Mathematical Modeling of a Robot Vacuum Cleaner Suction with Matlab Simulink

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Abstract- The suction of a Robot Vacuum Cleaner (RVC) is not to be taken slightly, because it determines how well a room will be clean. Researchers over the years have tried to come up with a better way for the RVCs to suck up dust and dirt without losing the pressure for suction, but there have always been rooms for improvement. This study shows another simple and better way to rectifying issues bothering on the suction failure of a RVC. It presents the mathematical modeling of the centrifugal fan's speed via the DC motor through the equations of moment of inertia, torque and Newtons second law of rotation. The equations were modeled with certain parameters in MATLAB® SIMULINK® to check if there is a drop in the speed of the centrifugal fan. The velocity of the fan in 10 seconds was 60m/s without any form of retardation. Again the airflow rate which was an ideal way of checking the suction of a RVC was modeled using the same MATLAB® SIMULINK® with the same equation. The graphical result of the airflow rate in the model showed that the airflow increased to a point where it was constant, indicating a perfect suction without dropping.

Keyword- Suction; Robot; Mathematical modeling; Centrifugal fan; Airflow rate

I. INTRODUCTION

A vacuum cleaner can be referred to as a Hoover or a sweeper, and it is a device that uses an air pump (a centrifugal fan in all but some of the very oldest models), that creates a partial vacuum to suck up dust and dirt, usually from the floor, and other surfaces like the upholstery and draperies, [6]. The dust and dirt is collected by either a cyclone or dust bag for disposal after vacuuming. RVCs which are commonly used in homes, offices and industries, exist in various kinds of sizes and models: small battery-powered hand-held devices, wheeled canister models for home use, domestic central vacuum cleaners, huge stationary industrial appliances that can handle several hundred liters of dust before being emptied, and self-propelled vacuum trucks for recovery of large spills or removal of contaminated soil. Specialized shop vacuums can be used to suck up both dust and liquid. The sucked up dust and liquid passes through a passage called the duct. The duct of the RVC is the length of the flow channel between the inlet of the rolling brush blower and the outlet of the vacuum blower, which includes the dust bag and other intermediaries, [10].

According to Reference [1], suction is created by the RVC's rotating fan which creates a flowing stream of air moving through the intake port and out of the exhaust port.

The flow of the stream of air acts just exactly like the flow of a stream of water. This flow is supposed to be regular and constant in order to achieve a perfect suction and a perfect cleaning. The reverse is the case, because most of these RVC's used at homes, offices or industries tend to have pressure drop for some of the reasons which is resolved in this paper work.

Gathering facts about consumers' sincere displeasure over the RVC used in their homes and offices from the irobot consumers affairs, [2], it shows that the consumers are losing interest and at the same time discouraging others from using the RVC, which is not good for the stability of a robot industry. This can lead to the liquidation of a robot industry. It equally shows that the robot industries need researchers to look into the major causes of the RVC short-coming, to model and to provide a feasible solution.

This paper focuses on one of the many problems common to a RVC which is the weak suction. The paper suggests to provide a mathematical model for the RVC to tackle the suction area. Performing experiments to understand and solve real-life problems may be very expensive and risky. Some times it may not be feasible at all to perform experiments. Mathematical modeling is the only recourse in such situations and it is very inexpensive to represent a real problem in terms of appropriate equations and solve them, [4].

The robotic industries and their consumers would benefit from the model provided by this paper. Relationship, trust and reliability will be built between the robotic industries and their consumers. Production of RVC will be substantial and their demand in the robotic market will yield a notable increase. Advertisement of the RVC products would grow faster and cheaper since satisfied consumers would love to introduce their friends, colleagues and loved ones to the product.

II. REVIEW OF RELATED LITERATURE

Over the years researchers have been trying to come up with a better solution to the suction problem of the RVC, using diverse methods ranging from data collections, factory experimentation, mechanical designs, performance analysis, force control, mathematical modeling, simulations and so on. This section will present related works in the area of improving the suction, starting with the major paradigm shift in the RVC.

