Cahit-9-Equitability of Coronas $C_n ok_1$

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***Abstract*. For any graph G and any positive integer k assign vertex labels from {0, 1, 2,…, *k*-1} so that when the edge labels are induced by the absolute value of difference of the vertex labels, the number of vertices labeled with i and the number of vertices labeled with j differ by atmost one and the number of edges labeled with i and the number of edges labeled with j differ by at most one. Cahit called a graph with such an assignment of labels *k*-equitable. In this paper, we show that the corona graphs C_nOK_1 are 9-equitable as per cahit's definition of k-equitability.**

***Keywords*: labeling, Cahit-k-equitable, Corona Graph.**

2010 Mathematics Subject Classification: Primary 05C78; Secondary 05C76.

I. INTRODUCTION

A labelling of the vertices of a graph *G* is an assignment of distinct natural numbers to the vertices of *G*. every labeling induces a natural labeling of the edges: The label of an edge υν is the absolute value of the difference of the labels of υ and ν. Bloom [3] defined a labelling of the vertices of a graph to be *k*-equitable if in the induced labelling of its edges, every label occurs exactly *k*-times, if at all. Furthermore a *k*-equitable labeling of a graph of order p is said to be minimal if the vertices are labeled with 1,2,…,p.

Bloom [3] posed the following question: Is the condition that *k* is a proper divisor of *p* sufficient for the cycle C_p to have a minimal *k*-equitable labeling. Wojeiechowski [7] gave a positive answer to this question. Barrientos, Dejter and Hevia [4] have shown that forests of even size are 2-equitable. They also prove that for *k* =3 or *k* =4 a forest F of size *kw* is *k*-equitable if and only if the maximum degree of F is at most 2*w* and that if 3 divides the size of the double star $S_{m,n}$($1\leq m \leq n$), then $S_{m,n}$ is 3-equitable if and only if $\frac{q}{3}\leq m \leq \lceil \frac{q-1}{2} \rceil$. Here $S_{m,n}$ is K_2 with *n* pendent edges attached at one end and *n* pendent edges attached at the other end. They discussed the *k*-equitability of forests for $k \geq 5$ and characterized all caterpillars of diameter 2 that are *k*-equitable for all possible values of *k*.

The corona G_1oG_2 of two graphs G_1 and G_2 was defined Frucht and Harary [5] as the graph G obtained by taking one copy of G_1 which has p_1 vertices and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

In [6] Vasanti Bhat-Nayak and Shanta Telang prove that the corona graphs C_nOK_1 are *k*-equitables as per cahit's definition of *k*-equitability *k=2,3,4,5,6*. In [1] D G Akka and Sanjay Roy proved that the corona graphs C_nOK_1 are 7-equitable. In [2] D G Akka and et al proved that the corona graphs C_nOK_1 are 8-equitable. Here we prove that the corona C_noK_1 is cahit-9-equitable.

II. CAHIT-9-EQUITABILITY OF CORONAS

In this paper we will use the following notations

$V(C_noK_1) = \{u_1,u_2, \dots, u_n,\upsilon_1,\upsilon_2, \dots, \upsilon_n\}$ where $u_1,u_2, \dots, u_n, u_1$ is the cycle C_n and υ_i is the pendent vertex adjacent to u_i, $1 \leq i \leq n$.

Theorem. All coronas are Cahit-9-equitable.

Proof. For Cahit-9-equitability the label set as well as the edge weight set is {0,1,2,…,8}. We have $p=q=2n$ where $p=|V(C_noK_1)|$, $q = |E(C_noK_1)|$. We consider nine different cases.

CASE 1. $2n \equiv 0 \pmod 9$

Let p = q = 2n =9t, $t \geq 2$. Note that 9t = 2n implies t is even. We describe labeling at the end of the proof for t = 2, so let $t \geq 4$. For Cahit-9-equitability of (C_noK_1) each label will have to be used *t*-times such that each edge weight occur *t* times.

We define the labelling function $f:V(C_noK_1)\rightarrow \{0,1,2,\dots,8\}$ as follows.

