Development of Four Degree of Freedom Vibratory Model of Human Lower Limbs

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***Abstract*— Frequent and prolonged expose of human body to vibrations can induce back pain, physical disorder and degeneration of tissue. To estimate forces and frequencies, a biomechanical model is developed. A four-degree of freedom discrete mass system is developed using Lagrange's energy method. In this paper a three degree of freedom system using linear motion is developed first then model was coupled with another angular motion using co-ordinate coupling to develop four degree of freedom model. Different case studies are presented for static and dynamic coupling and combination of both for the same model. Mathematical model for all the cases was developed for the study of human lower limbs.**

Keywords—Biomechanics; vibrations; lower limbs

I. Introduction

Vibration is the motion of a particle or a body or system of connected bodies displaced from a position of equilibrium. Most vibrations are undesirable in machines and structures because they produce increased stresses, energy losses, causes added wear, increase bearing loads, induced fatigue, create passenger discomfort in vehicles, and absorb energy from the system. Rotating machine parts need careful balancing in order to prevent damage from vibrations.

When people are exposed to vibration in vehicles i.e. cars, buses, trains and various machinery their performance is affected causing back pain, fatigue stresses and disorder. Obviously there is a relationship between human body and vibration environment, particularly at frequencies near the principal resonance. There are several reports describing how vibration interferes with people's working efficiency, safety and health. However it is difficult to accurately estimate the behavior of the human body under vibration because it is complex dynamic system. However (4-DOF) a model consists of masses, springs and dampers is developed simulating the lower limb.

II. 3-DOF LINEAR VIBRATION Model under consideration

Figure-1 simulates lumped mass system of human lower limbs. In which Mass m_1 and m_2 simulate the mass of lower limbs and mass m_3 is the upper body mass.Stiffness of lower limbs are denoted by k_1, k_2, k_3 and k_4.

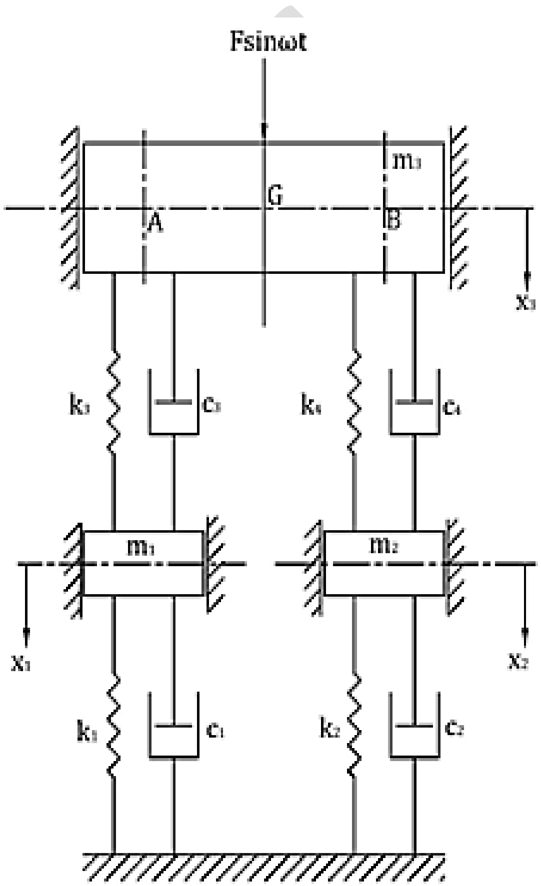

Figure 1 The physical model of a three degree of freedom discrete system

Also dampingcoefficients are depicted by c_1, c_2, c_3 and c_4. Considering physical system in to a three degree of freedom system with three masses m_1, m_2 andm_3. m_1 and m_2 tied to ground through spring k_1 and k_2 and damper c_1 and c_2 similarly both are connected to mass m_3 through spring k_3 and k_4 and damper c_3 and c_4.

One simple harmonic excitation force is acting on the centroid of mass m_3. The system is shown in its mean equilibrium position. Three coordinates x_1, x_2, and x_3 are enough to define the position of system at any instant of time of the masses m_1, m_2 andm_3 respectively.The system is assumed to be free in executing oscillations in the vertical direction only, also clearance between mass and guide is negligible. The springs and dampers are assumed mass less and deformation of spring and damper is linear.