According to Reference [7], this paradigm shift started in the year 1978 by James Dyson, when bagged robot vacuum cleaners began to lose suction as soon as the bags are filled with dust. Dyson had the idea of using cyclonic separation to create a vacuum cleaner that would not lose suction as it picked up dirt. The idea came to him after seeing a local sawmill which used a conical centrifuge technology that spin dust out of the air. He invented this RVC that doesn't have traditional bag or filter system, instead, he constructed the RVC to send the air stream through one or more cylinders, along a high-speed spiral path. As the air stream shoots around in a spiral, all of the dirt particles experience a powerful centrifugal force. They are whipped outward, away from the air stream. In this way, the dirt is extracted from the air without using any sort of filter. It simply collects at the bottom of the cylinder. The cyclone system is a marked improvement on the traditional RVCs - there are no bags to replace and the suction doesn't decrease as other RVCs. Dyson did not use mathematical model to achieve this but factory experimentation. The thing about factory experimentation is that it is very expensive. Dyson achieved this after his 5,127 prototype which almost made him bankrupt, [3].

Reference [10], researched on the duct flow of an RVC. In other to cope with the pressure drop or suction problem of the duct flow in a RVC, they introduced an algorithm and the optimisation design of the duct flow field was implemented by a method based on Pressure Implicit with Splitting of Operators (PRISO). Firstly, the duct structure in a RVC is taken as a research object, with the Computational Fluid Dynamics (CFD) theories adopted; a three-dimensional fluid model of the duct is established by means of the FLUENT solver of the CFD software. Secondly, the $k-\epsilon$ turbulence model of three dimensional in-compressible fluid was considered and the PRISO pressure modification algorithm was employed, then the flow field numerical simulations inside the duct of the RVC were carried out. They then plotted the velocity vector on the arbitrary plane of the duct flow field. Finally, they investigated the dynamics characteristics of the duct flow field. Experimental results show that the duct flow field after optimisation can effectively reduce pressure drop, the feasibility as well as the correctness of the theoretical modeling and optimisation approaches are validated.

Reference [9], in their study, a new type of RVC was designed and prototyped. The RVC has functionality that cleans four sides of the inner duct simultaneously with a constant pressure by using a force compliance device. However, there are still room for improvement.

Reference [8], used performance analysis to improve the duct cleaning robot. He made use of the MATLAB and SIMULINK tool to simulate the general DC motor dynamic model. The model consist of linear differential equations of both mechanical and electrical system, which are:

$$i_a(t) = \frac{1}{L_a} \int_0^t [v_a(\tau) - R_a i_a(\tau) - e_a(\tau)] d\tau \quad \text{-----1}$$

$$w_m = \frac{1}{J_m} \int_0^t [T_e(\tau) - B W_m(\tau) - T_L(\tau)] d\tau \quad \text{----2}$$

$$e_a = k_b w_m(t) \quad \text{-----3}$$

$$T_m = k_t i_a(t) \quad \text{-----4}$$

Where T_L is the motor load, T_e is torque proportional to the current i_a , and e_a is back electro-motive force from the coil of the motor. $J_m, B, k_t, k_b, L_a, R_a$ are motor constants indicating the motor inertia, speed/torque gradient, torque constant, speed constant, terminal inductance, and terminal resistance respectively. He conducted a dust cleaning performance with the model. Deposit Thickness Test (DTT) and Vacuum Test (VT) was used to measure the thickness and weight of dust before and after cleaning with the developed robot system. The data collected showed an improvement and possibility of using the mobile duct cleaning robot. Still there exist more room for improvement.

Then Reference [5], argued that they pioneer a new future in robotic dust collection by introducing passive dust-collecting robots that, unlike their predecessors, do not require locomotion to collect dust. They imply that the previous research has exclusively focused on active dust-collecting robots, and their paper showed that these active dust-collecting robots fail with respect to practical and theoretical aspects, as well as human factors. They present a mathematical formalism of both paradigms (active and passive) followed by a user study and field study. Active RVCs are dynamic, they move from place to place while passive RVCs are static, they are fixed on a position. The thing about the passive RVCs is that they might succeed in collecting more dust but they cannot completely clean the entire room or do the job of moving from one room to another performing their cleaning task.

Continuous and constant flow of air through the duct is a sure way of tackling the suction problem. This paper work presents that best way of a continuous and constant flow of air to maintain a perfect suction.

III. METHODOLOGY

A. Analysis of RVC Suction

According to Reference [1], the regular RVC is actually made-up of only six essential components:

1. A duct or intake port which contains varieties of cleaning accessories
2. An exhaust port
3. A dc motor
4. A fan with blades (centrifugal)
5. A container or bag that is porous

6. A housing that contains all the other components.

B. Analysis of the Operation of Suction

When the RVC is turned on, this is what happens:

1. The dc motor begins to rotate. The motor is operated by the electric current supplied directly by the battery or alternatively. A fan which has angled blades like an airplane propeller is attached to the motor.
2. As the fan with blades turn, they force air forward, toward the exhaust port.
3. When the air particles are driven forward, the pressure of particles increases in front of the fan and decreases behind the fan.
4. As the pressure level behind the fan drops below the pressure level outside the RVC it creates a suction, a partial vacuum, inside the RVC. The air outside pushes itself into the RVC through the intake port because the air pressure inside the RVC is lower than the pressure outside.

