Generalized Tetranacci Sequence and Its Period

Mansi N. Zaveri[1], Dr. Jayant K.Patel[2]

Asst. Prof., A.S.H. Dept., S.N.P.I.T & RC, GTU, Umrakh, Bardoli, Gujarat, India [1]
Asso. Prof., Mathematics Dept., Navyug Science College, V.N.S.G.U., Surat, Gujarat, India [2]

***Abstract:* Every Tetranacci Sequence starting with an arbitrary quadruple of integers is periodic modulo m for any modulus m > 1. In this paper we derive relations between $h(p^e)[a,b,c,d]$ and $h(p)[a,b,c,d]$ which also depend on the form of the factorization of polynomial $g(x)$ over the field $\mathbb{F}_p$.**

***Keywords* — Modular periodicity, Periodic sequence, Tetranacci sequence,**

I. Introduction

Let $(t_n)_{n=0}^{\infty}$ be a Tetranacci sequence 0, 0, 0, 1, 1, 2, 4, 8. 15, 29, 56, 108, 208, 401, 773,……. defined by the recurrence relation $t_{n+4} = t_{n+3} + t_{n+2} + t_{n+1} + t_n$ and with initial values $[t_0, t_1, t_2, t_3] = [0, 0, 0, 1]$ and let $(T_n)_{n=0}^{\infty}$ be a Tetranacci sequence defined by the recurrence relation $T_{n+4} = T_{n+3} + T_{n+2} + T_{n+1} + T_n$ with an arbitrary quadruple of integers $[a,b,c,d]$. Mansi & Dr. Patel [5] has proved that both the above sequences are periodic for any arbitrary modulus $m > 1$. . We denote $h(m)$ and $h(m)[a,b,c,d]$ the primitive periods of these sequences respectively. Let L be the splitting field of the Tetranacci polynomial $g(x) = x^4 - x^3 - x^2 - x - 1$ over the field $\mathbb{F}_p$. Here, we derive relationship between $h(p)[a,b,c,d]$ and $h(p^e)[a,b,c,d]$, where p is an arbitrary prime, and $e \in N$.

II. Tetranacci Sequence – Conceptual Framework

Let Tetranacci matrix T be defined by

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and}$$

$$T^n = \begin{bmatrix} t_{n-1} & s_{n-1} & m_{n-1} & t_n \\ t_n & s_n & m_n & t_{n+1} \\ t_{n+1} & s_{n+1} & m_{n+1} & t_{n+2} \\ t_{n+2} & s_{n+2} & m_{n+2} & t_{n+3} \end{bmatrix} \quad \text{for } n > 1$$

The Tetranacci sequence $\{t_n\}$ is defined by:

$$t_{n+4} = t_{n+3} + t_{n+2} + t_{n+1} + t_n$$
$$t_0 = t_1 = t_2 = 0,\ t_3 = 1$$

Two companion sequences $\{s_n\}$ *and* $\{m_n\}$ are defined as:

$$s_{n+4} = s_{n+3} + s_{n+2} + s_{n+1} + s_n$$
$$s_0 = s_3 = 1,\ s_1 = s_2 = 0$$
$$m_{n+4} = m_{n+3} + m_{n+2} + m_{n+1} + m_n$$
$$m_0 = m_2 = 0,\ m_1 = m_3 = 1$$

For any arbitrary $n \in N$ and an arbitrary modulus m, T^n assumes a unique form $T^n = B + mA$, where $A = [a_{ij}]$ and $B = [b_{ij}]$ are integer matrices such that $0 \le b_{ij} \le m - 1$ and $[a_{ij}]$ are nonnegative integers. If $n = h(m)$, then $T^{h(m)} \equiv E \ (mod\ m)$, where E is an identity matrix. Therefore, we can express $T^{h(m)}$ as $T^{h(m)} = E + mA$. ([3], [5])

III. Relation Between Tetranacci Sequence $(T_N)_{N=0}^{\infty}$ And Generalized Tetranacci Sequence $(T_N)_{N=0}^{\infty}$

A formula for the n^th^ term of generalized Tetranacci sequence $(T_n)_{n=0}^{\infty}$ was derived from the given terms designated as T_0, T_1, T_2, T_3 only.