$f(u_1) = 0$ $\quad f(\upsilon_1) = 2$

$f(u_{2i}) = 8$ $\quad f(\upsilon_{2i}) = 8$ $\quad 1 \leq i \leq t/2$

$f(u_{2i+1}) = 0$ $\quad f(\upsilon_{2i+1}) = 3$ $\quad 1 \leq i \leq t/2$

$f(u_{2i}) = 7,$ $\quad f(\upsilon_{2i}) = 1,$ $\quad \frac{t}{2}+1 \leq i \leq t$

$f(u_{2i+1}) = 0,$ $\quad f(\upsilon_{2i+1}) = 6,$ $\quad \frac{t}{2}+1 \leq i \leq t-1$

$f(u_{2i+1}) = 1,$ $\quad f(\upsilon_{2i+1}) = 5,$ $\quad t \leq i \leq \frac{3t}{2}-2$

$f(u_{2i}) = 6,$ $\quad f(\upsilon_{2i}) = 2,$ $\quad t+1 \leq i \leq \frac{3t}{2}$

$f(u_{3t-1}) = 1$ $\quad f(\upsilon_{3t-1}) = 6$

$f(u_{2i+1}) = 4,$ $\quad f(\upsilon_{2i+1}) = 5,$ $\quad \frac{3t}{2} \leq i \leq 2t-2$

$f(u_{2i}) = 2,$ $\quad f(\upsilon_{2i}) = 3,$ $\quad \frac{3t}{2}+1 \leq i \leq 2t-1$

$f(u_i) = 4,$ $\quad f(\upsilon_i) = 5,$ $\quad 4t-1 \leq i \leq 4t$

$f(u_i) = 4,$ $\quad f(\upsilon_i) = 7,$ $\quad 4t+1 \leq i \leq \frac{9t}{2}-1$

$f(u_{\frac{9t}{2}}) = 7, \quad f(v_{\frac{9t}{2}}) = 3$

It is not hard to verify that each label and each edge-weight occurs exactly t times. We obtain a suitable labelling for $t = 2$ which corresponds to $n = 9$ as follows.

CAHIT-9-EQUITABLE LABELING OF C_9oK_1

Here $p = q = 18$, $t = 2$, $n = 9$

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
2	8	3	1	6	2	5	5	3
0	8	0	7	1	6	4	4	7
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9

CASE 2. $2n \equiv 1 \pmod 9$

Suppose $p = q = 2n = 9t + 1$, $t \geq 1$. It is easy to see that as $2n = 9t + 1$, t is an odd integer. We select suitable labeling at the end of the proof for t = 1 and 3. So let $t \geq 5$ for cahit-9-equitability of C_noK_1, 8 labels will have to be utilised t times each and one label will have to be utilised $'t + 1'$ times such that eight edge weight will occur $'t'$ times each and one edge weight will occur $'t + 1'$ times.

We given below the labeling function $f: V(C_noK_1) \rightarrow \{0,1,\ldots,8\}$

$f(u_1) = 8, \quad f(v_1) = 3$

$f(u_2) = 0, \quad f(v_2) = 2$

$f(u_{2i+1}) = 8, \quad f(v_{2i+1}) = 8, \quad 1 \leq i \leq \frac{t-1}{2}$

$f(u_{2i+2}) = 0, \quad f(v_{2i+2}) = 3, \quad 1 \leq i \leq \frac{t+1}{2}$

$f(u_{2i+1}) = 7, \quad f(v_{2i+1}) = 1, \quad \frac{t+1}{2} \leq i \leq t$

$f(u_{2i}) = 0, \quad f(v_{2i}) = 6, \quad \frac{t+5}{2} \leq i \leq t$

$f(u_{2i}) = 1, \quad f(v_{2i}) = 5, \quad (t+1) \leq i \leq 3(\frac{t-1}{2})$

$f(u_{2i+1}) = 6, \quad f(v_{2i+1}) = 2, \quad (t+1) \leq i \leq (\frac{3t+1}{2})$

$f(u_{3t-1}) = 1, \quad f(v_{3t-1}) = 6,$

$f(v_{2i}) = 4, \quad f(v_{2i}) = 5, \quad \frac{3t+1}{2} \leq i \leq 2t$

$f(u_{2i-1}) = 2, \quad f(v_{2i-1}) = 3, \quad \frac{3t+5}{2} \leq i \leq 2t$

$f(u_{4t+1}) = 4, \quad f(v_{4t+1}) = 5$

$f(u_i) = 4, \quad f(v_i) = 7, \quad (4t+2) \leq i \leq \frac{9t+1}{2}$

Any one can verify that eight labels and eight edge-weight occurs 't' times each and one label and one edge weight occurs *'t+1'* times each. We give below a suitable labelling for t= 1,3 which corresponds n=5, 14 respectively.