As long as the fan keeps rotating with a constant speed and the passageway through the RVC remains open, there is a constant stream of air moving through the intake port and out of the exhaust port.

C. Problems Inherent in the Existing RVC Suction

According to Reference [4], the suction which is the cleaning system of the vacuum determines how well the dirt is collected. Current RVCs (Roomba for instance) have less suction, because they are smaller than a normal vacuum cleaners (Dyson), and rather than depending on a main power source, they have to use (and conserve) a battery pack. This actually is a problem since it can end up not vacuuming the whole room properly. With all technology available there must be a better way of solving the less suction problems of the RVC.

D. Analysis of the Proposed Solution

There are two solutions proposed by this paper work. They are:

1. To mathematically model the centrifugal fan speed via the DC motor using Newtons second law of rotation and then observe the result of the equation using MATLAB and SIMULINK. The centrifugal fan speed is very important for a perfect suction, but the DC motor determines that. This is because the centrifugal fan cannot rotate without the DC motor. It is the centrifugal fan that coordinates the pressure inside and outside the RVC.
2. To model the airflow rate in the suction, using MATLAB and SIMULINK. There is a relationship between the airflow rate and the pressure that produces the suction. When the airflow rate keep increasing, the pressure outside the RVC will keep

increasing thereby increasing the suction. When the airflow rate increase to a point where it is constant, the pressure outside the RVC will be constant as well as the suction.

IV. IMPLEMENTATION, EVALUTIONS, RESULTS AND DISCUSSION

The two proposed solutions will be implemented, evaluated and discussed in this section.

A. Mathematical modeling of the Centrifugal Fan Speed Via the DC Motor

To set a top spinning, the object must be twisted. The rod (shaft) of the DC motor is set spinning by the forces \vec{F}_1 and \vec{F}_2 exerted at the edges of the rod. The perpendicular distance between the line of action of a force and the axis of rotation is the lever arm ℓ of the force. A force times its lever arm is the magnitude of the torque τ_i :

$$\tau_i = F_i \ell \text{ -----1}$$

In Figure 2 the vertical ash line is F_{ir} and the horizontal ash line is F_{iu} . The red line is \vec{F}_i and the black line is \vec{r}_i . Then the angle between the vertical ash line and the red line is θ . the point where the red line originate from is m_i . So we have:

$$\tau_i = F_i \ell = F_i r_i \sin \theta = F_{iu} r_i \text{ -----2}$$

To show that a rigid body's (the fan) angular acceleration is proportional to the net torque acting on it. Let \vec{F}_i be the net external force acting on the *ith* particle. The tangential acceleration of the *ith* particle is Newtons second law,

$$F_{iu} = m_i a_{iu} = m_i r_i \alpha \text{ -----3}$$

Multiply each side by r_i : we get; $r_i F_{iu} = m_i r_i^2 \alpha$ -----4

$$\tau_i = m_i r_i^2 \alpha \text{ -----5}$$

Summing over all the particles in the object gives:

$$\sum_i \tau_i = \sum_i m_i r_i^2 \alpha \text{ -----6}$$

For continuous object in moment of inertia, the sum is replace by an integral.

$$I = \sum_i m_i r_i^2 \text{ (system of particles)-----7}$$

$$I = \int r^2 dm \quad (\text{continuous object}) \text{-----}8$$

B. Evaluation and Result

For example, assuming that the input of the suction system is the voltage source (V) applied to the motor’s armature, while the output is the rotational speed of the shaft (rod) θ . Let the fan and the shaft (rod) be rigid. Let the friction torque be proportional to the shaft angular velocity. Then let the physical parameters for the example be,

1. Moment of inertia of the fan (I) 0.01 kgm^{-2}
2. Motor viscous friction constant (b) 0.1 Nm/s
3. Electromotive force constant and motor torque constant (K) 0.01

Evaluating the mathematical formula with the given values, using MATLAB and SIMULINK, the model and results are shown below.