To find the general formula for the n^th^ term T_n of the generalized Tetranacci sequence, a following pattern is recognized. All the equations derived are listed to find recognizable patterns easily.

$$T_4 = T_0 + T_1 + T_2 + T_3$$
$$T_5 = T_0 + 2T_1 + 2T_2 + 2T_3$$
$$T_6 = 2T_0 + 3T_1 + 4T_2 + 4T_3$$
$$T_7 = 4T_0 + 6T_1 + 7T_2 + 8T_3$$
$$T_8 = 8T_0 + 12T_1 + 14T_2 + 15T_3$$
$$T_9 = 15T_0 + 23T_1 + 27T_2 + 29T_3$$

After careful observation and investigation, all the numerical coefficients of T_0, T_1, T_2 and T_3 are as shown in the Table.

A. Table 3.1

Number of terms	n^th^ term of Tetranacci Sequence	Coefficients			
		T_0	T_1	T_2	T_3
4	T_4	1	1	1	1
5	T_5	1	2	2	2
6	T_6	2	3	4	4
7	T_7	4	6	7	8
8	T_8	8	12	14	15
9	T_9	15	23	27	29
….	……	……	……	…….	…….
n	T_n	t_{n-1}	$t_{n-2} + t_{n-1}$	$t_{n-3} + t_{n-2} + t_{n-1}$	t_n

It can be noted that coefficients of T_0, T_1, T_2 and T_3 follows the given Tetranacci sequence [1].

B. Table 3.2

nth term	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9…
Tetranacci Number	0	0	0	1	1	2	4	8	15	29……

From these observations we have the following Theorem.

Theorem 3.3: For any real numbers T_0, T_1, T_2 and T_3 the formula for finding nth term T_n of generalized Tetranacci sequence is:

$$T_n = t_{n-1}(T_0 + T_1 + T_2) + t_{n-2}(T_1 + T_2) + t_{n-3}T_2 + t_nT_3$$

Proof: We will prove the Theorem by Induction on $n \geq 3$
From Table (3.2), we get, $T_3 = T_3$ (Which is easy to verify).
For $n = 4$, $T_4 = T_0 + T_1 + T_2 + T_3$ which is true from the definition of Tetranacci sequence.
Next, we will show that the formula is valid for $n > 4$.
Assuming that

(3.3.1) $P(k) = T_k = t_{k-1}(T_0 + T_1 + T_2) + t_{k-2}(T_1 + T_2) + t_{k-3}T_2 + t_kT_3$ is true.

We shall find the $(k+1)^{th}$ term of the sequence, i.e. we shall prove that

(3.3.2) $P(k+1) = T_{k+1} = t_k(T_0 + T_1 + T_2) + t_{k-1}(T_1 + T_2) + t_{k-2}T_2 + t_{k+1}T_3$

We will add t_{k-1}, t_{k-2} and t_{k-3} to both sides of $P(k)$. The left side of $P(k)$ will become,
$t_{k-3} + t_{k-2} + t_{k-1} + t_k$, Which is equal to t_{k+1}, that is the left side of $P(k+1)$.
After adding t_{k-1}, t_{k-2} and t_{k-3} the right side of $P(k)$ will become,

(3.3.3) $t_{k-1}(T_0 + T_1 + T_2) + t_{k-2}(T_1 + T_2) + t_{k-3}T_2 + t_kT_3 + t_{k-1} + t_{k-2} + t_{k-3}$

But from (3.3.1),

$$T_{k-1} = t_{k-2}(T_0 + T_1 + T_2) + t_{k-3}(T_1 + T_2) + t_{k-4}T_2 + t_{k-1}T_3$$

$$T_{k-2} = t_{k-3}(T_0 + T_1 + T_2) + t_{k-4}(T_1 + T_2) + t_{k-5}T_2 + t_{k-2}T_3$$

$$T_{k-3} = t_{k-4}(T_0 + T_1 + T_2) + t_{k-5}(T_1 + T_2) + t_{k-6}T_2 + t_{k-3}T_3$$

The equation (3.3.3) becomes,

$$(t_{k-1} + t_{k-2} + t_{k-3} + t_{k-4})(T_0 + T_1 + T_2) + (t_{k-2} + t_{k-3} + t_{k-4} + t_{k-5})(T_1 + T_2) + (t_{k-3} + t_{k-4} + t_{k-5} + t_{k-6})(T_2) + (t_k + t_{k-1} + t_{k-2} + t_{k-3})(T_3)$$

$$= (t_k)(T_0 + T_1 + T_2) + (t_{k-1})(T_1 + T_2) + (t_{k-2})(T_2) + (t_{k+1})(T_3)$$

Which is equal to the right side of $P(k+1)$.
Therefore through Mathematical induction conclusion follows.

