Service Surrender Queuing Model Applicable To Travel Tickets Reservation System of Indian Railways

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***Abstract*- The existing queuing theory provides various models which are inadequate to handle "Secondary queues" generated out of the situation of the availability of the 'Service Surrender' facility. The present paper deals with such secondary queues as applied particularly in Indian Railways ticket reservation system.**

Keywords: Secondary queues, Service Surrender Facility.

I. INTRODUCTION

Indian Railways (IR) is the principal mode of transport in the country. It is one of the world's largest rail networks under a single management. The route length is around 63,332 km with more than 8000 stations. As it is the backbone of nation's transport system, IR owns more than 25,000 wagons, 45,000 different types of coaches and 8000 locomotives. The system carries about 5,000 million passengers generating a traffic output of 340 billion passenger kms.

A] Need for Computerized Passenger Reservation System (PRS)

The seat/ berth reservation on trains is pretty complex activity, not only because of the volumes involving around lac seats/ berth reservations per day, but also because of several different categories of trains operating, using some 72 types of coaches with seven classes of reservation, more than 40 types of quotas and more than 80 kinds of concessional tickets. The method of calculation of fare is also quite complex as charges are based on distance, comfort level and transit time. Also there were many infirmities with the manual system like the current status did not get updated, it was slow and time consuming, inadvertent errors and malpractices in reservation were there. Because of the complexity and sheer volumes involved, there was a need for development of computerized reservation system.

RAILWAY RESERVATION SYSTEM should be able to manage all the reservation related functions. The system should be distributed in nature. This system is divided into five zones.

NORTH ZONE,
SOUTH ZONE,
EAST ZONE,
WEST ZONE,
CENTRAL ZONE

Each zone should have same functionalities. Each zone will stores the information about train name, train schedules, availability. The administrator should be able to enter any change related to the train information like change in train name, number etc. The system should be able to reserve seat in a train for a passenger. First the clerk will check for availability for the seats in a particular train on a specified date of journey. If it is available the clerk will reserve seats. The passenger will be given a unique PNR no. The system should be able to cancel a reservation. The clerk will delete the entries in the system. The passenger can check their reservation status online by entering their PNR no. The system will display his current status like confirmed, Reservation against cancellation (RAC) or waiting list. They are also able to see information related to the train schedules. The system should be able to print the report like it should be able to generate reservation chart, train report, reservation ticket which will have train no and name, date of journey, boarding station, destination station, person name, age, [censored], total fare and a unique PNR no. The system should be able to print the cancellation ticket which will have total fare and the amount deducted.

B] PURPOSE

The purpose of this source is to describe the railway reservation system which provides the train timing details, reservation, billing and cancellation on various types of reservation namely, Confirm Reservation for confirm Seat.

Reservation against cancellation.
Waiting list Reservation.
Online Reservation.
Tatkal Reservation.

C] Type of quotas: -

Quotas in Indian Railways

S. No.	Quota	Description
1.	GN	General Quota
2.	LD	Ladies Quota
3.	HO	Headquarters/high official Quota
4.	DF	Defence Quota
5.	PH	Parliament house Quota
6.	FT	Foreign Tourist Quota
7.	DP	Duty Pass Quota
8.	CK	Tatkal Quota

9. PT Premium Tatkal Quota
10. SS Female (above 45 Year)/Senior Citizen/Travelling alone
11. HP Physically Handicapped Quota
12. RE Railway Employee Staff on Duty for the Train
13. GNRS General Quota Road Side
14. OS out Station
15. PQ Pooled Quota
16. RAC Reservation Against Cancellation
17. RS Road Side
18. YU Yuva
19. LB Lower Berth

NOTE:

RAC is a special provision to 'split' a berth into two or more seats. This is really speaking not a quota but it is a predefined allocation to take care of the above mentioned special provision.

In case of each road-side station quota, berths/seats are booked by the originating station for journeys up-to the road-side station only up-to the extent of accommodation earmarked for that Road-Side quota .In such cases, distance restriction does not apply. If that berths/seat is redefined from a remote location, then booking can also be done beyond the road side station, within the limits defined for the remote location quota. In no case can a through passenger be given accommodation in the RS berth and the redefined portion. For e.g., a passenger travelling from NDLS to MAS cannot be given accommodation in a berth defined as GNRS up-to BPL and subsequently redefined as GN from BPL to MAS.

