A Comparative Study on Stress Concentration Factor in a Centrally Notched Glass-Fibre/Epoxy Plate by Theoretical and Finite Element Analysis

Yathisha.N [#], Ravikumar.S [*], Arjun.M.S[*], Devaraj.M.R [*], Dr Suresha S[#]

[#] *Department of Mechanical Engineering, ATMECE, Bannur road, Mysore - 570028, India.*

[*] *Department of Mechanical Engineering, ATMECE, Bannur road, Mysore - 570028, India.*

Abstract— **Polymer matrix composites (PMCs) are replacing metals in structural applications because of its directional properties and high strength to weight ratio. In a structure there will be many discontinuities created for the purpose of design and assembly. When these structures are subjected to loading conditions, the failure of the structure usually starts from the discontinuities because of stress concentration. Therefore understanding of mechanical behaviour of the composite materials with stress concentration is of greater importance because of the resulting reduced strength of components and higher amount of damage around this region. The aim of this study is to investigate the effects of fibre orientation and the type of continuous fibre reinforcement on the stress concentration factor (SCF) in a composite lamina. Both theoretical results and finite element results for SCF were compared for unidirectional and woven fabric lamina having a central notch for fibre angles 0°, 15°, 30°, 45°, 60°, 75°& 90°. It was found that for unidirectional fibre reinforcement the stress concentration region is perpendicular to the specified fibre orientations up to 60° fibre angle, and the stress concentration factor decreases as fibre angle increases to 90°, but for the woven fabric reinforcement the stress concentration region was found perpendicular to the fibre orientation up to 45° fibre angle, and stress concentration factor decreases till 45° fibre angle ,but increases for fibre angle above 45°. The stress contour obtained from finite element analysis reveals that the maximum stress region is found on the periphery of the hole but depends on the fibre angle.**

I. INTRODUCTION

Composite materials are slowly and steadily replacing isotropic metals, because of their tailored properties and high strength to weigh ratio. Composite materials are the combination of two different materials in a way to obtain desirable properties.

Glass fibre reinforced plastic (GFRP) is a type of polymer matrix composite made up of plastic matrix reinforced by continuous fibres of glass. GFRP is very light weighted and strong material hence they are used in automobiles, aerospace, electrical appliances etc [3].

In a structure there will be many discontinuities created for the purpose of design and assembly. When these structures are subjected to loading conditions, the failure of the structure usually starts from the discontinuities because of stress concentration. Therefore understanding of mechanical behaviour of the composite materials with stress concentration is of greater importance because of the resulting reduced strength of components and higher amount of damage around this region. The stress concentration factor (SCF) for an anisotropic material is different from an isotropic material. The SCF value for an isotropic material depend only on the geometry of the component , but for an anisotropic material it depends on the geometry of the component, stacking sequence and number of layers in the laminate because in an anisotropic materials the material properties are different in all directions[1].

In this study, theoretical calculation for stress concentration factor was carried out for both unidirectional lamina and woven fabric lamina for different fibre angles. These results are compared with the finite element results. Finite element analysis is carried out using ansys workbench 12.1.

II. THEORETICAL CALCULATION

In this study, the fibre used is E-Glass fibre in unidirectional form and fabric form, the matrix material used is epoxy resin. The mechanical properties of the fibre and matrix are given in table -1[8].

TABLE I
Mechanical properties of constituents of FRP materials[8].

Properties	Epoxy	Glass
Elastic modulus (Gpa)	4.3	74
Density (kg/m^3)	1270	2600
Shear modulus (Gpa)	1.50	30
Poisson ratio	0.35	0.25

Elastic properties of the lamina are determined through the simple rule-of-mixture calculations. Fig.1 defines the material principal axes for a typical woven fibre reinforced lamina. Axis 1 along the fibre length and represents the longitudinal direction of the lamina; axes 2 and 3 represents the transverse in-plane and through the thickness directions respectively.