Remark 3.4: Replace n by $n+4$, in the result of Theorem (3.3) we get,

$$T_{n+4} = t_{n+3}(T_0 + T_1 + T_2) + t_{n+2}(T_1 + T_2) + t_{n+1}T_2 + t_{n+4}T_3$$

Let, $T_0 = a, T_1 = b, T_2 = c, T_3 = d$, in above result, we get,

$$T_{n+4} = (a+b+c)t_{n+3} + (b+c)t_{n+2} + c.t_{n+1} + d.t_{n+4}$$

IV. IRREDUCIBLE CASE

In this case we define quartic form to investigate the primitive periods of Tetranacci sequences beginning with the arbitrary quadruples $[a, b, c, d]$. Here we define integer representation by a quartic form associated with a forth order recurrence known as the Tetranacci recurrence. The technique of derived sequences is used to define an invariant quartic forms, computed as third derived sequences ([4], [9]) of a forth order linear recurrence sequences defined by $T_{n+4} = T_{n+3} + T_{n+2} + T_{n+1} + T_n$, $n \geq 0$.

$$T_n^{(3)} = \begin{vmatrix} T_n & T_{n+1} & T_{n+2} & T_{n+3} \\ T_{n+1} & T_{n+2} & T_{n+3} & T_{n+4} \\ T_{n+2} & T_{n+3} & T_{n+4} & T_{n+5} \\ T_{n+3} & T_{n+4} & T_{n+5} & T_{n+6} \end{vmatrix}$$

Let, $T_n = a$, $T_{n+1} = b$, $T_{n+2} = c$ and $T_{n+3} = d$,

we get,

$T_n^{(3)} =$

$$\begin{vmatrix} a & b & c & d \\ b & c & d & a+b+c+d \\ c & d & a+b+c+d & a+2b+2c+2d \\ d & a+b+c+d & a+2b+2c+2d & 2a+3b+4c+4d \end{vmatrix}$$

$= -a^4 - 2ab^3 - 4a^2b^2 - 3a^3b - ac^3 - 8abc^2 - 2a^3c - 12ab^2c - 7a^2bc - ad^3 + a^2d^2 + 4abd^2 - 2acd^2 - a^3d + 3ab^2d + 5ac^2d + 2a^2cd + 3abcd + b^4 - 5bc^3 - 3b^3c - 2bd^3 + bcd^2 + 2a^2bd + 3b^3d + 8bc^2d + 2b^2cd - 3c^4 - 7b^2c^2 - 3cd^3 + 2c^2d^2 + 2c^3d + d^4$

$= D(a, b, c, d)$ (say)

Theorem 4.1.: If a quadruple of initial values $[a, b, c, d]$ of a Tetranacci sequence $\{T_n\}_{n=0}^{\infty}$ satisfies $(D(a, b, c, d), m) = 1$ then $h(m)[a, b, c, d] = h(m)$.

Proof. For $n \geq 0$, the sequences $\{t_n\}_{n=0}^{\infty}$ and $\{T_n\}_{n=0}^{\infty}$ satisfy the relation by Theorem 3.3,

(4.1.1) $T_n = t_{n-1}(T_0 + T_1 + T_2) + t_{n-2}(T_1 + T_2) + t_{n-3}T_2 + t_nT_3$

Put, $T_0 = a, T_1 = b, T_2 = c, T_3 = d$, we get, (4.1.2) $T_n = (a+b+c)t_{n-1} + (b+c)t_{n-2} + c.t_{n-3} + d.t_n$

Let $h(m)[\mathrm{a,b,c,d}] = \mathrm{h}$ and therefore,

$$[T_h, T_{h+1}, T_{h+2}, T_{h+3}] \equiv [a,b,c,d](mod\ m)$$

Case 1: Put $n = h$ in equation (4.1.2) we get,

(4.1.3) $T_h = (a+b+c)t_{h-1} + (b+c)t_{h-2} + c.t_{h-3} + d.t_h$

We know that, $t_{h-1} = t_{h+3} - t_h - t_{h+1} - t_{h+2}$

$$t_{h-2} = 2t_{h+2} - t_{h+3}$$

Also we have, $t_{h-3} = 2t_{h+1} - t_{h+2}$

From equation (4.1.3) we get,

$$T_h = (a+b+c)(t_{h+3} - t_h - t_{h+1} - t_{h+2}) + (b+c)(2t_{h+2} - t_{h+3}) + c.(2t_{h+1} - t_{h+2}) + d.t_h$$