D] Role ofQueuing Theory in Indian Railway Reservation System: -

The IR reservation tickets can be obtained by the passengers by approaching to the service counter forming a queue. There are two different modes available for the passengers for getting reservation tickets,

1) By directly approaching to the service counters located at different railway stations.
2) By utilizing internet facility for obtaining online reservation tickets.

In any of the above cases the passengers have to join only one single queue indirectly. There are three special features which can be easily noticed. These are;

1) Once the passenger (customer) receives the service (i.e. confirmed railway reservation tickets) it is possible for the customer to surrender the service (i.e. tickets). Hence the service type is such that it can be surrendered any time.
2) Secondly we noticed that, the queuing system for a particular train starts only after a specific point of time that is say 120 days prior to the date of journey.
3) The date of journey is a time point say ‘T’ when the entire system collapses. The system starts functioning a units of time briar to this point T. (i.e. we can say that A = 120 for any train and correspond T.)

When the customer approach the service counter and come to know that the confirmed tickets are all sold out, he/she has to join by choice the ‘waiting list’ queue. As such the formation of queue of waiting list customers starts as and when the earlier customer who got the confirmed reservation ticket surrendered it, the ‘waiting list’ queue customer get the surrendered service by first come first served way of queue discipline.

This creates uncertainty in the minds of the customer in ‘waiting list’ about getting the confirmed ticket.

And as such the customers on ‘waiting list’ now called here onwards as customers in the ‘Secondary queue’ are a part of such a queue which has the customers without any guarantee of getting the service.

A study of such a queue becomes an interesting problem.

In the present project work we developed some queuing models for ‘Secondary queues’. These mathematical models can be used to study some characteristics of the secondary queues.

Hence queuing theory plays an important role in understanding and analysing the Indian Railway Reservation System.

II. SERVICE SURRENDER QUEUING MODELS

Up till now in the literature of queuing analysis the entire study is focused on the one and only one queue which is formed in front of a service counter. We know that depending on the input distribution and the output distribution the results differ in the corresponding models.

We have noticed that there are many situations where the customers have to wait not only in one single queue but they have to wait in the second queue also. The analysis of the first queue differs from the analysis of the second queue. Further, it is always not necessary that the customers have to go through two different queues. The existence of second queue is possible only when the system has a characteristic of ‘service surrender facility’. That is, returning back the utilized service by the customer. In this situation we note that the availability of the service with the server is limited.

Here we mainly concentrate on the study of the second queue which is not carried out so far in the queuing literature.

Here we analysed a queuing system in which a facility of surrending the service is available to the customers. We explain the situation as follows.

Let there be a service counter where the service is available only to a finite number (say N) of customers. The system works as per (M/M/1): (∞/FIFO) pattern. After having served N customer from this queue, service is not available to any of the next arriving customers. But such customers which arrive after the N^{th} one, can now register their names with the server, who now starts preparing a ‘waiting list’. These customers are served as and when any of the previously served customers surrenders his service. This waiting list one becomes an another queue. We analyse such types of queues in this chapter. We note that those customers who have received the service can utilize it for some period of time and have a facility to surrender the service as and when they wish.

Here, this time span during which he utilizes a service is a random variable.

We further note here that the customers on 'waiting list' do not know how long they will have to wait and even whether they will get the service at all.

Therefore when customer joins the system he can either get a service or has a chance of being registered on 'waiting list' if he wishes, otherwise he can quit the system.

This system can generate various models depending on some assumptions. We consider these models one by one.

A) SOME DEFINITIONS:

Primary queue:

It is the waiting line of the units in front of the service counter before they are registered on the 'waiting list'.

Secondary queue:

It is the waiting line of the units which are registered on the 'waiting list'.

Here we note that a customer can join the secondary queue only if he comes through the primary queue. The customers have an option whether to join the secondary queue or not after having come through the primary queue.

B) ELEMENTARY MODEL BUILDING:

In this model a few assumptions are made so as to make the analysis simple. As we go on changing some assumptions, the analysis becomes more and more complex, which we shall study in forth coming sections.

ASSUMPTIONS:

The system starts with non-empty queue as per (M/M/1): (∞/FIFO) pattern.

Service Surrender facility is available to the customer.

Service can be given only to N customers. (N is finite).

Service Surrender facility begins only after N services are delivered.

Once the customer surrenders the service he is not allowed to join the system again.

A customer arriving at a service counter after N services are over, has one of the following options:

To join a waiting list.

Not to join a waiting list and quit the system.

Once the service is received by the customer he enjoys the service continuously until he surrenders it.