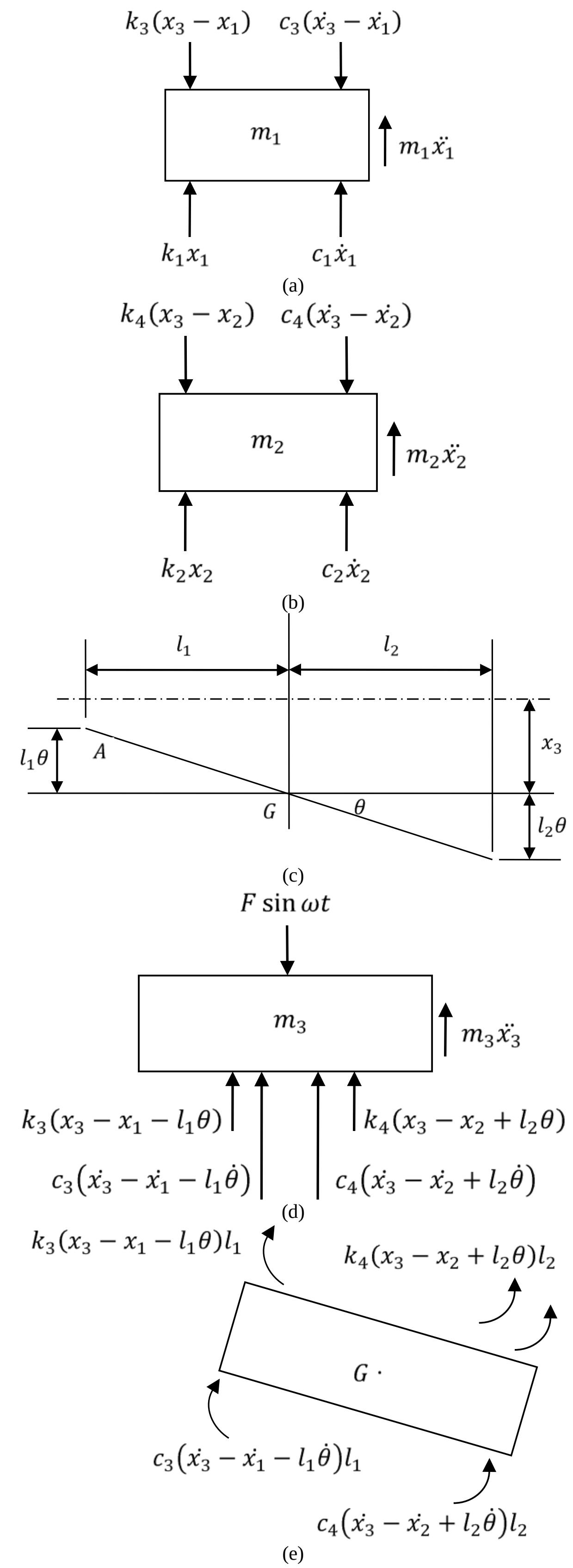

Figure 2 Free body diagram of (a) mass m_1 (b) mass m_2 (c) mass m_3 (d) moment on m_3

The inertia, moment and restoring forces in free body diagram of all the mass is depicted in the figure 2. Measuring the displacement, velocity and acceleration quantities positive downwards and taking moment CCW positive, in system of three masses in the equilibrium. By applying energy based Lagrangian equation to this system,

$$K.E. = \frac{1}{2}m_1\dot{x}_1^{\,2} + \frac{1}{2}m_2\dot{x}_2^{\,2} + \frac{1}{2}m_3\dot{x}_3^{\,2}$$
$$P.E. = \frac{1}{2}k_1x_1^{\,2} + \frac{1}{2}k_2x_2^{\,2} + \frac{1}{2}k_3(x_3 - x_1)^2 + \frac{1}{2}k_4(x_3 - x_2)^2$$
$$D.E. = \frac{1}{2}c_1\dot{x}_1^{\,2} + \frac{1}{2}c_2\dot{x}_2^{\,2} + \frac{1}{2}c_3(\dot{x}_3 - \dot{x}_1)^2 + \frac{1}{2}c_4(\dot{x}_3 - \dot{x}_2)^2 \quad (1)$$