(4.1.4) $T_h = (d-a-b-c)t_h + (c-a-b)t_{h+1} + (b-a)t_{h+2} + at_{h+3}$

Case 2: Take, $n = h+1$, we get,

$$T_{h+1} = (a+b+c)t_h + (b+c)t_{h-1} + c.t_{h-2} + d.t_{h+1}$$
$$= (a+b+c)t_h + (b+c)(t_{h+3} - t_h - t_{h+1} - t_{h+2}) + c.(2t_{h+2} - t_{h+3}) + d.t_{h+1}$$

(4.1.5) $T_{h+1} = a.t_h + (d-b-c)t_{h+1} + (c-b).t_{h+2} + +b.t_{h+3}$

Case 3: Put $n = h+2$ we get,

$$T_{h+2} = (a+b+c)t_{h+1} + (b+c)t_h + c.t_{h-1} + d.t_{h+2}$$

(4.1.6) $T_{h+2} = bt_h + (a+b)t_{h+1} + (d-c)t_{h+2} + c.t_{h+3}$

Case 4: Put $n = h+3$, we get,

(4.1.7) $T_{h+3} = (a+b+c)t_{h+2} + (b+c)t_{h+1} + c.t_h + d.t_{h+3}$

The equations (4.1.4), (4.1.5), (4.1.6) and (4.1.7) can be written in the matrix form as:

$$\begin{bmatrix} T_h \\ T_{h+1} \\ T_{h+2} \\ T_{h+3} \end{bmatrix} = \begin{bmatrix} d-a-b-c & c-a-b & b-a & a \\ a & d-b-c & c-b & b \\ b & a+b & d-c & c \\ c & b+c & a+b+c & d \end{bmatrix} \begin{bmatrix} t_h \\ t_{h+1} \\ t_{h+2} \\ t_{h+3} \end{bmatrix}$$

We have, $[T_h, T_{h+1}, T_{h+2}, T_{h+3}] \equiv [a,b,c,d](mod\ m)$

By above equation, we can write,

$$\begin{bmatrix} d-a-b-c & c-a-b & b-a & a \\ a & d-b-c & c-b & b \\ b & a+b & d-c & c \\ c & b+c & a+b+c & d \end{bmatrix} \begin{bmatrix} t_h \\ t_{h+1} \\ t_{h+2} \\ t_{h+3} \end{bmatrix} \equiv \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} (mod\ m)$$

which can be modified into the form

(4.1.8) $$\begin{bmatrix} d-a-b-c & c-a-b & b-a & a \\ a & d-b-c & c-b & b \\ b & a+b & d-c & c \\ c & b+c & a+b+c & d \end{bmatrix}.$$

$$\begin{bmatrix} t_h \\ t_{h+1} \\ t_{h+2} \\ t_{h+3}-1 \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} (mod\ m)$$

Here the determinant of the matrix (4.1.8) depends only on $a, b, c,$ and d and is equal to $D(a,b,c,d)$. System has only one solution if and only if the numbers $D(a,b,c,d)$, m are coprime. In this case we have

$[t_h,\ t_{h+1},\ t_{h+2},\ t_{h+3}] \equiv [0,0,0,1](mod\ m)$ and hence $h(m)|h$. since also $h|h(m)$.

Therefore $h = h(m)$ follows.

Theorem 4.2: let $v_1 = [a,\ b,\ c,\ d]$, $v_2 = [b,\ c,\ d,\ a+b+c+d$, $v3{=}c,\ d,\ a{+}b{+}c{+}d,\ a{+}2b{+}2c{+}2d,\ v4{=}d,\ a{+}b{+}c{+}d,\ a{+}2b{+}2c{+}2d,\ 2a{+}3b{+}4c{+}4d$. Then $v1$, $v2$, $v3$, $v4$ are linearly independent over $\mathbb{F}p$ if and only if $Da,\ b,\ c,\ d \not\equiv 0 mod\ p$. Moreover, the linear independence of $v1$, $v2$, $v3$, $v4$ implies $hpa,\ b,\ c,\ d{=}hp$.