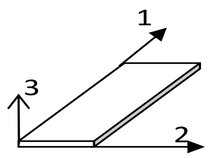


Fig.1. Lamina reference axes.

The elastic constants of the unidirectional composite are calculated using the simple rule of mixture equations (*Eqn.1*) listed below [1]:

$E_{11} = E_f v_f + E_m v_m$

$E_{22}=E_{33}= E_f E_m/(E_f v_m+ E_m v_f)$

$u_{12}= u_{13}= u_f v_f+ u_m v_m$

$u_{21}= (E_{22}/E_{11})u_{12}$ **(1)**

$u_{13} = u_{12}[(1-u_{21})/(1-u_{12})]$

$G_{12}=G_{13}= G_f G_m/(G_f v_m+ G_m v_f)$

$G_{23}= E_{22}/2(1+u_{23})$.

After calculating these elastic constants of the unidirectional composite, elastic constants of the woven fabric composite material are estimated using relations below [8]:

$$\left(\frac{2}{E_1}\frac{E_1(E_1+(1-\mu_{12}^2)E_2-\mu_{12}^2E_2^2)}{E_1(E_1+2E_2)+(1+2\mu_{12}^2)E_2^2}\right)^{UD}=\left(\frac{1}{E_1}\right)^{WF}$$

$$\left(\frac{4}{E_1}\frac{\mu_{12}E_2(E_1-\mu_{12}^2E_2)}{E_1(E_1+2E_2)+(1+2\mu_{12}^2)E_2^2}\right)^{UD}=\left(\frac{\mu_{12}}{E_1}\right)^{WF}$$

$$\left(\frac{1}{E_1}\frac{E_1(\mu_{12}+\mu_{23}+\mu_{12}\mu_{23})+\mu_{12}^2E_2^2}{E_1+(1+2\mu_{12})E_2}\right)^{UD}=\left(\frac{\mu_{13}}{E_1}\right)^{WF}$$

$$\left(\frac{(1-\mu_{23}^2)E_1^2+(1+2\mu_{12}+2\mu_{12}\mu_{23})E_1E_2-\mu_{12}^2E_2^2}{E_1E_2(E_1+(1+2\mu_{12})E_2)}\right)^{UD}=\left(\frac{1}{E_3}\right)^{WF}$$

$$\left(\frac{1}{G_{12}}\right)^{UD}=\left(\frac{1}{G_{12}}\right)^{WF}$$

$$\left(\frac{1+\mu_{23}}{E_2}+\frac{1}{2G_{12}}\right)^{UD}=\left(\frac{1}{G_{13}}\right)^{WF}$$

Where

E_{11}-Young's modulus in direction 1

E_{22}-Young's modulus in direction 2

E_{33}-Young's modulus in direction 3

u_{12}, u_{21}-Major and minor Poisson' ratios

G_{12} and G_{12} -In plane shear moduli

G_{23}-Out of plane shear modulus and

UD and WF denote unidirectional fibre and woven fibre, respectively.

The stress concentration factor at the edge of the hole of an infinite plate is given by[1],

$$K_T^{\infty}=1+\sqrt{\frac{2}{A_{22}}\left[\sqrt{A_{11}A_{22}}-A_{12}+\frac{A_{11}A_{22}-A_{12}^2}{2A_{66}}\right]}$$

Where K_T^{∞} denotes the stress concentration factor at the edge of the hole: A_{ij}, i.j=1, 2, 6, are the components of the in-plane stiffness matrix with 1 and 2 parallel and transverse to the loading direction, respectively.

Approximate orthotropic finite width correction factor for circular hole is given by [1],

$$\frac{K_T^{\infty}}{K_T}=\frac{3\left(1-\frac{2a}{W}\right)}{2+\left(1-\frac{2a}{W}\right)^3}+\frac{1}{2}\left(\frac{2a}{W}M\right)^6(K_T^{\infty}-3)\left[1-\left(\frac{2a}{W}M\right)^2\right]$$

Where,

K_T = stress concentration at the hole in finite plate.

a = radius of the hole.