CAHIT-9-EQUITABLE LABELING OF C_5oK_1

Here $p = q = 10$, $t = 1$, $n = 5$

v_1	u_2	u_3	u_4	u_5
4	7	6	2	3
8	0	6	1	5
u_1	u_2	u_3	u_4	u_5

CAHIT-9-EQUITABLE LABELING OF $C_{14}oK_1$

Here $p = q = 28$, $t = 3$, $n = 14$

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}
3	2	8	3	1	3	1	6	2	5	2	5	5	7
8	0	8	0	7	0	7	1	6	4	6	4	4	4
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_1	u_2	u_3	u_4	u_5

CASE 3. $2n \equiv 2 \pmod 9$

Let $p = q = 2n = 9t+2$, $t \geq 2$. Implies that t is even number. We give a suitable labeling at the end of the proof for t = 2. So let $t \geq 4$. For Cahit-9-equitability of C_noK_1 seven labels will have to be used $'t'$ times each and two labels will have to e used $'t + 1'$ times each such that seven edge weight will occur t times each and two edge weights will occur $'t + 1'$ times each.

We describe below labelling function $f : V(C_noK_1) \rightarrow \{0,1,2, \ldots, 8\}$ as follows.

$f(u_1) = 0, \quad f(v_1) = 2$

$f(u_{2i}) = 8, \quad f(v_{2i}) = 8, \quad 1 \leq i \leq t/2$

$f(u_{2i+1}) = 0, \quad f(v_{2i+1}) = 3, \quad 1 \leq i \leq t/2$

$f(u_{2i}) = 7, \quad f(v_{2i}) = 1, \quad \frac{t+2}{2} \leq i \leq t$

$f(u_{2i+1}) = 0, \quad f(v_{2i+1}) = 6, \quad \frac{t+2}{2} \leq i \leq t\text{-}1$

$f(u_{2i+1}) = 1, \quad f(v_{2i+1}) = 5, \quad t \leq i \leq \frac{3t-4}{2}$

$f(u_{2i}) = 6, \quad f(v_{2i}) = 2, \quad t+1 \leq i \leq \frac{3t}{2}$

$f(u_{3t-1}) = 1, \quad f(v_{3t-1}) = 6$

$f(u_i) = 2, \quad f(v_i) = 3, \quad 3t+2 \leq i \leq 4t\text{-}2$

$f(u_{2i+1}) = 4, \quad f(v_{2i+1}) = 5, \quad \frac{3t}{2} \leq i \leq 2t$

$f(u_{4t}) = 4, \quad f(v_{4t}) = 3$

$f(u_i) = 4, \quad f(v_i) = 7, \quad 4t + 2 \leq i \leq \frac{9t}{2}$

$f(u_{\frac{9t}{2}+1}) = 7, \quad f(v_{\frac{9t}{2}+1}) = 3$

It is easy to see that seven labels and seven edge weights occur $'t'$ times each and two labels and two edge weights occur ′t+1′ times each. We give below a suitable labeling for t= 2 which corresponds to n = 10 respectively.

CAHIT-9-EQUITABLE LABELING OF $C_{10}oK_1$

Here $p = q = 20$, $t = 2$, $n = 10$

v1	v2	v3	v4	v5	v6	v7	v8	v9	V10
2	8	7	1	6	2	5	4	4	7
0	8	0	7	1	6	3	3	5	3
u1	u2	u3	u4	u5	u6	u7	u8	u9	U10

CASE 4. $2n \equiv 3 \pmod 9$

Let $p = q = 2n = 9t+3$, $t \geq 1$. Note that as $2n \equiv 9t+3$, t is an odd number. We give suitable labelings at the end of the proof for t = 1,3. So let $t \geq 5$. For cahit-9-equitability of C_noK_1, six labels will have to be used ′t′ times each and three labels will have to be used ′t+1′ times each such that six edge weights will occur t times each and three edge weights will occur ′t+1′ times each.