Proof: From the above result (4.1.8) we have a matrix,

$$\begin{bmatrix} d-a-b-c & c-a-b & b-a & a \\ a & d-b-c & c-b & b \\ b & a+b & d-c & c \\ c & b+c & a+b+c & d \end{bmatrix}$$

By applying elementary column transformation, the matrix can be converted the form:

$$A = \begin{bmatrix} a & b & c & d \\ b & c & d & a+b+c+d \\ c & d & a+b+c+d & a+2b+2c+2d \\ d & a+b+c+d & a+2b+2c+2d & 2a+3b+4c+4d \end{bmatrix}$$

Where $\det A = D(a,b,c,d)$. Hence it follows that the rows of A are linearly independent over $\mathbb{F}_p$ if and only if $D(a,b,c,d) \not\equiv 0\ (mod\ p)$. Therefore from Theorem (4.1) it follows that $h(p)[a,\ b,\ c,\ d] = h(p)$.

Remark 4.3

Generally the equality of periods $h(p)[a,\ b,\ c,\ d] = h(p)$ does not imply linear independence of v_1, v_2, v_3, v_4 over $\mathbb{F}_p$. For example, $h(3)[1,2,3,4] = h(3)[0,\ 0,\ 0,\ 1] = 26$ and $D(1,2,3,4) = 111 \equiv 0\ (mod\ 3)$. Therefore, the rows of $A = D(1,2,3,4)$ are not linearly independent over $\mathbb{F}_p$.

Theorem 4.4

A quadruple $[a,\ b,\ c,\ d]$ satisfies the congruence $D(a,b,c,d) \equiv 0\ (mod\ p)$ if and only if the sequence

$\{T_n mod\ p\}_{n=0}^{\infty}$ with beginning value $[a,\ b,\ c,\ d]$ can be defined as a first, second or third order recurrence formula.

Proof. If $D(a,b,c,d) \equiv 0\ (mod\ p)$ then from the Theorem (4.2) it follows that $v_1,\ v_2,\ v_3, v_4$ are linearly dependent.

Let v_1 and v_2 are linearly dependent. Therefore there exist an integer $k \in Z$ such that

(4.4.1) $k[a,b,c,d] \equiv [b,c,d,a+b+c+d](mod\ p)$

By matching the terms, we get, $T_n \equiv ak^n(mod\ p)$, from (4.4.1) by induction. This means that $\{T_n mod\ p\}_{n=0}^{\infty}$ can be defined over $\mathbb{F}_p$ by the first order recurrence formula,

$$T_{n+1} \equiv kT_n(mod\ p), \quad \text{Where } T_0 = a$$

Suppose that, v_1, v_2 are linearly independent and $v_1,\ v_2,\ v_3$ are linearly dependent. This means that there exists integers $k_1, k_2 \in Z$ such that

(4.4.2) $k_1[a,b,c,d] + k_2[b,c,d,a+b+c+d]$

$$\equiv [c,d,a+b+c+d,a+2b+2c+2d](mod\ p)$$

By analogy it follows from (4.4.2) that

$$T_{n+2} \equiv k_1T_n + k_2T_{n+1}(mod\ p),$$

Where $T_0 = a$ and $T_1 = b$.

Suppose that $v_1, v_2,\ v_3$ are linearly independent and $v_1, v_2, v_3,\ v_4$ are linearly dependent. This means that there exists integers $k_1, k_2, k_3 \in Z$ such that

(4.4.3) $k_1[a,b,c,d] + k_2[b,c,d,a+b+c+d] + k_3[c,d,a+b+c+d,a+2b+2c+2d]$
$\equiv [d,a+b+c+d,a+2b+2c+2d,2a+3b+4c+4d\ ](mod\ p)$

Using analogy, (4.4.3) can be written in the form as,

$$T_{n+3} \equiv k_1T_n + k_2T_{n+1} + k_3T_{n+2}\ (mod\ p)$$

Where $T_0 = a, T_1 = b$ and $T_2 = c$.

Conversely suppose that $\{T_n mod\ p\}_{n=0}^{\infty}$ can be defined by a recurrence of at most three. This means that $v_1, v_2, v_3,\ v_4$ are linearly dependent over $\mathbb{F}_p$ and by Theorem (4.2) we can write,

(4.4.4) $D(a,b,c,d) \equiv 0\ (mod\ p)$.

Let us now investigate the number of all solutions of the congruence (4.4.4).

We know that the number of solutions of (4.4.4) depends on the form of factorization of $g(x) = x^4 - x^3 - x^2 - x - 1$ over the field $\mathbb{F}_p$.