W = width of the finite plate.

M = magnification factor, it is given by

$$M^2=\frac{\sqrt{1-8\left[\frac{3\left(1-\frac{2a}{W}\right)}{2+\left(1-\frac{2a}{W}\right)^3}-1\right]}-1}{2\left(\frac{2a}{W}\right)^2}$$

III. FINITE ELEMENT MODELLING

Finite element analysis is carried out using ansys workbench 12.1. A 3D model is developed using modeling software. This model is imported to ansys and boundary conditions are applied, i.e. 100MPa remote tensile stress is applied to the plate as shown in fig.2.

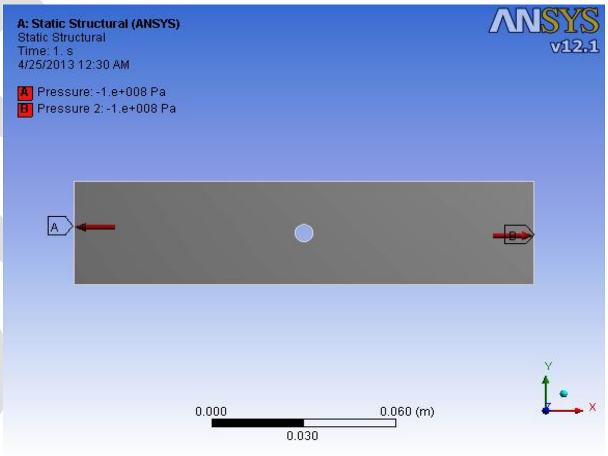


Fig.2. Single hole plate model.

IV. RESULTS AND DISCUSSION.

The main objective of this study is to find out the effect of fibre orientation and continuous type reinforcement on the stress concentration factor in a finite width composite lamina with central notch. Table 2 gives the theoretically calculated and finite element results of stress concentration factor for unidirectional reinforced lamina and woven fabric reinforced lamina.

TABLE 2

Theoretical and FEA results for stress concentration factor.

Fibre orientation	Stress concentration factor			
	Unidirectional		Woven fabric	
	Theoretical	FEA	Theoretical	FEA
0°	5.284	4.80	4.291	4.04
15°	4.2684	4.12	3.550	3.5
30°	3.0244	3.37	2.724	2.74
45°	2.3439	2.33	2.3378	2.33
60°	2.294	2.66	2.724	2.77
75°	2.622	2.90	3.550	3.54
90°	2.997	2.98	4.291	4.04

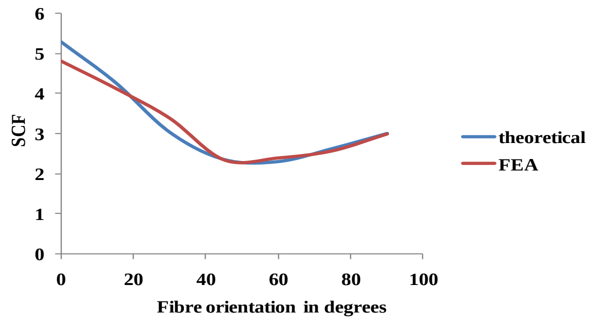


Fig.3.Comparison for theoretical results and FEA results for stress concentration factor in unidirectional lamina.

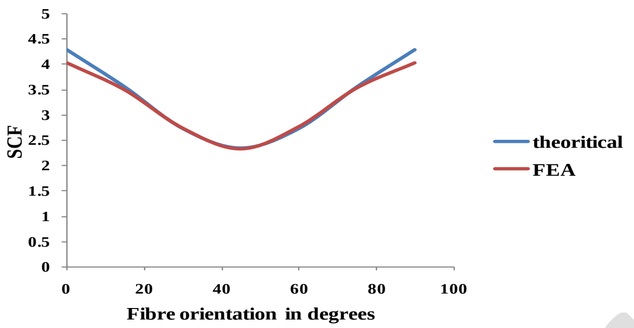


Fig.4. Comparison for theoretical results and FEA results for stress concentration factor in woven fabric lamina.