We define the labelling function f: $V(C_noK_1) \rightarrow \{0,1,2,\ldots,8\}$ as follows

$f(u_{2i-1}) = 0$, $f(v_{2i-1}) = 3$, $1 \leq i \leq \frac{t+1}{2}$

$f(u_{2i}) = 8$ $f(v_{2i}) = 8$, $1 \leq i \leq \frac{t-1}{2}$

$f(u_{2i}) = 7$, $f(v_{2i}) = 1$, $\frac{t+1}{2} \leq i \leq t$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 6$, $\frac{t+1}{2} \leq i \leq t-1$

$f(u_{2i+1}) = 1$, $f(v_{2i+1}) = 5$, $t \leq i \leq \frac{3(t-1)}{2}$

$f(u_{2i}) = 6$, $f(v_{2i}) = 2$, $t+1 \leq i \leq \frac{3t+1}{2}$

$f(u_{3t}) = 1$, $f(v_{3t}) = 1$

$f(u_{2i+1}) = 4$, $f(v_{2i+1}) = 5$, $\frac{3t+1}{2} \leq i \leq 2t-1$

$f(u_{2i}) = 2$, $f(v_{2i}) = 3$, $\frac{3(t+1)}{2} \leq i \leq 2t$

$f(u_i) = 4$, $f(v_i) = 7$, $4t+1 \leq i \leq \frac{9t+1}{2}$

$f(u_{\frac{9t+3}{2}}) = 8$ $f(v_{\frac{9t+3}{2}}) = 5$

verify easy that six labels and six edge weights occur ′t′ times each and three labels and three edge weights occur ′t+1′ times each.

We present below suitable labelling for t=1,3 which correspond to n=6,15.

CAHIT-9-EQUITABLE LABELING OF C_6oK_1

Here $p = q = 12$, $t = 1$, $n = 6$

v_1	v_2	v_3	v_4	v_5	v_6
3	1	0	2	4	5
0	7	1	6	4	8
u_1	u_2	u_3	u_4	u_5	u_6

CAHIT-9-EQUITABLE LABELING OF $C_{15}oK_1$

Here$p = q = 30$, t =3, n = 15

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
3	8	3	1	6	1	5	2
0	8	0	7	0	7	1	6
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8

v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}
0	2	5	3	4	7	5
2	6	4	2	4	4	8
u_9	u_1	u_2	u_3	u_4	u_5	u_5

CASE 5. $2n \equiv 4 \pmod 9$

Let $p = q = 2n = 9t+4$, ′t′ is even and number $t \geq 2$. We obtain suitable labeling at the end of the proof for t = 2. So let $t \geq 4$. For Cahit-9-equitability of C_noK_1 five labels will have to be used ′t′ times each and four labels will have to be used t+1 times each such that five edge weight will occur ′t′ times each four edge weights will occur t+1 times each.

We define the labeing function f: $V(C_noK_1) \rightarrow \{0,1,2, \ldots, 8\}$ as follows:

$f(u_1) = 0$, $f(v_1) = 2$,

$f(u_{2i}) = 8$, $f(v_{2i}) = 8$, $1 \leq i \leq t/2$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 3$, $1 \leq i \leq t/2$

$f(u_{2i}) = 7$, $f(v_{2i}) = 1$, $\frac{t}{2}+1 \leq i \leq t+1$

$f(u_{2i+1}) = 0$, $f(v_{2i+1})$=6 $\frac{t}{2}+1 \leq i \leq t$

$f(u_{2i+1}) = 1$, $f(v_{2i+1})$=5, $t+1 \leq i \leq \frac{3t}{2}-1$

$f(u_{2i}) = 6$, $f(v_{2i}) = 2$, $t+2 \leq i \leq \frac{3t}{2}+1$

$f(u_{3t+1}) = 1$, $f(v_{3t+1}) = 6$,

$f(u_{2i+1}) = 4$, $f(v_{2i+1}) = 5$, $\frac{3t}{2}+1 \leq i \leq 2t$

$f(u_{2i}) = 2$, $f(v_{2i}) = 3$, $\frac{3t}{2}+2 \leq i \leq 2t$

$f(u_{4t+2}) = 4$, $f(v_{4t+2})=3$

$f(u_i) = 4$, $f(v_i) = 7$, $4t+3 \leq i \leq \frac{9t+4}{2}$

It is easy to verify that five labels and five edge weights occurs 't' times each and four labels and four edge weights occur 't+1' times each. We establishing suitable labeling for $t = 2$ which correspond to $n = 11$.