The time taken by the server to dispose of the customer from the counter is same before and the start of the 'waiting list'.

ANALYSIS:

We consider a queuing system having single service channel, Poisson input, Exponential service and there is no limit on the system capacity while the customers are served on a "first in, first out" basis i.e. we consider (M/M/1): (∞/FIFO) model.

The existing analysis of such a model from the literature gives us some of the results as under:

The probability of n customers in the system at an arbitrary point of time denoted by Pn is given by

$$Pn = p^n (1-p) \quad \text{.... (1)}$$

Where $p = \left(\frac{\lambda}{\mu}\right) < 1$ and $n \geq 0$

λ = Average number of customers arriving per unit of time.

μ = Average number of customers being served per unit of time.

Average number of customers in the system denoted by E(n) is given by

$$E(n) = \frac{p}{(1-p)} \quad \text{..... (2)}$$

Average queue length denoted by E (m) is given by

$$E(m) = \frac{p^2}{(1-p)} \quad \text{....... (3)}$$

Average waiting time of a customer in the queue is denoted by

$$E(w) = \frac{p}{\mu\,(1-p)} \quad \text{........ (4)}$$

We mainly analyse our model from 'waiting time' point of view.

Since we consider only that queue which is formed by the customers who are on the waiting list, we try to find out the waiting time distribution of such customers.

Further, those customers which are registered on waiting list may have already waited for some time in the queue to reach the service counter just only to find that the service is not available and such on an average they have waited for $\frac{p}{\mu\,(1-p)}$ units of time (from equation 4 and assumption 8 of elementary model).

Now since they are registered on the waiting list, they may get the service as and when the previously served customers surrender their services and this second phase of waiting may be for unpredictable time span. Now we attempt to find out this time span. For this, first we define the following:

SERVICE HOLDING TIME (SHT) OF THE QUEUEING SYSTEM:

Service holding time of a queuing system is the average duration of time for which customer holds the service, before it is surrendered.

We can find out the SHT distribution as a probability distribution. We note that SHT is a continuous random variable.

For the sake of analytical convenience we convert SHT in to a discrete random variable as follows.

Let p be the probability that a customer enjoys the service for a unit time. The probability that he surrenders the service some time during a unit time is (1-p).

Therefore probability that a customer completes't' units of time before he surrenders the service i.e. P(T=t) is as follows:

$$P(T = t) = p^{t-1}(1 - p) \qquad t = 1,2,3,\ldots$$

But this is the probability mass function of a Geometric Distribution. Hence T is distributed as a Geometric variable.

Now since 'p' differs from person to person randomly, taking any value in the range [0, 1], it is appropriate to consider it as a random variable. The appropriate distribution of p is generally considered as Beta Distribution with parameters 'a' and 'b' which is given by

$f(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1} a, b > 0; 0 < p < 1$

Where B (a, b) is Beta function with parameters a and b given by

$$B(a\,,b) = \int_0^1 p^{a-1}\,(1-p)^{b-1}\,dp$$
$$= \frac{\Gamma a \Gamma b}{\Gamma(a+b)}$$
$$= \frac{(a-1)!\,(b-1)!}{(a+b-1)!}$$

Here T is distributed as Geometric and p is distributed as Beta.

Thus according to D. J. Bartholomew(1963), SHT(T) follows a Compound Beta distribution as follows.

$$P[T=t] = \int_0^1 f(p) p^{t-1}(1-p)dp$$

$$= \frac{1}{B(a,b)} \int_0^1 p^{a-1}(1-p)^{b-1} p^{t-1}(1-p)dp$$
$$= \frac{1}{B(a,b)} \int_0^1 p^{a+t-2}(1-p)^b dp$$
$$= \frac{1}{B(a,b)} B(a+t-1, b+1)$$
$$= \frac{B(a+t-1, b+1)}{B(a,b)} \qquad a, b > 0; t = 1, 2, \dots, \infty$$

C)

(i) Analysis of secondary queueing systems with infinite range of SHT

We note that, here't' is having an infinite range. Now we consider a queueing system having SHT ranging from 1 to ∞. The event of surrendering the service by a customer and allotting the service to a customer on waiting list is allowed to occur only at discrete time points.