From Figs 3&4 it can be concluded that the theoretical results and FEA results both satisfy each other. Hence from theoretical results, i.e., from Fig.5 we can see that the stress concentration factor for a unidirectional lamina decreases with increase in fibre angle, up to around 60° fibre angle and remains constant up to 90°.Whereas in woven fabric lamina the stress concentration value is least at 45° fibre angle and have maximum value for 0° and 90° fibre angle as shown in Fig.5.

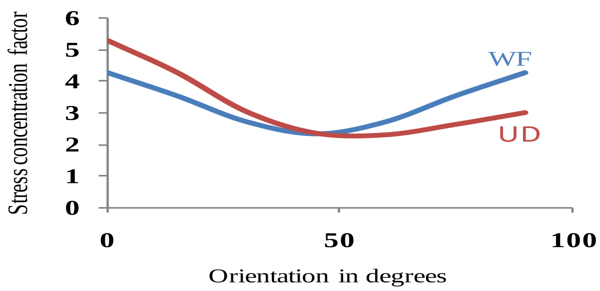


Fig.5. Comparison of theoretical results for stress concentration factor in
Unidirectional lamina (UD) and woven fabric lamina (WF).

Stress contour around the hole for different fibre angle is obtained from finite element analysis is shown in Figs 6&7. From these figures it can be seen that the maximum stress region is found on the edge of the hole for all fibre angles, in both unidirectional and woven fabric lamina.

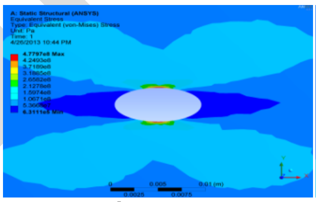
From Fig.6, in unidirectional lamina the maximum stress region found at the edge of the hole changes its position as the as the fibre angle is changed in the lamina. This maximum stress region at the hole will be approximately perpendicular to the fibre angle till 60° fibre angle.

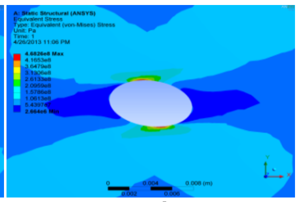
From Fig.7, in woven fabric lamina, the maximum stress region at the hole will be approximately perpendicular to the specified fibre angle up to 45° fibre angle, but after 45° fibre angle the maximum stress region at the hole edge is approximately parallel to the specified fibre angle.

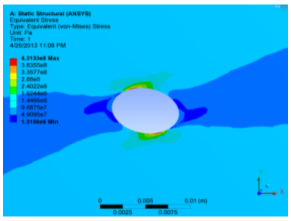
The reason for this difference in unidirectional and woven fabric lamina as mentioned in above two paragraphs is;

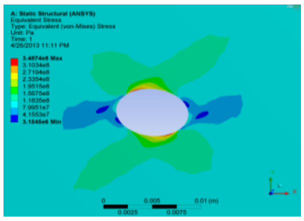
In unidirectional lamina the fibre are in single direction only, so the maximum stress region will always be perpendicular to the fibre angle until the fibre length is reasonably longer in the lamina to carry the applied load.

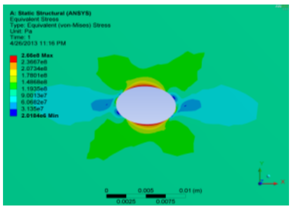
In woven fabric lamina the fibres are in plain weave form, i.e. the fibre will be present in 2 direction which are perpendicular to each other, therefore up to 44° fibre angle the longitudinal fibres will have greater length than the lateral fibres, but for fibre angle above 45° fibre angle, the lateral fibres earlier will have greater length than longitudinal fibres. Hence the maximum stress region will be found parallel to the specified fibre angle up to 45° fibre angle. In 45° fibre angle we can see maximum stress region on the hole is found at 4 spots, the reason is, here both longitudinal fibres and lateral fibres have the same length.