After simplifying these equations, Equation of motion can be written as,

$$m_1\ddot{x}_1 + (c_1 + c_3)\dot{x}_1 + (k_1 + k_3)x_1 - c_3\dot{x}_3 - k_3x_3 = 0$$
$$m_2\ddot{x}_2 + (c_2 + c_4)\dot{x}_2 + (k_2 + k_4)x_2 - c_4\dot{x}_3 - k_4x_3 = 0$$
$$m_3\ddot{x}_3 + (c_3 + c_4)\dot{x}_3 + (k_3 + k_4)x_3 - c_3\dot{x}_1 - k_3x_1 - c_4\dot{x}_2 - k_4x_2 = F \sin \omega t \quad (2)$$

III. 4-DOF LINEAR AND ANGULAR VIBRATIONS

To predict behavior of human lower limb more precisely 3-DOF is not enough. So after getting equation of motion for 3-DOF in previous section, add another angular motion to the same system as shown in figure 3. Now it is required to have two coordinates x_3 and θ of the massm_3 to define the position of system at any instant of time.

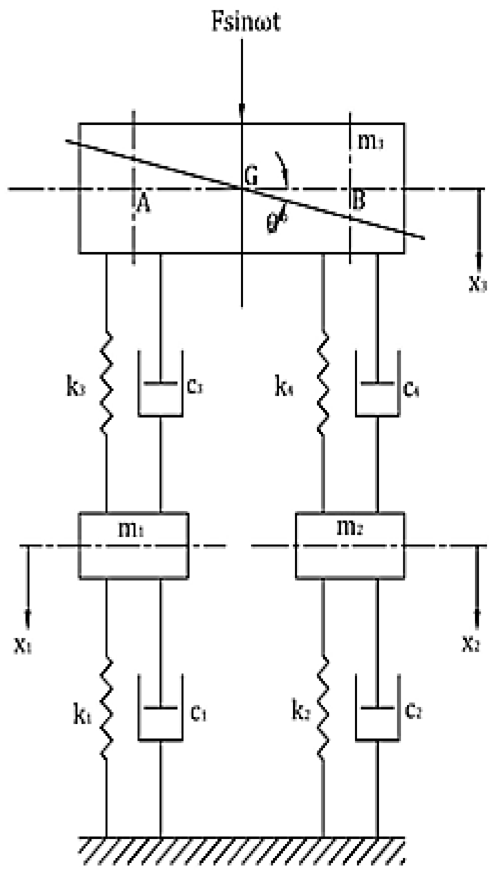

Figure 3 The physical model of a four degree of freedom with linear and angular co-ordinate coupling

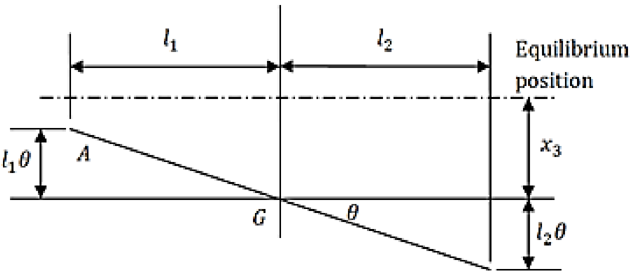

Figure 4 FBD of Mass m_3

Figure 4 describes the mechanics of the mass m_3 with two variables x_3 and θ. Here considering force acting at center of gravity (C.G) G, system can be represented by,