CAHIT-9-EQUITABLE LABELING OF $C_{11}oK_1$

Here $p = q = 22$, $t = 2$, $n = 11$

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	V_{10}	v_{11}
2	8	4	1	3	1	6	2	3	5	8
0	8	0	7	0	7	1	6	4	4	5
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	U_{10}	u_{11}

CASE 6.2n ≡ 5 (mod 9)

Let $p = q = 2n = 9t+5$, $t \geq 1$ and t is an odd integer. We give suitable labeling at the end of the proof for t = 1,3. So let $t \geq 5$. For Cahit-9-equitability of C_noK_1 four labels will have to be used 't' times each and five labels will have to be used 't+1' times each such that four edge weights will occur 't' times each and five edge weights will occur 't+1' times each.

We define the labelling function $f:V(C_noK_1) \rightarrow \{0,1,2, \ldots,8\}$ as follows.

$f(u_1) = 0$, $f(v_1) = 2$,

$f(u_{2i}) = 8$, $f(v_{2i}) = 8$, $1 \leq i \leq \frac{t-1}{2}$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 3$, $1 \leq i \leq \frac{t-1}{2}$

$f(u_{2i}) = 7$, $f(v_{2i}) = 1$, $\frac{t+1}{2} \leq i \leq t$

$f(u_{2i+1}) = 0$, $f(v_{2i+1})=6$ $\frac{t+1}{2} \leq i \leq t-1$

$f(u_{2i+1}) = 1$, $f(v_{2i+1})=5$, $t \leq i \leq 3(\frac{t-1}{2})$

$f(u_{2i}) = 6$, $f(v_{2i}) = 2$, $t+1 \leq i \leq \frac{3t+1}{2}$

$fuv_{3t}) = 1$, $f(v_{3t}) = 6$,

$f(u_{2i+1}) = 4$, $f(v_{2i+1}) = 5$, $\frac{3t+1}{2} \leq i \leq 2t$-1

$f(u_{2i}) = 2$, $f(v_{2i}) = 3$, $3(\frac{(t+1)}{2}) \leq i \leq 2t$

$f(u_{4t+1}) = 4$, $f(v_{4t+1})=3$

$f(u_i) = 4$, $f(v_i) = 7$, $4t+2 \leq i \leq \frac{9t+3}{2}$

$f(u_{\frac{9t+5}{2}}) = 8$, $f(v_{\frac{9t+5}{2}}) = 5$

It can be directly verified that four labels and four edge weights occurs 't' times each and five labels and five edge weights occur 't+1' times each. We give below suitable labelling for t= 1, 3 which correspond to n =7, 16.

CAHIT-9-EQUITABLE LABELING OF C_7oK_1

Here $p = q = 14$, $t = 1$, $n = 7$

v_1	v_2	v_3	v_4	v_5	v_6	v_7
2	1	6	2	3	7	5
0	7	1	6	4	4	8
u_1	u_2	u_3	u_4	u_5	u_6	u_7

CAHIT-9-EQUITABLE LABELING OF$C_{16}oK_1$

Here$p = q = 32$, $t = 3$, $n = 16$

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
2	8	3	1	6	1	5	2	6
0	8	0	7	0	7	1	6	1
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9

u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}
2	5	3	3	7	7	5
6	4	2	4	4	4	8
u_{10}	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}

CASE 7. 2n ≡ 6 (mod 9)

Let $p = q = 2n = 9t + 6$, t is even number $t \geq 2$. We give suitable labeling at the end of the proof t =2, 4. So let $t \geq 6$.For Cahit-9-equitability of C_noK_1 three labels will have to be used 't' times and six labels will have to be used 't+1' times each such that three edge weights will occur 't' times and six edge weights will occur 't+1' times each.

We define the labelling function $f:V(C_noK_1) \rightarrow \{0,1,2, \ldots, 8\}$ as follows.