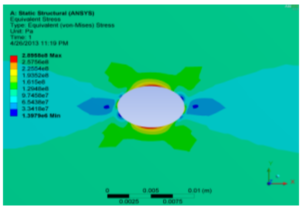

0° lamina


15° lamina


30° lamina


45° lamina


60° lamina


75° lamina

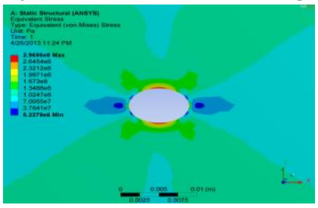
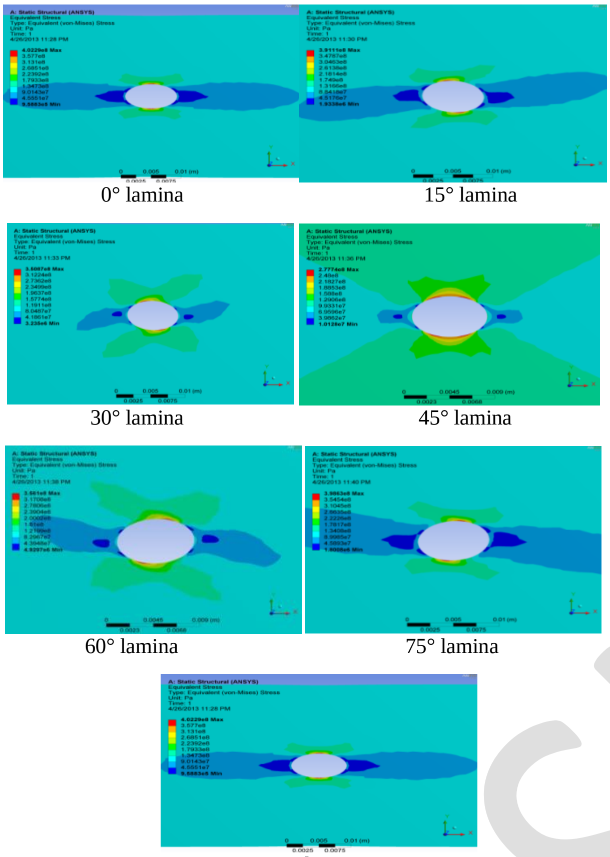

90° lamina

Fig.6. Stress contour around the hole in UD lamina with different fibre angle.



0° lamina 15° lamina

30° lamina 45° lamina

60° lamina 75° lamina

90° lamina

Fig.7. Stress contour around the hole in WF lamina with different fibre angle.

V. Conclusion

This study was conducted to investigate the effects of fibre orientation and type of continuous fibre reinforcement on the stress concentration factor of the composite lamina. Theoretical calculations were compared with finite element results. From the results we can conclude that the stress concentration value is not same for a unidirectional lamina and woven fabric lamina. For all lamina with a hole the maximum stress region is found at the periphery of the hole, this region is approximately perpendicular to the fibre angle specified in unidirectional lamina up to 60° fibre angle and from 60°to 90° fibre angle this maximum stress region is parallel to fibre direction. Whereas in woven fabric lamina the maximum stress region is perpendicular to the fibre angle up to 45° and parallel to the fibre direction for 45° to 90° fibre angle. Therefore from this analysis we can say that stress concentration factor in an orthotropic material depends on geometry, fibre orientation and type of reinforcement, whereas in isotropic materials it depends only on the geometry.

Comparing with unidirectional reinforced lamina and woven fabric reinforced lamina, we can say that the stress concentration factor is less in woven fabric reinforced lamina for 0° to 45° fibre angle, but for fibre angle 45° to 90° the unidirectional reinforced lamina will have least stress concentration factor compared to woven fabric reinforced lamina.

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