Lemma 4.5 Let $g(x)$ be irreducible over $\mathbb{F}_p$. Then the only solution of (4.4.4) is $(a,b,c,d) \equiv [0,0,0,0](mod\ p)$

Proof: Let L be the splitting field of $g(x)$ over $\mathbb{F}_p$. The irreducibility of $g(x)$ gives that $[L{:}\ \mathbb{F}_p\ ] = 4$. The Galois group of $L/\ \mathbb{F}_p$ is generated by the Frobenius Automorphism $\sigma: L \to L$ determined by $\sigma(t) = t^p$, for any $t \in L$.

Let $\alpha \in L$ be a root of $g(x)$. Then $\beta = \sigma(\alpha),\ \gamma = \sigma(\beta),\ \delta = \sigma(\gamma)$ are the other roots of $g(x)$ and we have $\alpha^p = \beta,\ \beta^p = \gamma,\ \gamma^p = \delta,\ \delta^p = \alpha.$ There are unique $A, B, C, D \in L$ such that

(4.5.1) $T_n mod\ p = A\alpha^n + B\beta^n + C\gamma^n + D\delta^n$

For each $n \in N$, $A\alpha^n + B\beta^n + C\gamma^n + D\delta^n = \sigma(A\alpha^n + B\beta n+C\gamma n+D\delta n=\sigma A\beta n+\sigma B\gamma n+\sigma C\delta n+\sigma D\alpha n$, which gives

(4.5.2) $B = \sigma(A) = A^p,\ c = \sigma(B) = B^p,\ D = \sigma(C) = C^p,\ A = \sigma(D) = D^p,$

It follows from (4.5.2) that A, B, C, D are either all non zero or $A = B = C = D = 0.$ Hence by (4.5.1) the sequence $(T_n\ mod\ p)_{n=0}^{\infty}$ cannot be, with the exception of the sequence beginning with $[0,0,0,0]$, defined by a recurrence formula of first, second or third order. Lemma (4.5) now follows from 4.4.

Lemma 4.6. Let $g(x)$ be factorized over $\mathbb{F}_p$ **, (** $p \neq 563$) into the product of linear factor and an irreducible cubic factor, two linear factors and one irreducible quadratic factor, or four linear factors then (4.4.4) has exactly $p^3 + p - 1,\ 2p^3 - 2p + 1,\ 4p^3 - 6p^2 + 4p - 1$ solutions respectively.

Proof: If $p \neq 563$ then $g(x)$ has only simple roots in the splitting field L of $g(x)$ over $\mathbb{F}_p$. And so a Tetranacci sequence can be expressed in the form

$$T_n = k_1\alpha^n + k_2\beta^n + k_3\gamma^n + k_4\delta^n$$

Where $\alpha, \beta, \gamma, \delta$ are the roots of $g(x)$ in L and $k_i \in L$. It is evident that $D(a,b,c,d) \equiv 0\ (mod\ p)$ if and only if $k_i = 0$, for $i = 1,2,3,4$. The assertion of lemma can now be proved by a suitable use of inclusion exclusion principle.

Remark 4.7. Two irreducible quadratic factors of $g(x)$ are not possible.

REFERENCES

[1]. A novel formula in solving Tribonacci like sequence, Lexter R. Natividad and Paola B. Policarpio, Gen. Math. Notes, Vol. 17, No. 1, July, 2013, pp. 82-87

[2]. David M.Burton, Elementary Number Theory, Seventh Edition

[3]. J. klaska, Tribonacci modulo p^t. Mathematica Bohemica, 133(2008) No. 3, 267 – 288.

[4]. L. E. Dickson, History of theory of numbers, Newyork, Dover, 1971.

[5]. M.N.Zaveri, Dr J.K.Patel, Some properties of generalized Tetranacci sequence modulo m, IJCMS, Volume 4, Issue 6, June 2015, 58- 64

[6]. M. E. Waddill: some properties of a generalised Fibonaccci sequence modulo m. The Fibonacci Quarterly 16 (Aug 1978) no.4, 344-353.

[7]. M. E. Waddill, The Tetranacci sequence and generalizations, Wake Forest university, Winston – salem, NC 27109, Jan 1990.

[8]. M. E. Waddill, Some properties of generalized tetranacci sequence modulo m, Wake Forest university, Winston – salem, NC 27109, October 1990.

[9]. Michele Elia, Derived sequences, The Tribonacci recurrence and cubic forms, dept of elecronica, Politechnico di Torino, Italy, Submitted Jan 1999.

[10]. www.google.com