$f(u_1) = 0$, $f(v_1) = 2$,

$f(u_{2i}) = 8$, $f(v_{2i}) = 8$, $1 \leq i \leq t/2$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 3$, $1 \leq i \leq \frac{t}{2}+1$

$f(u_{2i}) = 7$, $f(v_{2i})= 1$, $\frac{t}{2}+1 \leq i \leq t+1$

$f(u_{2i+1}) = 0$, $f(v_{2i+1})=6$, $\frac{t}{2}+2 \leq i \leq t$

$f(u_{2i+1}) = 1$, $f(v_{2i+1})=5$, $t+1 \leq i \leq \frac{3t}{2}-1$

$f(u_{2i}) = 6$, $f(v_{2i}) = 2$, $t+2 \leq i \leq \frac{3t}{2}+1$

$f(u_{3t+1}) = 1$, $f(v_{3t+1}) = 6$,

$f(u_{2i+1}) = 4$, $f(v_{2i+1}) = 5$, $\frac{3t}{2}+1 \leq i \leq 2t$

$f(u_{2i}) = 2,\quad f(v_{2i}) = 3,\quad \frac{3t}{2} + 2 \le i \le 2t$

$f(u_{4t+2}) = 4,\quad f(v_{4t+2})=5$

$f(u_i) = 4,\quad f(v_i) = 7,\quad 4t+3 \le i \le \frac{9t+4}{2}$

$f(u_{\frac{9t+6}{2}}) = 8,\quad f(v_{\frac{9t+6}{2}}) = 3$

It is not hard to verify that three labels and three edge weights occurs ′t′ times each and six labels and six edge weights occur ′t+1′ times each. We have below a suitable labeling for t = 2, 4 which correspond to n = 12, 21.

CAHIT-9-EQUITABLE LABELING OF $C_{12}oK_1$

Here $p = q = 24$, $t = 2$, $n = 12$

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
2	8	3	1	3	1	6	2	5	5	7	3
0	8	0	7	0	7	1	6	7	4	4	8
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}

CAHIT-9-EQUITABLE LABELING OF $C_{21}oK_1$

Here $p = q = 42$, $t = 4$, $n = 21$

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
2	8	3	8	3	1	3	1	6	1	5	2
0	8	0	8	0	7	0	7	0	7	1	6
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}

u_{13}	u_{14}	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	u_{20}	u_{21}
6	2	5	3	5	5	7	7	3
1	6	4	2	4	4	4	4	8
u_{13}	u_{14}	u_{15}	u_{16}	u_{17}	u_{18}	u_{19}	u_{20}	u_{21}

CASE 8. $2n \equiv 7 \pmod 9$

Let $p = q = 2n = 9t+7$,then clearly ′t′ is odd ≥ 1. We establish suitable labeling at the end of the proof for t = 1, 3. So let $t \ge 5$. For Cahit-9-equitability of C_noK_1 two labels will have to be used ′t′ times and seven labels will have to be used ′t+1′ times each such that two edge weights will occur ′t′ times and seven edge weights will occur ′t+1′ times each.

We define a function $f:V(C_noK_1) \rightarrow \{0,1,2,\ldots,8\}$ as follows:

$f(u_1) = 0,\quad f(v_1) = 2,$

$f(u_{2i}) = 8,\quad f(v_{2i}) = 8,\quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i+1}) = 0,\quad f(v_{2i+1}) = 3,\quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i}) = 7,\quad f(v_{2i}) = 1,\quad \frac{t+3}{2} \le i \le t+1$

$f(u_{2i+1}) = 0,\quad f(v_{2i+1})=6,\quad \frac{t+3}{2} \le i \le t$

$f(u_{2i+1}) = 1,\quad f(v_{2i+1})=5,\quad t+1 \le i \le \frac{3t-1}{2}$

$fu_{2i}) = 6,\quad f(v_{2i}) = 2,\quad t+2 \le i \le \frac{3(t+1)}{2}$

$f(u_{3t+2}) = 1,\quad f(v_{3t+2}) = 6$

$,f(u_{2i+1}) = 4,\quad f(v_{2i+1}) = 5,\quad \frac{3(t+1)}{2} \le i \le 2t+1$

$f(u_{2i}) = 2,\quad f(v_{2i}) = 3,\quad \frac{3(t+1)}{2} + 1 \le i \le 2t+1$

$f(u_i) = 4,\quad f(v_i) = 7,\quad 4(t+1) \le i \le \frac{9t+7}{2}$

One can easily verify that two labels and two edge weights occur ′t′ times each and seven labels and seven edge weights occur ′t+1′ times each. We obtain below a suitable labeling for t=1,3 which corresponds to n=8,17.