Then the probability mass function of T having probability structure as defined above is obtained as follows

P(T = t) = ft

$$= \frac{B(a+t-1\,, b+1)}{B(a,b)} \qquad t \;=\; 1,2,3\,, \dots, \infty \qquad a\,, b > 0$$
$$= \frac{(a+t-2)!\,(b)!\,(a+b-1)!}{(a+b+t-1)!\,(a-1)!\,(b-1)!}$$

$$= \frac{\{(\mathbf{b})[(\mathbf{a}+\mathbf{t}-\mathbf{2})(\mathbf{a}+\mathbf{t}-\mathbf{3})(\mathbf{a}+\mathbf{t}-\mathbf{4}) \dots \dots .\mathbf{1}].[(\mathbf{a}+\mathbf{b}-\mathbf{1})(\mathbf{a}+\mathbf{b}-\mathbf{2}) \dots .\mathbf{1}]\}}{\{[(\mathbf{a}+\mathbf{b}+\mathbf{t}-\mathbf{1})(\mathbf{a}+\mathbf{b}+\mathbf{t}-\mathbf{2}) \dots \dots (\mathbf{a}+\mathbf{b}+\mathbf{t}-\mathbf{t})(\mathbf{a}+\mathbf{b}+\mathbf{t}-(\mathbf{t}-\mathbf{1})) \dots \dots].[(\mathbf{a}-\mathbf{1})(\mathbf{a}-\mathbf{2}) \dots .]\}}$$

$$= \frac{b[(a+t-2)(a+t-3) \dots .1]}{[(a+b+t-1)(a+b+t-2) \dots .(a+b)].[(a-1)(a-2) \dots .1]}$$

$$= \frac{b[(a+t-2)(a+t-3) \dots (a+t-t)(a+t-(t+1)) \dots .]}{[(a+b+t-1)(a+b+t-2) \dots (a+b)].[(a-1)(a-2) \dots .1]}$$

$$= \frac{b[(a+t-2)(a+t-3) \dots (a)(a-1)(a-2) \dots 1]}{[(a+b+t-1)(a+b+t-2) \dots (a+b)].[(a-1)(a-2) \dots 1]}$$
$$= \frac{b[(a+t-2)(a+t-3) \dots .a]}{[(a+b+t-1) \dots .(a+b)]}$$

Using the relation$x^{(t)} = x(x-1)(x-2) \dots (x-t+1)$ in above we get

$$P(T=t) = \frac{b(a+t-2)^{(t-1)}}{(a+b+t-1)^{(t)}}$$

(*ii) Average waiting time of a customer in the secondary queue:*

When SHT follows compound Geometric distribution, the p.m.f. is

$$F_t = \frac{B(a+t-1, b+1)}{B(a,b)} \qquad a, b > 0; t = 1, 2, \dots, \infty$$

The average duration of utilizing the service by a customer is given by

$$E(t) = \sum_{t=1}^{\infty} t.f_t$$
$$= \frac{1}{B(a,b)} \sum_{t=1}^{\infty} t.B(a+t-1, b+1)$$

$$= \frac{1}{B(a,b)} [1.B(a,b+1) + 2.B(a+1,b+1) + \cdots]$$

$$= \frac{1}{B(a,b)} \left[\int_0^1 p^{a-1}(1-p)^b dp + 2 \int_0^1 p^a (1-p)^b dp + \cdots \right]$$

$$= \frac{1}{B(a,b)} \left[\int_0^1 (1-p)^b (p^{a-1} 2p^a + 3p^{a+1} + \cdots) dp \right]$$
$$= \frac{1}{B(a,b)} \left[\int_0^1 p^{a-1}(1+2p+3p^2+\cdots)(1-p)^b dp \right]$$

$$= \frac{1}{B(a,b)} \left[\int_0^1 p^{a-1}(1-p)^{-2}(1-p)^b dp \right]$$

$$= \frac{1}{B(a,b)} \left[\int_0^1 p^{a-1}(1-p)^{b-2} dp \right] \qquad \because B(a,b)$$
$$= B(b,a)$$

$$\because B(a+1,b)=\frac{a}{a+b}B(a,b)$$

$$\therefore B(b,a)=\frac{b-1}{a+b-1}B(b-1,a)$$

$$E(t)=\frac{a+b-1}{b-1}.\frac{1}{B(b-1,a)}\left[\int_0^1 p^{a-1}(1-p)^{b-2}dp\right]$$

$$=\frac{a+b-1}{b-1}.\frac{1}{B(b-1,a)}[B(a,b-1)]$$

$$E(t)=\frac{(a+b-1)}{(b-1)} \qquad \text{where } b\neq 1, b>a$$

Remark: from above we observe that for E(t) to be valid, we must have to impose some restriction on the choice of b such that b > 1.