$$
\begin{aligned}
&m_1\ddot{x}_1 + (c_1 + c_3)\dot{x}_1 + (k_1 + k_3)x_1 - c_3\dot{x}_3 - k_3x_3 = 0\\
&m_2\ddot{x}_2 + (c_2 + c_4)\dot{x}_2 + (k_2 + k_4)x_2 - c_4\dot{x}_3 - k_4x_3 = 0\\
&m_3\ddot{x}_3 + (c_3 + c_4)\dot{x}_3 + (k_3 + k_4)x_3 + (c_4l_2 - c_3l_1)\dot{\theta}\\
&\qquad + (k_4l_2 - k_3l_1)\theta - c_3\dot{x}_1 - k_3x_1 - c_4\dot{x}_2\\
&\qquad - k_4x_2 = F\sin\omega t\\
&m_3r_g^2\ddot{\theta} + (c_4l_2 - c_3l_1)\dot{x}_3 + (k_4l_2 - k_3l_1)x_3\\
&\qquad + \left(c_3{l_1}^2 + c_4{l_2}^2\right)\dot{\theta} + (k_3{l_1}^2 + k_4{l_2}^2)\theta\\
&\qquad + c_3l_1\dot{x}_1 + k_3l_1x_1 - c_4l_2\dot{x}_2 - k_4l_2x_2\\
&\qquad = 0
\end{aligned}
\tag{3}
$$

Equation (3) represents the equation of motion for given system with 4-DOF. These equations can be written in matrix form as below,

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{4}$$

Where,

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0\\ 0 & 0 & m_3 & 0\\ 0 & 0 & 0 & m_3r_g^2 \end{bmatrix}$$

$$\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\\ \ddot{\theta} \end{Bmatrix}$$

$$[C] = \begin{bmatrix} c_1 + c_3 & 0 & -c_3 & 0\\ 0 & c_2 + c_4 & -c_4 & 0\\ -c_3 & -c_4 & c_3 + c_4 & c_4l_2 - c_3l_1\\ c_3l_1 & -c_4l_2 & c_4l_2 - c_3l_1 & c_3{l_1}^2 + c_4{l_2}^2 \end{bmatrix}$$

$$\{\dot{x}\} = \begin{Bmatrix} \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{\theta} \end{Bmatrix}$$

$$[K] = \begin{bmatrix} k_1 + k_3 & 0 & -k_3 & 0\\ 0 & k_2 + k_4 & -k_4 & 0\\ -k_3 & -k_4 & k_3 + k_4 & k_4l_2 - k_3l_1\\ k_3l_1 & -k_4l_2 & k_4l_2 - k_3l_1 & k_3{l_1}^2 + k_4{l_2}^2 \end{bmatrix}$$

$$\{x\} = \begin{Bmatrix} x_1\\ x_2\\ x_3\\ \theta \end{Bmatrix}$$

$$\{F(t)\} = \begin{Bmatrix} 0\\ 0\\ F\sin\omega t\\ 0 \end{Bmatrix}$$

This equation represents the behavior of the system under consideration. Where [M], [C] and [K] are mass matrix, damping coefficient matrix and stiffness matrix of the system and$\{\ddot{x}\}$,$\{\dot{x}\}\{x\}$are the acceleration, velocity and displacement vector respectively at each joints.

A. *case-1: Dynamic coupling of linear and angular vibration*

In previous model it is assumed applied force at C.G. but now,considering force applied at point *C* as shown in Figure 5 and it produce pure transmission x_c. Eccentricity of force from centroid is taken as *e*.

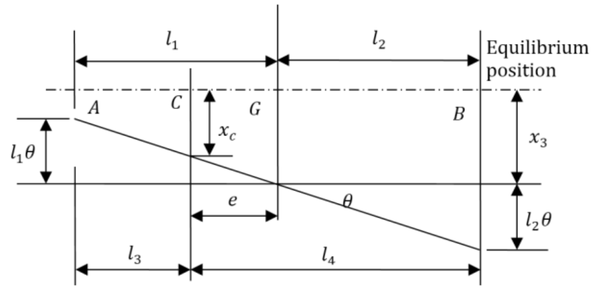

Figure 5 Dynamic Coupling of Coordinates

Considering θ is to be very small so equations can be written from free body diagram shown in figure 5 as,

$$
\begin{aligned}
x_3 &= x_c + e\theta\\
l_3 &= l_1 - e\\
l_4 &= l_2 + e
\end{aligned}
\tag{5}
$$