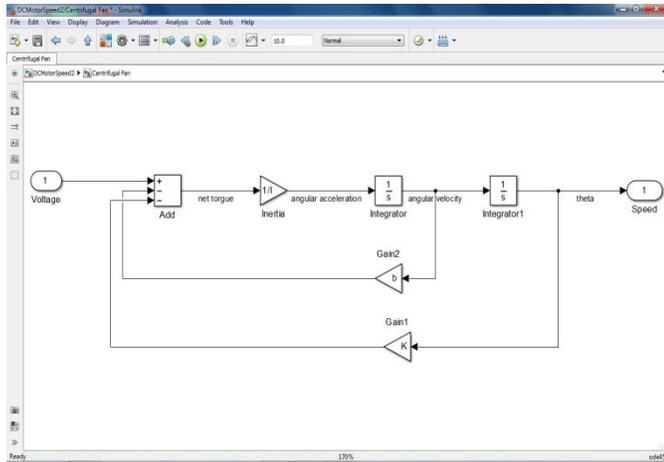


Fig.3 Model of the Centrifugal Fan using Newtons Second law of Rotation

In other to view the result of the model above, all the components were saved as a single subsystem block. The input block (voltage) became the step block and the output block (speed) became the scope, which showed the behaviour of the model.

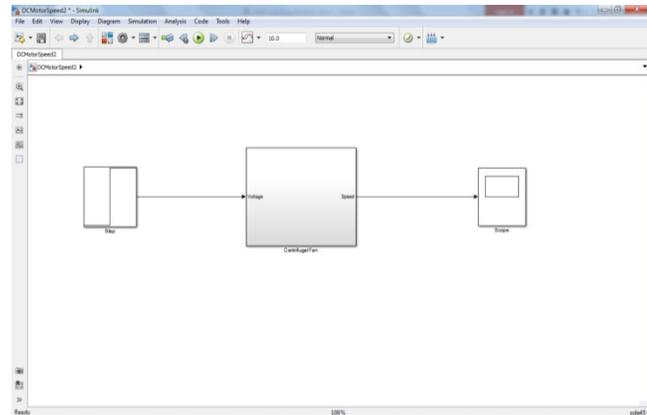


Fig.4 Created Subsystem, in-order to view the result.

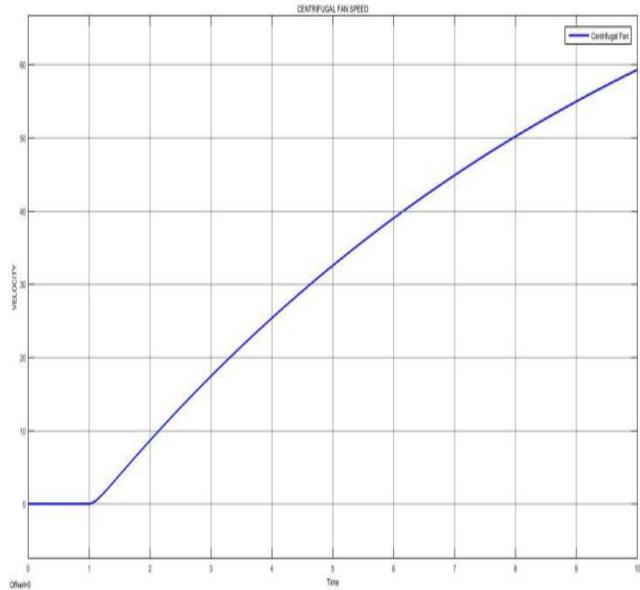


Fig.5 The graph of the centrifugal fan speed

C. Modeling the Airflow Rate

This section of modeling the airflow rate is achieved by using the SIMULINK with the consciousness of the mathematical equations above. The model comprises of both the mechanical and electrical parts of the suction system. The model is an ideal rotational motion system (the centrifugal fan) that shows whether the airflow rate is constant with the equation introduced.

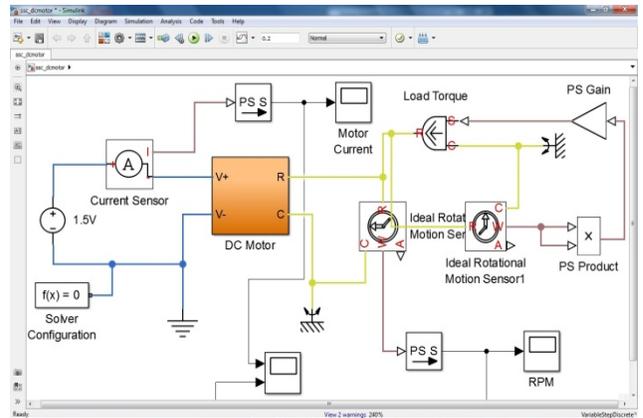


Fig.6 Airflow rate SIMULINK model

D. Evaluation and Result

In the model, current is supplied to the DC motor which becomes operational with a particular load torque. There is an attached centrifugal fan (ideal rotational motion sensor) to the shaft of the DC motor (another ideal rotational motion sensor). The equations were introduced to both the electrical part (solver configuration) and the mechanical part (PS Product). A scope is attached to both the DC motor current scope (electrical) and the centrifugal fan scope (mechanical), to view the graphical result of the both.