(iii) Other Characteristics: -

The Distribution Function of T:

The distribution function of T is given by

$$F_T=\sum_{t=1}^{T} f_t$$

Using we get

$$F_T=\sum_{t=1}^{T}\frac{B(a+t-1,b+1)}{B(a,b)}$$

$$=\frac{1}{B(a,b)}\sum_{t=1}^{T}B(a+t-1,b+1)$$

$$=\frac{1}{B(a,b)}[B(a,b+1)+B(a+1,b+1)+B(a+2,b+1)+\cdots+B(a+T-1,b+1)]$$

$$=\frac{1}{B(a,b)}\left[\int_0^1 p^{a-1}(1-p)^b dp+\int_0^1 p^{a}(1-p)^b dp+\cdots+\int_0^1 p^{a+T-2}(1-p)^b dp\right]$$

$$=\frac{1}{B(a,b)}\left[\int_0^1 p^{a-1}\{1+p+p^2+\cdots+p^{T-1}\}(1-p)^b dp\right]$$

If p ≠ 1,

$$F_T=\frac{1}{B(a,b)}\left[\int_0^1 p^{a-1}\frac{(1-p^T)}{(1-p)}(1-p)^b dp\right]$$

$$=\frac{1}{B(a,b)}\left[\int_0^1 p^{a-1}(1-p)^{b-1}(1-p^T)dp\right]$$

$$=\frac{1}{B(a,b)}\left[\int_0^1 p^{a-1}(1-p)^{b-1}dp.\int_0^1 p^{a+T-1}(1-p)^{b-1}dp\right]$$

$$=\frac{1}{B(a,b)}[B(a,b)-B(a+T,b)]$$

Consider from the above equation B (a + T, b)

Now B (a + T, b) = B (b, a + T)

$$=\frac{a+T+b}{B(a,b)}B(b+1,a+T)$$

$$F_T=\frac{1}{B(a,b)}\left[B(a,b)-\frac{(a+T+b)}{b}B(b+1,a+T)\right]$$

$$=\frac{1}{bB(a,b)}[bB(a,b)-(a+T+b)B(b+1,a+T)]$$

$$=1-\frac{(a+T+b)}{b}.\frac{B(b+1,a+T)}{B(a,b)}$$

$$=1-\frac{(a+T+b)}{b}.\frac{B(a+T,b+1)}{B(a,b)}$$

$$=1-\frac{(a+T+b)}{b}.f_{(T+1)}$$

$$F_T=1-\frac{(a+b+T)}{b}f_{(T+1)}$$

Survival function

Let T > 0 have a density fT and distribution function FT. The survival function GT is given by

$$G_T=\begin{cases}1-F_T & \text{t is continuous}\\ 1-F_T & \text{t is discrete}\end{cases}$$

In our model obviously GT becomes

$$G_T=\frac{(a+b+T-1)}{b}f_T$$

Hazard rate or Hazard function:

The Hazard rate or Hazard function S_T is given by

$$S_T=\frac{f_T}{G_T}$$

The Hazard rate has the interpretation

$S_T dt = P\{t < T < t + dt / T > t\}$

= P{surrendering in interval (t, t + dt) / not surrendered till time T}

In our case instead of calling Hazard rate we call it by the terminology 'Service wastage' of the model.

Thus service wastage ST of the model is given by

$$S_T=\frac{f_T}{G_T}=\frac{b}{(a+b+t-1)}$$

Thus service wastage indicates the rate of surrender of the service. If service wastage ST is more then there is more rate of service surrender. Hence ST is one of the most important characteristics of the model.

III. TABLES

t	f_t			
	a = 15			
	b = 10	b = 10	b = 10	b = 10
2	0.221893	0.231481	0.212665	0.191327
4	0.079931	0.047124	0.027765	0.017366
6	0.033966	0.01209	0.004754	0.002123
8	0.016381	0.00372	0.001011	0.000330
10	0.008725	0.001325	0.000256	0.000062

t	f_t			
	a = 15			
	b = 10	b = 10	b = 10	b = 10
2	0.221893	0.231481	0.212665	0.191327
4	0.079931	0.047124	0.027765	0.017366
6	0.033966	0.01209	0.004754	0.002123
8	0.016381	0.00372	0.001011	0.000330
10	0.008725	0.001325	0.000256	0.000062