Substituting Equation (5) in the Equation (3),

$$
\begin{aligned}
&m_1\ddot{x}_1 + (c_1 + c_3)\dot{x}_1 + (k_1 + k_3)x_1 - c_3\dot{x}_c - c_3e\dot{\theta} - k_3x_c\\
&\qquad - k_3e\theta = 0\\
&m_2\ddot{x}_2 + (c_2 + c_4)\dot{x}_2 + (k_2 + k_4)x_2 - c_4\dot{x}_c - c_4e\dot{\theta} - k_4x_c\\
&\qquad - k_4e\theta = 0\\
&m_3\ddot{x}_c + m_3e\ddot{\theta} + (c_3 + c_4)\dot{x}_c + [2c_4e + (c_4l_2 - c_3l_1)]\dot{\theta}\\
&\qquad + (k_3 + k_4)x_c\\
&\qquad + [2k_4e + (k_4l_2 - k_3l_1)]\theta - c_3\dot{x}_1\\
&\qquad - k_3x_1 - c_4\dot{x}_2 - k_4x_2 = F\sin\omega t\\
&m_3r_c^2\ddot{\theta} + m_3e\ddot{x}_c + [(c_4l_2 - c_3l_3) + (c_4 + c_3)e]\dot{x}_c\\
&\qquad + [(k_4l_2 - k_3l_3) + (k_3 + k_4)e]x_c\\
&\qquad + \left[(c_4 + c_3)2e^2 + (c_4l_2 - c_3l_1)3e\right.\\
&\qquad \left.+ (c_3{l_1}^2 + c_4{l_2}^2)\right]\dot{\theta}\\
&\qquad + \left[(k_4 + k_3)2e^2 + (k_4l_2 - k_3l_1)3e\right.\\
&\qquad \left.+ (k_3{l_1}^2 + k_4{l_2}^2)\right]\dot{\theta} + c_3(l_1 - e)\dot{x}_1\\
&\qquad + k_3(l_1 - e)x_1 - c_4(l_2 + e)\dot{x}_2 - k_4(l_2\\
&\qquad + e)x_2 = 0
\end{aligned}
\tag{6}
$$

This is the equation of motion for dynamic coupling of linear and angular vibration for given system.

B. *case-2: static and Dynamic coupling of linear and angular vibration*

In previous sessions coordinates are either statically or dynamically coupled. In this section both static and dynamic coupling of coordinates is considered. If point of action is to be taken at a point shown in Figure 6, then we have both static and dynamic coupling.

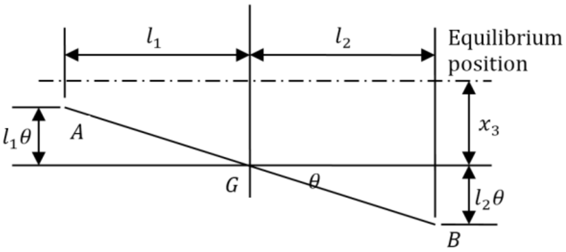

Figure 6 Dynamic Coupling of Vibration

From Figure 6,

$$e=l_1;$$

$$l_2=l-l_1$$

Put this value in equation 6 and writing in matrix form,

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & m_3 l_1 \\ 0 & 0 & m_3 l_1 & m_3 r_c^2 \end{bmatrix} \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{x_c} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 + c_3 & 0 & -c_3 & -c_3 l_1 \\ 0 & c_2 + c_4 & -c_4 & -c_4 l_1 \\ -c_3 & -c_4 & c_3 + c_4 & (c_4 l - c_3 l_1) \\ 0 & -c_4 l & c_4 l & c_4 {l_1}^2 + c_4 l_1 l \end{bmatrix} \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_c} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 + k_3 & 0 & -k_3 & -k_3 l_1 \\ 0 & k_2 + k_4 & -k_4 & -k_4 l_1 \\ -k_3 & -k_4 & k_3 + k_4 & (k_4 l - k_3 l_1) \\ 0 & -k_4 l & k_4 l & k_4 {l_1}^2 + k_4 l_1 l \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_c \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F \sin \omega t \\ 0 \end{Bmatrix} \quad (7)$$

Equation 7 is the equation of motion in case of combination of static and dynamic coupling of coordinates.