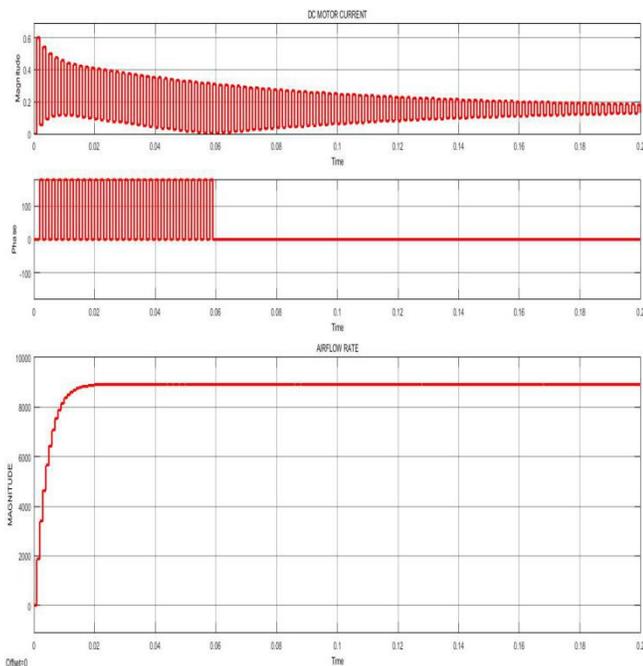


Fig.7 The graph of the DC motor current and the airflow rate

E. Discussion

1). Mathematical modeling of the centrifugal fan via the DC motor

In the literature review, James Dyson idea of resolving the suction problem by tackling the rotation pattern is almost similar with the mathematical modeling of the centrifugal fan in this paper. The difference is the approach used in getting the desired result. Dyson approach was factory experimentation of 5,127 prototype of the vacuum cleaner which caused him and his wife their entire savings, [3]. This paper presents a non-cost effective and reliable approach of modeling the centrifugal fan speed with use of mathematical equations induced into MATLAB for simulation and observation of result before implementation in the robotic industry. This method and section of implementation was possible by first providing the mathematical equations of the torque, moment of inertia and Newtons second law of rotation for the centrifugal fan via the DC motor. An example was given for evaluation and a result obtained. The graphical result shows the increase of velocity of the centrifugal fan. In 10 seconds the velocity of the centrifugal fan was 60m/s.

2). Modeling the airflow rate

Reference [8], on the performance analysis of the vacuum cleaner, with this paper work share the same approach of using the MATLAB® SIMULINK® to model the airflow considering the electrical and mechanical parts. The difference is the pattern of model and the kind of mathematical equations induced into the SIMULINK model. Wootae made use of linear differential equations while this paper made use of Newton second law of rotation. Wootae

work presented the result of a force control, while this work presented airflow rate at constant flow. The graphical result of this section is divided into two parts, considering the DC motor current and the airflow rate. The first part (DC motor current) is divided into two, firstly, the magnitude of the DC motor current showing the sinusoidal (perfect) flow of current and secondly the phase of the DC motor current. The second part (airflow rate) showed how the pressure of the suction increased to a particular level and became constant without fluctuating to lose suction. Ideally with this model, the suction problem is solved.

V. CONCLUSION

The RVC was first introduced in this study as a Hoover or sweeper, a device that uses centrifugal fan that creates a partial vacuum to suck up dust and dirt, usually from the floor, and other surfaces like the upholstery and draperies. The act of sucking up the dust and dirt was seen as the suction. This suction determines how well a RVC cleans up a room. Firstly, the centrifugal fan was mathematically modeled via the DC motor and then simulated through MATLAB SIMULINK to show that the Newton second law of rotation can be used to configure the speed of the fan. Secondly, the airflow rate was modeled to be constant for a perfect suction. The study is an improvement on modeling the suction of an RVC with a mathematical and MATLAB SIMULINK model, but there are still rooms for a better improvement.

VI. RECOMMENDATIONS

This research was conducted to study and investigate a better way to construct vacuum cleaners with a better suction for proper cleaning. The study recommends that the Robotic Companies will be able to apply these mathematical models in producing vacuum cleaners.

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