CAHIT-9-EQUITABLE LABELING OF C_8oK_1

Here $p = q = 16$, $t = 1$, $n = 8$

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
2	8	3	1	6	2	5	7
0	8	0	7	1	6	4	4
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8

CAHIT-9-EQUITABLE LABELING OF $C_{17}oK_1$

Here $p = q = 34$, $t = 3$, $n = 17$

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
2	8	3	8	3	1	6	1	5	2
0	8	0	8	0	7	0	7	1	6
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}

u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}
6	2	5	3	5	7	7
1	6	4	2	4	4	4
u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}

CASE 9. $2n \equiv 8 \pmod 9$

Let p = q = 2n = 9t+8. It shows that ′t′ is even ≥ 2. We give suitable labeling at the end proof for t = 2. So let $t \ge 4$. For Cahit-9-equitability of C_noK_1 one label will have to be used ′t′ times and eight labels will have to be used ′t+1′ times each such that one edge weight will occur ′t′ times and eight edge weights will occur ′t+1′ times each.

We define a function $f:V(C_noK_1) \rightarrow \{0,1,2,\ldots,8\}$ as follows:

$f(u_1) = 0,\quad f(v_1) = 2,$

$f(u_{2i}) = 8$, $f(v_{2i}) = 8$, $1 \le i \le \frac{t}{2}$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 3$, $1 \le i \le \frac{t}{2} + 1$

$f(u_{2i}) = 7$, $f(v_{2i}) = 1$, $\frac{t}{2} + 1 \le i \le t+1$

$f(u_{2i+1}) = 0$, $f(v_{2i+1}) = 6$, $\frac{t}{2} + 2 \le i \le t$

$f(u_{2i+1}) = 1$, $f(v_{2i+1}) = 5$, $t + 1 \le i \le \frac{3t}{2} - 1$

$f(u_{2i}) = 6$, $f(v_{2i}) = 2$, $t + 2 \le i \le \frac{3t}{2} + 1$

$f(u_{3t+1}) = 1$, $f(v_{3t+1}) = 6$

$f(u_{2i+1}) = 2$, $f(v_{2i+1}) = 3$, $\frac{3t}{2} + 1 \le i \le 2t$

$f(u_{2i}) = 4$, $f(v_{2i}) = 5$, $\frac{3t}{2} + 2 \le i \le 2t + 1$

$f(u_{4t+3}) = 4$, $f(v_{4t+3}) = 5$

$f(u_i) = 4$, $f(v_i) = 7$, $4(t+1) \le i \le \frac{9t+6}{2}$

$f(u_{\frac{9t+8}{2}}) = 8$, $f(v_{\frac{9t+8}{2}}) = 6$

It is easily verify that one label and one edge weight occur 't' times each and eight labels and eight edge weight occur 't+1' times each. We give below a suitable labeling for t=2 which correspond on n = 13.

CAHIT-9-EQUITABLE LABELING OF $C_{13}oK_1$

Here $p = q = 2n$, t = 2, n = 13

u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
2	8	3	1	3	1	6	2	3	5	5	7
0	8	0	7	0	7	1	6	2	4	4	4
u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}

REFERENCES

[1]. D G Akka and Sanjay Roy, Cahit-7-equitability of corona $C_n oK_1$.International journal of mathematical sciences and Applications, Vol. 1 No. 3 (2011) 1609-1618.

[2]. D G Akka and Dayanand G.K. Cahit-8-equitability of corona $C_n oK_1$. International journal of research in Engineering and Technology, Vol.03 NCRIET (2014)

[3]. Bloom, problem posed at the Graph theory meeting of the New York Academy of Science, Nov. (1989).

[4]. Barrientos, Dejter and Hevia, Equitable labelings of forests Comin, and Graph theory I (1995) 1-26.

[5]. R Frucht and F Haray, On the corona of two graphs, Aequationes Mathematicae (1971).

[6]. Vasanti N Bhat Nayak and ShantaTelang, Cahit-equitability of Coronas, Ars Combinatoria, Vol. 71 (2004) 3-32.

[7]. J. Wojeiechowski, Equitable of cycles, J. of Graph Theory Vol. 17, No. 1 (1993) 531-547.