CONCLUSION

The vibration analysis was carried out to study the behaviour of the human lower limb under different conditions. Hense in this paper, firstly three degree of freedom sysem was developed for human lower limbs. Then by adding one angular motion at the combined mass, four degree of freedom model was developed. To reach to the actual condition statically coupled, dynamically coupled and combinly coupled cordinates of four degree of freedom system was analysed. A equation of motion for the model under consideration was developed for further analysis.

Further, developed equation will be use to find out the natural frequency of the system using state space analysis at each mass location. Which will help in predicting the behaviour of the human lower limb for development of diffent applications. This study may be further extended for more no of degree of freedom syatem. In future results obtained by this analysis may be validated through the experements.

REFERENCES

[1] G. K Grover, "Mechanical Vibrations", Chapter 4.2, 119-121

[2] J. S. Rao, K. Gupta, "Introductory Course on Theory and Practice of Mechanical Vibrations".

[3] W. T. Thomson, "Theory of vibration with applications", Third Edition, CBS publication.

[4] William W. Seto, "Theory and Problems of Mechanical Vibrations", McGraw-Hill International.

[5] Michael R. Hatch, "Vibration simulation using MATLAB"

[6] M. H. Mohammadi, A. Mohammadi, A.Kimiaeifarand H.Tabaei, Application of HPEM to Investigate the Response and Stability of Nonlinear Problems in Vibration, Australian Journal of Basic and Applied Sciences, 4(4): 557-563, 2010

[7] Ibrahim I.Esat, Moudar Saud and Seref Naci Engin, A novel method to obtain a real-time control force strategy using genetic algorithms for dynamic systems subjected to external arbitrary excitations, Journal of Sound and Vibration, 330 (2011) 27–48

[8] Wassim M. Haddad, Ali Ra.zavi, David C. Hyland, Active Vibration Isolation of Multi-Degree of Freedom Systems, the American Control Conference, Albuquerque, New Mexico June 1997.

[9] Mehmet Bulent Ozer and Thomas J. Royston, Application of Sherman–Morrison matrix inversion formula to damped vibration absorbers attached to multi-degree of freedom systems, Journal of Sound and Vibration 283 (2005) 1235–1249

[10] Z. ONISZCZUK, damped vibration analysis of a two-degree-of-freedom discrete system, Journal of Sound and vibration (2002) 257(2), 391-403

[11] Seon J. Jang and Yong J. Choi, Geometrical design method of multi-dof dynamic vibration absorbers, Journal of Sound and Vibration 303 (2007) 343–356.

[12] Y. Matsumoto and M.J. Griffin, Mathematical models for the apparent masses of standing subjects exposed to vertical whole-body vibration, Journal of Sound and Vibration 260 (2003) 431–451.

[13] DEVENDRA P. GARG AND MICHAEL A. ROSS, Vertical Mode Human Body VibrationTransmissibility, IEEE transactions on systems, man, and cybernetics, vol. Smc-6, no. 2,

[14] M. Tylee, M. R. Popovic, S.Yu, and C. Craven, Human Responses to Vibration Therapy.

[15] T. C. Gupta, Identification and Experimental Validation of Damping Ratios of Different Human Body Segments Through Anthropometric Vibratory Model in Standing Posture, ASME Vol. 129, AUGUST 2007.

[16] M. Fritz, Three-dimensional biomechanical model for simulating the response of the human body to vibration stress , Med. Biol. Eng. Comput., 1998, 36, 686-692

[17] Tae-Hyeong Kim, Development of a biomechanical model of the human body in a sitting posture with vibration transmissibility in the vertical direction, International Journal of Industrial Ergonomics 35 (2005) 817–829

[18] S KITAZAKI and M J GRIFFIN, a modal analysis of whole-body vertical vibration, using a finite, element model of the human body, Journal of Sound and Vibration(1997) 200(1), 83-103,

[19] G.H.M.J. Subashia, Y. Matsumotoa, M.J. Griffin, Modelling resonances of the standing body exposed to vertical whole-body vibration: Effects of posture, Journal of Sound and Vibration 317 (2008) 400–418.