E (t)						
B \ A	5	10	15	20	25	30
5	2.250	1.555	1.357	1.263	1.208	1.172
10	3.500	2.111	1.714	1.526	1.416	1.344
15	4.750	2.666	2.071	1.789	1.625	1.517
20	6.000	3.222	2.428	2.052	1.833	1.689
25	7.250	3.777	2.785	2.315	2.041	1.862
30	8.500	4.333	3.142	2.578	2.250	2.034

T	S_t				
	b=10				
	a=20	a=40	a=60	a=80	a=100
5	0.294118	0.185185	0.135135	0.106383	0.087719
10	0.25641	0.169492	0.126582	0.10101	0.084034
15	0.227273	0.15625	0.119048	0.119048	0.080645
20	0.204082	0.144928	0.11236	0.091743	0.077519
25	0.185185	0.135135	0.106383	0.087719	0.074627
30	0.169492	0.126582	0.10101	0.084034	0.071942
35	0.15625	0.119048	0.096154	0.080645	0.069444
40	0.144928	0.11236	0.091743	0.077519	0.067114
45	0.135135	0.106383	0.087719	0.074627	0.064935
50	0.126582	0.10101	0.084034	0.071942	0.062893
55	0.119048	0.096154	0.080645	0.069444	0.060976
60	0.11236	0.091743	0.077519	0.067114	0.059172
65	0.106383	0.087719	0.074627	0.064935	0.057471
70	0.10101	0.084034	0.071942	0.062893	0.055866
75	0.096154	0.080645	0.069444	0.060976	0.054348
80	0.091743	0.077519	0.067114	0.059172	0.05291
85	0.087719	0.074627	0.064935	0.057471	0.051546
90	0.084034	0.071942	0.062893	0.055866	0.050251
95	0.080645	0.069444	0.060976	0.054348	0.04902
100	0.077519	0.067114	0.059172	0.05291	0.047847

T	S_t				
	a=10				
	b=20	b=40	b=60	b=80	b=100
5	0.588235	0.740741	0.810811	0.851064	0.877193
10	0.512821	0.677966	0.759494	0.808081	0.840336
15	0.454545	0.625	0.714286	0.769231	0.806452
20	0.408163	0.57971	0.674157	0.733945	0.775194
25	0.37037	0.540541	0.638298	0.701754	0.746269
30	0.338983	0.506329	0.606061	0.672269	0.719424
35	0.3125	0.47619	0.576923	0.645161	0.694444
40	0.289855	0.449438	0.550459	0.620155	0.671141
45	0.27027	0.425532	0.526316	0.597015	0.649351
50	0.253165	0.40404	0.504202	0.57554	0.628931
55	0.238095	0.384615	0.483871	0.555556	0.609756
60	0.224719	0.366972	0.465116	0.536913	0.591716
65	0.212766	0.350877	0.447761	0.519481	0.574713
70	0.20202	0.336134	0.431655	0.503145	0.558659
75	0.192308	0.322581	0.416667	0.487805	0.543478
80	0.183486	0.310078	0.402685	0.473373	0.529101
85	0.175439	0.298507	0.38961	0.45977	0.515464
90	0.168067	0.28777	0.377358	0.446927	0.502513
95	0.16129	0.277778	0.365854	0.434783	0.490196
100	0.155039	0.268456	0.35503	0.42328	0.478469

IV. CONCLUSIONS

Analysis of queuing system with service surrender facility is dominated mainly by secondary queue. The main focus is on finding the expected waiting time of a customer in the entire system. In such systems a customer faces one of the following situations while joining the system. These are:

1) At the time of joining the system there is a primary queue and service is available with the server. There can be two possibilities in this case.
   (i) The customer after reaching the service counter gets the service and
   (ii) Service is not available with the server and hence the customer has to decide whether to join a secondary queue or not.

2) At the time of joining the system there is a primary queue but the service is not available with the server and as such the formation of secondary queue is in progress.

3) At the time of joining the system there is no primary queue and as such a customer directly approaches the server for registering himself in the secondary queue.

In all these cases the average waiting time of a customer varies as the service rate is different from case to case. The average time taken by a customer to reach the server i.e. E is different in all these cases but the average waiting time in the secondary queue remains to be equal to (a+b-1)/(b-1). So to find the expected waiting time of a customer in the system it is necessary to take into account the status of the system at the time of joining it by a customer. The status of the system at that time will decide the value of E.

The service holding time throws some light on the load of the work a server has to carry while the secondary queue is in existence.

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